Graph Reduction for Hierarchical Control of Energy Integrated Process Networks

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Abstract—In this paper, we propose a graph-theoretic algorithm that can be used to analyze complex chemical processes comprising of multiple energy integration loops. Such networks are known to exhibit dynamics in multiple time scales. The algorithm uses information on the order of magnitude of the different energy flows and determines automatically the time scales where the units evolve, the manipulated inputs acting in the different time scales and the form of the reduced order models in each time scale. The application of the algorithm is illustrated through a case study of a benchmark chemical process.

I. INTRODUCTION

Energy integration is ubiquitous in modern chemical plants (e.g. [1], [2]). It leads to significant cost savings through reduction of utilities consumption, but it also results in tightly integrated designs that are difficult to operate and control, especially in the context of transitions between different operating points. Extensive research activity is currently being pursued on the control of large-scale, integrated plants. Examples include the analysis of passivity and dissipativity of such networks (e.g. [3], [4]), the development of distributed control methods and architectures (e.g. [5], [6], [7], [8]) and the selection of the control structure for entire plants (e.g. [9], [10]).

In our previous work [11], [12], we have identified generic prototype energy integrated networks that exhibit unique dynamic features. Specifically, we first analyzed networks with large energy recycle compared to the input/output energy flows, and established that the underlying models exhibit a singularly perturbed form, resulting in the evolution of the individual units in a fast time scale and the overall network in a slower time scale. We also analyzed networks with high energy throughput and established that the energy balance variables evolve in a fast time scale, compared to the material balance variables. In both cases, singular perturbations were used for model reduction and composite (hierarchical) control.

In the present work, we focus on complex process networks comprising of interconnections of these prototype ones, which is indeed the norm in integrated chemical plants. Such networks can in principle be analyzed within a singular perturbation framework, although as their size and complexity increases such an analysis becomes cumbersome. Instead we adopt a graph-theoretic formalism, aiming at a scalable and efficient framework for reduction of such complex process networks that can be used as the basis for hierarchical control. The developed reduction framework relies on information about the order of magnitude of the different energy flows in the networks and automatically generates information on i) the time scales where each unit evolves, ii) manipulated variables that act in each time scale and can be used to enforce control objectives, and iii) form of the reduced order models in each time scale. The developed framework is applied to a benchmark chemical plant example, the toluene hydrodealkylation process [13].

II. GRAPH THEORETIC REPRESENTATION OF COMPLEX ENERGY INTEGRATED NETWORKS

A. Basic building blocks and their dynamic equations

Complex energy integrated networks can be represented as energy flow graphs with nodes corresponding to individual units in the system, with directed and weighted edges corresponding to energy flows between units. The two basic building blocks of such networks are recycles and throughputs of energy, which have graph theoretic analogues directly derived from common parlance [14]. Specifically, in the energy flow graphs, throughputs are energy flow paths, while recycles are energy flow cycles.

Large energy recycles are obtained when the magnitude of the recycle is larger than the magnitude of the inlet and outlet energy flows. Large energy throughputs are obtained when the magnitude of the throughput is larger than any other energy input/output. These blocks can be replaced in the graph by a single composite unit to reduce the complexity of the representation and to analyze the dynamics of the energy balance variables in the process network.

Fig. 1(a) shows a large throughput of energy over N units starting from a source supplying \(Q_{in}\) and ending with a
sink taking $Q_{out}$. Energy flows of different orders of magnitude are distinguished by lines of different thickness. The equivalent representation is a unit labeled $T_{L(N)}^{h_1}$. $T$ denotes throughout, the superscript represents the order of magnitude of the energy flows within the block and the subscript is a set of labels of the units constituting the composite unit. We have identified, in our previous work [12], that energy throughputs do not lead to time-scale multiplicity in the energy dynamics. Therefore, further details on the models of such networks are not considered here.

Fig. 1(b) shows a simple network with large energy recycle and its equivalent representation as a unit labeled $R_{L(N)}^{h_1}$. Again, $R$ represents recycle, the superscript represents the order of magnitude of the energy recycle and the subscript represents the set of labels of the units in the recycle. The dynamics of this type of recycle can be described by the following equation:

$$\frac{d\mathbf{H}}{dt} = f(\mathbf{H}, \mathbf{u}_s) + \frac{1}{\varepsilon}g(\mathbf{H}, \mathbf{u}_l)$$  \hspace{1cm} (1)$$

Here, $\mathbf{H}$ is the vector of enthalpies of the units in the recycle loop. $\mathbf{u}_s$ and $\mathbf{u}_l$ are the scaled small and large energy flows respectively, and $f$ and $g$ are the energy balance vector functions associated with these flows. This is a nonstandard singularly perturbed system [15] with a small parameter $\varepsilon$ which is the ratio of the energy input through the feed stream to the energy recycle. Using singular perturbations, we can derive the reduced order dynamic models in the two time scales. The slow time scale dynamic model takes the form [11]:

$$\frac{d\mathbf{H}}{dt} = f(\mathbf{H}, \mathbf{u}_s) + \mathbf{Bz}$$

$$0 = \tilde{g}(\mathbf{H}, \mathbf{u}_l)$$  \hspace{1cm} (2)$$

The original quasi-steady state constraints are linearly dependent because the total enthalpy of the recycle loop does not change in the fast time scale. Therefore, in Eq.(2), we only considered a linearly independent subset, $\tilde{g}$, of the original quasi-steady state constraints obtained as:

$$g(\mathbf{H}, \mathbf{u}_l) = \mathbf{B}\tilde{g}(\mathbf{H}, \mathbf{u}_l)$$

with $\mathbf{B}$ being a transformation matrix with full column rank. Further, $z$ refers to “algebraic” variables defined as $\lim_{\varepsilon \to 0} \frac{\mathbf{g}(\mathbf{u})}{\varepsilon}$ which capture differences of large energy flows in the limit as these flows become infinitely large. Eq.(2) represents a differential algebraic equation (DAE) system with a non-trivial index and possibly a control dependent state space ([16], [17]). Using index reduction, it can be transformed into an ODE of dimension 1 capturing the slow evolution of the energy in the network [11].

It is important to note that the energy dynamics in the slow time scale depends on the small energy flows and the quasi-steady state constraints obtained from the fast dynamics are determined by the large energy flows. We can, thus, take advantage of the multi-time scale dynamics of a large energy recycle system to design a hierarchical control structure with the control objectives in each time scale being addressed at separate tiers using distinct sets of manipulated inputs. Further, the energy flow graph of a complex process network essentially comprises of multiple loops and paths involving many units with flow magnitudes varying over several orders of magnitude. The next section presents an algorithm that combines graphical identification of building blocks with application of appropriate reduced dynamic equations to design a hierarchical control structure for an energy-integrated process network.

B. Algorithm for the analysis of complex energy integrated networks

Algorithm 1 outlines the procedure for simplifying a complex energy flow graph and formulating a hierarchical control design by exploiting the multi-time scale dynamics. The inputs to the algorithm are the graph $G(N,E)$ and a vector $\mathbf{W}$ of the various orders of magnitude exhibited by different energy flows. Since one seeks the evolution of the system for times $t = 0 \to \infty$, the algorithm begins with the largest order of magnitude energy flows (corresponding to the fastest time scale) and proceeds to the smallest. For a given order of magnitude ‘m’, an induced subgraph ‘$H^m$’ is first formed from $G$ such that all the edges in $H^m$ are of the order ‘m’ (Line 3). In Line 4, the energy flow expressions of magnitude ‘m’ are obtained from a subroutine, $E_{balance}$, that exploits the connectivity of the graph to reconstruct the scaled energy balance. All the nodes in this subgraph are noted to evolve in

![Fig. 2. Process flow diagram of the reaction part of HDA process](image)

![Fig. 3. Energy flow graph of HDA process](image)
Algorithm 1 GraphAnalysis($G,W$)
1: Sort $W$ in descending order
2: for $i = 1$ to $\text{Size}(W)$ do
3: \hspace{5 pt} $m = W[i]$; $H = \text{InducedSubgraph}(G,m)$
4: \hspace{10 pt} $g_m(u_m) = h_{1,m} \times \text{Ebalance}(G,m)$
5: $T(\tau_m) = \text{nodes} \in H$
6: for each node $N \in H$ do
7: \hspace{10 pt} if $N$ is a composite node then
8: \hspace{15 pt} add $\sum N_i$ to $\mathcal{Y}(\tau_m)$
9: \hspace{10 pt} else
10: \hspace{15 pt} add $N$ to $\mathcal{Y}(\tau_m)$
11: $\mathcal{U}(\tau_m) = \text{Edges} \in H$
12: $C = \text{SmallestElementaryCycle}(H)$
13: while $C \neq \emptyset$ do
14: \hspace{5 pt} GraphReduce($G,C,m$)
15: \hspace{5 pt} $H = \text{GraphReduce}(H,C,m)$
16: \hspace{5 pt} $C = \text{SmallestElementaryCycle}(H)$
17: if $\text{size}(\text{RecycleTimes}) \neq 0$ then
18: \hspace{10 pt} $DAE_m = C_m B_m z_m$
19: \hspace{10 pt} $\text{Constraint}_m = \bar{g}_m - (u_m - )$
20: \hspace{10 pt} if $\text{size}(\text{RecycleTimes}) > 1$ then
21: \hspace{15 pt} $\text{AddConstraints}_m = \sum_{j=1}^{\text{size}(\text{RecycleTimes})} DAE_j$
22: \hspace{10 pt} if $\text{degree}(N) = 0$ for any node $N \in H$ then
23: \hspace{15 pt} Add $\tau_m$ to RecycleTimes
24: \hspace{15 pt} Add $N_i$ to PureRecycles
25: All but 1 out of $N_i$ should be controlled in this time scale
26: if $\text{degree}(N) \neq 0$ for all nodes $N \in H$ then
27: Clear RecycleTimes, PureRecycles
28: for all node $N \in H$ such that $\text{degree}(N) \neq 0$ do
29: if $N$ is a composite node then
30: Remove $N_i$ from PureRecycles
31: Remove $N$ from $G$
32: return $T, \mathcal{Y}, \mathcal{U}$
33: Energy balance equations are
34: \[ \frac{dH}{dt} = \sum_{i=W[1]}^{\text{size}(W)} \frac{1}{E_i} g_e(u_i) \]
35: for all $m \in W$ do
36: Reduced order model in $\tau_m$ is
37: \[ \frac{dH_m}{d\tau_m} = g_m(u_m) + DAE_m \]
\[ 0 = \text{Constraint}_m + \text{AddConstraints}_m \]

time scale ($\tau_m$). Lines 6-11 identify the control objectives, $\mathcal{Y}$, and manipulated inputs, $\mathcal{U}$. Since all simple (non-composite) nodes in this subgraph represent nodes evolving in that time scale, its enthalpy should be controlled in that time scale. On the other hand, for a composite node, which signifies a recycle from a faster time scale, the total enthalpy of this recycle block should be controlled in this time scale. The manipulated inputs are simply the edges of $H$. Lines 12-16 successively identify the smallest cycle in $H$ – corresponding to the smallest recycle – and replace it by a composite node in the manner described earlier. This graph reduction process ultimately removes all recycle blocks of the given magnitude ‘$m$’ from the parent graph $G$.

A pure recycle – identified as composite blocks that do not have any edges to/from it – will result in the corresponding quasi-steady state constraints being linearly dependent, as discussed in section II-A. Such dependent constraints result in DAE descriptions of the slower time scale dynamics. To this end, lines 22 to 24 identify such ‘pure recyclers’ in a time scale and add that time scale to the set RecycleTimes. They also add the components of that recycle ($N_i$) to PureRecycles. In the subsequent time scales, lines 17 to 21 search for such ‘pure recyclers’ in previous time scales and determine the corresponding contribution ($DAE_m$) to the dynamic equations. It should be noted that the subscript ‘$m$’ refers to the contributions of the different variables in the given time scale.

The matrix $C_m$ allows for the selection of the nodes evolving in $\tau_m$ from all the $N$ nodes. For example, $C_{ij} = 1$ iff the node $i$ represents the $j$th entry in PureRecycles. Lines 20 to 21 account for the contribution due to the presence of recycles spanning multiple time scales (for example, multiple large recycles inside intermediate recycle). Lines 26 to 27 ensure that the effect of recycle is limited only until a throughput is encountered. Line 25 suggests that if pure recyclers exist, then the enthalpies of all of the component nodes cannot be simultaneously controlled. Since the energy dynamics of all the units of a throughput block evolve in a single time scale, these units need not be considered while analyzing the subsequent time scales. Lines 28-31 ensure that such nodes are removed from the graph.

Once all the contributing terms are identified, the canonical form of the energy balance equations is given by the ‘weighted’ sum of contributions from energy flows of each order of magnitude. Similarly, the reduced order dynamic equations are constructed via summation of contributions from energy flows $g_m(u_m)$ and recycles in previous time scales $DAE_m, g_m(u_m)$ is a subset of $g_m(u_m)$ and includes only the entries corresponding to the nodes in $T(\tau_m)$.

### III. Case study - Process description

Let us consider a design alternative (alternative 1) of the toluene hydrodealkylation process proposed by Terrill and Douglas [13]. This process can be divided into two main parts, a reaction and a distillation one. We focus on the reaction part since it has a potential of multi-time scale

<table>
<thead>
<tr>
<th>Index</th>
<th>Unit</th>
<th>Index</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mixer</td>
<td>5</td>
<td>FEHE-Hot</td>
</tr>
<tr>
<td>2</td>
<td>FEHE-Cold</td>
<td>6</td>
<td>Cooler</td>
</tr>
<tr>
<td>3</td>
<td>Furnace</td>
<td>7</td>
<td>Separator</td>
</tr>
<tr>
<td>4</td>
<td>PFR</td>
<td>8</td>
<td>Compressor</td>
</tr>
</tbody>
</table>

**TABLE I**

**ENERGY FLOW GRAPH NODE LIST**
dynamics due to the presence of two recycle loops. Its
process flow diagram is shown in Fig. 2.

This network can be represented as a graph as shown in
Fig. 3. All the nodes of the energy flow graph are listed
in Table I. We note that the energy flow of the process
spans three orders of magnitude. There is an inner recycle
loop and the energy flows associated with it have energy of
large magnitude. We can see another recycle around it with
the intermediate energy flows associated. Also, several small
energy flows, mainly external energy flows, are present. The
orders of magnitude of the streams are determined based on
the sensible heat content of the flows as are shown in Table II.

IV. DYNAMICS OF THE NETWORK

Let us now investigate the dynamics of this network using
numerical simulations. The dynamic model of the system can
be written as follows:

\[
\frac{dH_m}{dt} = R\rho C_p(T_{comp} - Tin) + F_1\rho C_p(T_{F_1} - Tin) \\
+ F_2\rho C_p(T_{F_2} - Tin) + F_3\rho C_p(T_{F_3} - Tin) \\
\frac{\partial H_C}{\partial t} = \rho C_p v_C V_C \frac{\partial T_C}{\partial z} + UA(T_H - T_C) \\
\frac{\partial H_H}{\partial t} = -\rho C_p v_H V_H \frac{\partial T_H}{\partial z} - UA(T_H - T_C) \\
\frac{dH_f}{dt} = F_2\rho C_p(T_{C_{z=0}} - T_f) + Q_f \\
\frac{dH_r}{dt} = F_3\rho C_p(T_f - T_r) - \Delta H r V_r \\
\frac{dH_{cooler}}{dt} = F_3\rho C_p(T_{H_{z=L}} - T_{cooler}) - Q_c \\
\frac{dH_s}{dt} = F_3\rho C_p(T_{cooler} - T_s) \\
\frac{dH_{comp}}{dt} = R\rho C_p(T_s - T_{comp}) + W_c
\]  

(3)

\( R \) is the volumetric flow rate of the recycle stream, and \( F_1 \),
\( F_2 \) and \( F_3 \) are the volumetric flow rates of the toluene feed
stream, hydrogen feed stream and toluene recycle stream,
respectively. \( F \) is the volumetric flow rate of the inlet stream
of the feed effluent heat exchanger (FEHE) cold channel.

The FEHE transfers energy from the hot effluent stream of
a reactor to the cold reactor inlet stream. \( V_C, V_H \) and \( V_r \)
are the holdups of the FEHE cold and hot channels and the
reactor, respectively. \( Q_f \) and \( Q_c \) are the duties of the furnace
and the cooler, respectively, and \( W_c \) is the work input of
the compressor. \( T_{in} \) is the cold channel inlet temperature of
the FEHE. \( T_C \) and \( T_H \) are the temperatures of the cold
and hot channels of FEHE, respectively. \( T_f, T_r, T_{cooler}, T_s \)
and \( T_{comp} \) are the temperatures of the outlet streams of
the furnace, the reactor, the cooler, the separator and the
compressor, respectively.

For simplicity, we assume that there is no phase change in
the FEHE and the holdups for all units are constant. Also, we
assume constant heat capacity, density and reaction rate. A
finite difference approximation is used to simulate the FEHE.
The nominal values of the state variables and the process
parameters are given in Table III.

We consider the open-loop responses of the system by
increasing the initial condition for the compressor by 1 %. Fig. 4
shows the evolution of the furnace temperature and Fig. 5 the
evolution of the separator temperature. Notice that, in
the time scale shown, the system has not returned to steady
state. Furthermore, the furnace temperature exhibits a very
fast transient, not seen in the temperature of the separator.
This suggests an evolution over two time scales for the latter
and an evolution over three time scales for the former. In
what follows, we show how these features can be determined
systematically using the proposed algorithm.

V. GRAPH REDUCTION AND CONTROL STRUCTURE

We apply the graph-theoretic algorithm to the process
using information from Fig. 3 and Table II. The result on
the graph reduction is shown in Fig. 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta H )</td>
<td>-41.826 kJ/mol</td>
</tr>
<tr>
<td>( \rho C_p )</td>
<td>0.1772 mol/s</td>
</tr>
<tr>
<td>( U A )</td>
<td>6.58 \times 10^5 W/K</td>
</tr>
<tr>
<td>( V_m )</td>
<td>0.1 m³</td>
</tr>
<tr>
<td>( V_C )</td>
<td>14.16 m³</td>
</tr>
<tr>
<td>( V_H )</td>
<td>14.16 m³</td>
</tr>
<tr>
<td>( V_f )</td>
<td>8.5 m³</td>
</tr>
<tr>
<td>( V_r )</td>
<td>110 m³</td>
</tr>
<tr>
<td>( V_{cooler} )</td>
<td>8.5 m³</td>
</tr>
<tr>
<td>( V_s )</td>
<td>8.5 m³</td>
</tr>
<tr>
<td>( V_{comp} )</td>
<td>8.5 m³</td>
</tr>
<tr>
<td>( F_1 )</td>
<td>0.0035 m³</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>0.0056 m³</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>9.7 \times 10^{-4} m³</td>
</tr>
<tr>
<td>( T_{in} )</td>
<td>344.92 K</td>
</tr>
<tr>
<td>( T_{C_{z=0}} )</td>
<td>880.51 K</td>
</tr>
<tr>
<td>( T_f )</td>
<td>895.43 K</td>
</tr>
<tr>
<td>( T_r )</td>
<td>939.99 K</td>
</tr>
<tr>
<td>( T_{H_{z=L}} )</td>
<td>395.83 K</td>
</tr>
<tr>
<td>( T_{cooler} )</td>
<td>318.67</td>
</tr>
<tr>
<td>( T_s )</td>
<td>318.67</td>
</tr>
<tr>
<td>( T_{comp} )</td>
<td>349.83 K</td>
</tr>
</tbody>
</table>

6391
Fig. 4. Evolution of the furnace temperature

Fig. 5. Evolution of the separator temperature

Fig. 6(a) shows the subgraph of the energy flow graph for the energy flows with the largest magnitude ($O(h_2)$). The units evolving in the fastest time scale are $T(\tau_{h_2}) = \{2, 3, 4, 5\}$. This subgraph contains one prototype recycle and can be simplified to obtain a single composite recycle node $R_1 = R^{O(h_2)}_{2,3,4,5}$.

Fig. 6(b) shows the subgraph of the energy flow graph for the energy flows with the intermediate magnitude ($O(h_1)$). The units evolving in the intermediate time scale are $T(\tau_{h_1}) = \{1, R_1, 6, 7, 8\}$. Note that the composite node $R_1$ is included in the intermediate time scale. This subgraph also contains one prototype recycle and can be simplified to obtain a single composite recycle node $R_2 = R^{O(h_1)}_{1, R_1, 6, 7, 8}$.

Fig. 6(c) shows the subgraph of the energy flow graph for the energy flows with the smallest magnitude ($O(d_{1a})$). The unit evolving in the slow time scale is $T(\tau_{d_{1a}}) = \{R_2\}$. Note that the composite node $R_2$ is included in the slow time scale.

Based on this result, this network is expected to exhibit three-time scale energy dynamics as summarized in Table IV. Note that this is consistent with the numerical simulation results presented earlier.

The algorithm also gives us the canonical forms of the dynamic equations of the network for both the original and the reduced order models. The dynamics of the orginal order model (i.e. Eq.(3)) can be cast in a vector form:

$$\frac{dH}{dt} = f + \frac{1}{\varepsilon_\alpha} g_\alpha + \frac{1}{\varepsilon_\beta} g_\beta$$  \hspace{1cm} (4)

with

$$f = d_{1a,s} \begin{bmatrix} u_{d_{1a},a} + k_{d_{1a},b} u_{d_{1a},b} + k_{d_{1a},c} u_{d_{1a},c} \\ 0 \\ k_{d_{1a},d} u_{d_{1a},d} \\ k_{h_{1a}d} u_{h_{1a}d} \\ -k_{d_{1a},a} \\ -k_{h_{1a}b} u_{h_{1a}b} - k_{h_{1a}c} u_{h_{1a}c} - k_{h_{1a}d} u_{h_{1a}d} \\ k_{d_{1a},8} u_{d_{1a},8} \end{bmatrix}$$

$$g_\alpha = d_{1a,s} \begin{bmatrix} -k_{h_{1a}h_{1a}} \\ k_{h_{1a}h_{1a}} \\ 0 \\ 0 \\ -k_{h_{1a}h_{5a}} \\ k_{h_{1a}h_{5b}} - k_{h_{6a}b} \\ k_{h_{6a}b} - k_{h_{7a}c} u_{h_{7a}c} - k_{h_{8a}h_{8a}} \end{bmatrix}$$

$$g_\beta = d_{1a,s} \begin{bmatrix} 0 \\ -k_{h_{2a}h_{2a}} + k_{h_{5a}a} u_{h_{5a}} \\ k_{h_{2a}h_{2b}} - k_{h_{3a}h_{3b}} \\ k_{h_{3a}h_{3b}} - k_{h_{4a}h_{4a}} \\ k_{h_{4a}h_{4b}} - k_{h_{5a}a} u_{h_{5a}} \\ 0 \\ 0 \end{bmatrix}$$

$h_i$ represents the energy flow leaving the $i$-th unit and $d_i$ represents an external energy flow entering/leaving the $i$-th unit. $\varepsilon_\alpha$ is the ratio of the energy input through the toluene feed stream to the rate of energy recycle through gas recycle. $\varepsilon_\beta$ is the ratio of the energy input through the toluene feed stream to the rate of energy transferred inside the FEHE. $k_i$, which are the $O(1)$ steady state ratios, can be defined as:

$$k_{d_{1b}} = d_{1b,s} / d_{1a,s}, k_{h_{1a}} = \varepsilon_\alpha d_{1a,s}, k_{h_{2a}} = \varepsilon_\beta d_{1a,s}$$

and the deviations $u_i$ from the steady state as:

$$u_i = \frac{h_i}{h_{i,s}}$$

where the subscript $s$ represents a steady state value. The reduced order dynamic models are not presented here for brevity.

Finally, a hierarchical control structure of the process can be proposed using the algorithm. $Y(\tau_m)$ is the set of units whose enthalpies should be controlled in the $m$-th time scale. All simple nodes as well as composite recycle nodes in the subgraph corresponding to the $m$-th time scale should be included in $Y(\tau_m)$. We note that, in the case of the prototype recycle, all but one out of all units in this recycle must be controlled since their enthalpies are not linearly independent. $U(\tau_m)$ is the set of potential manipulated inputs available in
In this paper, we proposed a graph-theoretic algorithm which can be used to analyze complex energy integrated process networks. Specifically, it can be used to get information on the time scales exhibited by the network, the scaled form of the reduced-order dynamic models and a hierarchical control structure. We demonstrated the application of the algorithm through a benchmark chemical plant example.

Through the graph reduction using the algorithm, the system was shown to exhibit three-time scale dynamics, a result consistent with numerical simulations. Also, we identified the sets of units whose enthalpies should be controlled in the three different time scales. The sets of potential manipulated inputs in each time scale are identified as well.

The graph-theoretic algorithm that we have developed can be extended to more general complex process networks with mass and energy integration. Also, graph reduction of the type described here can be applied to a broader class of process networks containing flows with different orders of magnitude such as biological networks and reaction networks.

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