Dynamic Coupling between a Human User and Haptic Virtual Environment

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Abstract—In a teaching lab focused on embedded control, students create and interact with virtual environments using a haptic interface. Coupling physical (in particular physiological) environments to virtual environments gives rise to many interesting phenomena, one of which is the appearance of dissipativity in the coupled dynamics, the source of which is difficult to identify. Simple harmonic oscillators without damping exhibit damped behavior and diminished peak amplitudes when students excite them with their best manual approximations of step inputs. Motivated in part by our desire to develop teaching materials, we seek a simple human user model that describes the observed phenomena. We have found that a second order spring-mass-damper model describes the source impedance with which a human user is able to impose a position input on the haptic device, and that this impedance model can be incorporated into a model of the user’s neuromotor intent by placing the spring as a series elastic element with a motion source. We use simple models to describe smooth inputs generated by the user’s neuromotor system, and these are expressed as displacements of the motion source. We use the same haptic device to conduct system identification experiments using frequency domain techniques to estimate the driving point impedance of the human hand, and have recently incorporated these experiments into lab exercises.

I. INTRODUCTION

An embedded controls systems class taught at the University of Michigan since 2000 [1], [2], and at ETH Zurich since 2008, introduces real-time microprocessor-based control using the Freescale MPC5553 micro-controller and the single-axis haptic interface shown in Figure 1. Working in teams of two, students implement several virtual environments that can be manipulated through the haptic device, beginning with a virtual wall and finishing with a networked multi-vehicle driving simulator. The teaching laboratory features six workstations each with an MPC5553 development board, a host computer, a custom interface card, and a custom built single-axis haptic interface. The workstations are networked through a CAN bus.

The haptic interface is, essentially, a physical wheel driven directly through a DC motor. The wheel position is determined from an optical encoder mounted on the shaft; the motor is driven through a transconductance amplifier according to a command signal encoded using pulse width modulation (PWM). While a control law running on the microprocessor closes one feedback loop around the haptic device, another feedback loop is closed by a human user who grasps the wheel. These loops are shown in a block diagram in Figure 2. This simple force feedback system is capable of exhibiting a wide range of interesting phenomena. In the most elementary cases, a virtual spring can be created by generating a restoring torque proportional to the wheel angular displacement. A virtual wall is created by permitting the wheel to rotate freely up to some specified displacement, at which point a stiff virtual spring produces a strong reaction torque. More complex virtual environments are developed by...
incorporating various interconnections of walls, dampers and springs. A virtual knob is constructed with detents that can be felt at regular intervals by commanding torque as a sinusoidal function of angular displacement; a virtual spring-damper adds a retarding force proportional to wheel angular velocity, and virtual spring-mass and spring-mass-damper systems introduce dynamics that require dealing with implementation and stability issues associated with numerical integration. The virtual spring-mass system schematic is depicted in Figure 3. In one laboratory exercise, students implement this virtual spring-mass system, and apply a “step input” to the virtual system by rapidly turning the haptic wheel, holding it in its new position, and observing (sensing) the reaction torque. The reaction torque students feel, however, does not match the expected response of a simple spring-mass system to a step input. Explaining the difference between the actual and expected responses requires a model of the human user. Models of the driving point impedance of human limbs have previously been developed to support the analysis of virtual environments coupled to users through haptic devices [3]–[6]. Generally these models do not accommodate volitional control on the part of the user, which plays an important role even in our simple “step input” experiment. In this paper we use frequency domain identification techniques to identify components of a hand/arm model, then we elaborate this model to include volitional control.

The remainder of the paper is organized as follows. Section II provides a statement of the problem, a description of the observed phenomena, and the development of a model that includes a human user’s biomechanics coupled to the haptic device and in turn to the virtual environment. Section III describes the system identification of the human hand, and Section IV details the experimental verification of the proposed model and discusses the possible causes for the difference between simulation results and experimental data. A summary and conclusions are presented in Section V.

Fig. 3. Haptic wheel and virtual wheel with inertia $J_w$ coupled through virtual torsional spring of torsional stiffness $k$.

II. PROBLEM STATEMENT

Consider the virtual environment illustrated in Figure 3, consisting of a virtual inertia connected through a virtual spring to the haptic wheel. The system may be interchangeably represented in the translational domain as a virtual block of mass $J_w$ with displacement $\theta_w$ and spring of stiffness $k$ connected to a haptic “puck” with displacement $\theta_z$. The motion of the virtual wheel is related to that of the haptic wheel through the differential equation:

$$\dot{\theta}_w + \frac{k}{J_w} \theta_w = \frac{k}{J_w} \theta_z. \quad (1)$$

Note that this equation describes a harmonic oscillator with natural frequency $\omega_n = \sqrt{k/J_w}$ rad/sec. The torque $T$ exerted on the virtual wheel is given by:

$$T = k(\theta_z - \theta_w). \quad (2)$$

Assuming for now that the user imposes a desired position on the haptic wheel and senses torque, the user will feel a torque generated by the motor that is proportional to the relative positions of the virtual and haptic wheels. Suppose that the virtual wheel is initially at rest ($\theta_w = \theta_z = 0$), and that at time $t = 0$ the haptic wheel is moved by the user from $\theta_z(0) = 0$ to $\theta_z(0^+) = \theta_z^*$, and held constant thereafter. This action is equivalent to applying a step input to (1) with amplitude $\theta_z^*$. The resulting motion of the virtual wheel satisfies:

$$\theta_w(t) = \theta_z^*(1 - \cos \omega_n t), \quad t \geq 0. \quad (3)$$

The resulting torque exerted on the virtual mass is given by:

$$T(t) = k\theta_z^* \cos(\omega_n t), \quad t \geq 0. \quad (4)$$

We see from (3) that the oscillations in response to a step change in the haptic wheel position are sustained indefinitely since there is no damping in the system. If the user can hold the haptic wheel constant, he or she should be able to feel the oscillation of the torque.

In our laboratory exercises, however, human users cannot feel sustained oscillatory torque. What they feel is an oscillatory force that decays with time. Also, the oscillations with higher frequency die out more quickly while low frequency oscillations last longer. Figure 4 shows the experimental results with virtual spring-mass systems of different natural frequencies when human users attempt to move the haptic wheel to a final position and hold it constant. The natural frequency of the spring-mass on the left is lower than that of the spring-mass on the right. In both cases the oscillations eventually die out. In the case of the higher frequency on the right, the oscillation dies out so quickly that users barely feel the oscillatory torque. From the solid traces in Figure 4, we see that the human hand is backdriven by the virtual spring-mass system and exhibits an oscillatory displacement around the nominal final position.

Various causes may underlie the decay that we observe in the oscillations of the virtual spring-mass system. One assumption made implicitly in the above analysis that is not satisfied in practice is that the human user cannot instantaneously turn the haptic wheel to a new position, as we assumed when we modeled the change in wheel position as
a step change in $\theta_z$. Suppose instead that users can only turn
the haptic wheel at a constant rate $A$ radians/second. Then $t_f = \theta_z^*/A$ seconds are required to turn the wheel to its final
position of $\theta_z^*$. The input to the virtual wheel described by
(1) is thus given by a signal of the form shown in Figure 5.
We refer to this signal as a “saturated ramp,” because it is
described as a ramp of slope $A$ for the first $t_f$ seconds, and
then a constant value of $\theta_z^*$ thereafter.

![Fig. 5. Modeling the motion of the haptic wheel $\theta_z(t)$ as a saturated ramp signal with slope $A$, saturation time $t_f$, and saturation limit $\theta_z^* = At_f$.](image)

Whether the sampled motion of the haptic wheel appears
to the virtual environment as a step or a ramp signal will
depend upon the sample period, $T_s$. If sampling is so slow
that $T_s > t_f$, then at most one sample will be taken at a
wheel position that is intermediate between 0 and $\theta_z^*$. If no
such samples are taken, then the sampled input will appear
to be a step signal. Slow sampling, however, is inconsistent
with achieving other metrics of good performance. In our lab
exercises it usually takes 0.1 to 0.3 seconds to turn the haptic
wheel to its new position, and the sample period is $T_s = 2$
ms, consequently there will be many samples taken while
the wheel is being moved to its new position.
The deviation of the input waveform from a step signal
has implications, some of which might explain the observed
behavior of the virtual environment in the hands of the user.
To begin this analysis, note that the input signal in Figure 5
may be written as a difference of two ramp signals
$$\theta_z(t) = At_f(t) - A(t - t_f)1(t - t_f),$$
where $1(t)$ denotes the unit step function. The response of
the virtual wheel to zero initial conditions and the input (5)
satisfies
$$\theta_z(t) = \begin{cases}
\frac{1}{t_f} \left( t - \frac{1}{\omega_n} \sin \omega_n t \right), & 0 \leq t < t_f \\
1 - \frac{2}{\omega_n t_f} \sin \frac{\omega_n t_f}{2} \cos \left( \omega_n t - \frac{\omega_n t_f}{2} \right), & t \geq t_f.
\end{cases}$$
(6)
Hence the relative size of the oscillations is determined by
the coefficient $\frac{2}{\omega_n t_f} \sin \frac{\omega_n t_f}{2}$, which is plotted in Figure 6.

Intuitively, the response (6) to a saturated ramp should
approach the response to a step input of corresponding size,
as the slope of the ramp becomes large. To show that this is
indeed the case, recall that $t_f = \theta_z^*/A$ and thus, in the limit
as $A \to \infty$, $t_f \to 0$. This fact implies that as $A \to \infty$,
$$\cos \left( \omega_n t - \frac{\omega_n t_f}{2} \right) \to \cos (\omega_n t)$$
$$\frac{2}{\omega_n t_f} \sin \frac{\omega_n t_f}{2} \to 1,$$
where we use the fact that, for small values of $x$, $\sin x \approx x$.
It follows by comparing the limiting value of (6) to (3) that if
the ramp is sufficiently steep, then the response to the ramp
will approximate that to a step.

Although the “saturated ramp” movement of the human
hand shows that the relative size of oscillations depends
on the natural frequency of the spring-mass system and the slope
of the ramp, this does not explain why the oscillations decay
with time. Dissipation seems to be arising because of the
placement of the haptic wheel, induced by the action of
the motor. Evidently the human user is not able to impose a
position on the haptic wheel with infinite source impedance.
In simpler terms, it is not possible for a human to hold the
haptic wheel in an immovable position despite the reaction
torque exerted by the motor. Once displacements arise in the
haptic wheel, energy is dissipated. Dissipative elements may
include friction and damping in the rotor of the motor and
evidently damping effects in the mechanical impedance of
the human hand or in the control adopted by the user.
The control methods used by the human are difficult
to analyze because physiological signals are not generally
available for recording and the human is quite unpredictable.
It is also challenging to decide how to delineate
and capture in a model the active and passive features of human behavior, where active refers to features arising from feedback or feedforward control and passive refers to mechanical properties and features of the human body. One criterion for distinguishing between active and passive features in human mechanical behavior is the timescale. Neural conduction velocities impose significant delays on any feedback control—even reflex responses involve at least 30ms delay, and feedback control involving neural circuits in the central nervous system involves delays on the order of 60-100ms [7]. Feedforward control in contrast is not subject to delays but cannot account for dissipative behavior in the system involving the human user coupled to the virtual environment. The analysis of the assumed ramp input in Figure 6 and the data in Figure 4 concerns a time-scale parameter describing what is essentially a reference signal. We acknowledge that the saturated ramp signal is only a presumptive approximation of the actual signal applied to the haptic interface and thus to the virtual environment. The most likely explanation for dissipativity in the coupled system of the user, haptic device, and virtual environment is a damping component in the driving point impedance of the human hand. Parameters \( k \) and \( J_z \) as described above. The parameter \( J_z \) captures the combined inertia of the hand and haptic wheel. The variables \( \theta_z \) and \( \theta_w \) are the position of the haptic wheel and virtual wheel. The signal \( \theta_r \) is a step, ramp, or perhaps other smoother reference signal.

Further consider as a candidate Lyapunov function the absolute energy of the system, given by

\[
V = \frac{1}{2} \alpha (\theta_r - \theta_z)^2 + \frac{1}{2} k (\theta_z - \theta_w)^2 + \frac{1}{2} J_z \dot{\theta}_z^2 + \frac{1}{2} J_w \dot{\theta}_w^2. \tag{8}
\]

Taking the derivative of (8) along trajectories \( \theta_r \) yields

\[
\dot{V} = \alpha (\theta_r - \theta_z) (\dot{\theta}_r - \dot{\theta}_z) + k (\theta_z - \theta_w) (\dot{\theta}_z - \dot{\theta}_w) + J_z \dot{\theta}_z \ddot{\theta}_z + J_w \dot{\theta}_w \ddot{\theta}_w
\]

\[
= \alpha \theta_r \dot{\theta}_r - \beta \dot{\theta}_z^2.
\]

If the value of \( \theta_r \) is constant, such as \( \theta_r^* \) in Figure 5, then \( \dot{V} = -\beta \dot{\theta}_z^2 \). It follows that \( \dot{V} < 0 \) unless the human is holding the haptic wheel motionless \( (\theta_z = 0) \) and thus the human inability to do so will result in the oscillations of the virtual wheel decaying to zero.

In the following section we use a frequency-domain system-identification method to estimate the value of mechanical impedance \( J_z, \alpha, \beta \) of a human hand holding the haptic wheel.

III. IDENTIFICATION OF MECHANICAL IMPEDANCE OF HUMAN HAND

Mechanical impedance describes the relationship between motion and applied forces or torques. It can include a static component that relates forces and displacements (a spring, for example) and dynamic components relating forces to velocities and accelerations (for example a damper and mass). Mechanical impedance is an important consideration in the analysis of human-in-the-loop systems. The identification of human limb impedance has been reported in many places in the literature. In [3], a linear second-order model was fit to displacement, acceleration, and force data collected during a motion perturbation imposed on a human fingertip. A classic least squares method was used, and the authors showed consistent estimates of mass, viscous damping, and stiffness at the fingertip in short timescales. The short timescales precluded active control and even reflexive responses and thus the identified impedance can be considered passive, or inherent in the biomechanics (musculoskeletal and soft tissue mechanics) of the user. In [9], the impedance of the human arm was estimated when subjects held a robotic manipulandum with a 2D planar workspace and made horizontal point-to-point movements within a force field applied to the hand that included a negative stiffness perpendicular to the target direction. Their results showed that humans learn to stabilize unstable dynamics by actively shaping the 2D impedance ellipsoid in the plane of motion. In this paper, we will use a frequency-domain system-identification method to estimate the passive mechanical impedance of the human hand when grasping the haptic wheel. We have employed similar system-identification techniques to construct models of a piano action [10].

For purposes of system identification, white noise signals were generated by the embedded controller using the \texttt{rand()} function in C++ without filtering and used to drive the motor connected to the haptic wheel. The magnitude
of the white noise signal was 6 volts. At the same time, human users attempted to hold the haptic wheel stationary. Through calibration of the haptic system we determined that the actual torque that the motor applies to the human hand is proportional to the command voltage from the embedded controller with a gain of 0.0953 N-m/V.

The position of the haptic wheel was measured using the optical encoder with a resolution of 1000 cycles per revolution. The input white noise signal and the position of the haptic wheel were collected at 850 Hz over a 15-minute trial for each experiment. For comparison, we also performed several experiments that used white noise signals to drive the haptic wheel without a human hand holding it.

Fig. 8. Estimated frequency response curves.

The recordings of the haptic wheel displacement and the white noise commanded torque signals were processed in MATLAB using the tfestimate function to produce a transfer function estimate for each experiment. The algorithm behind tfestimate is Welch’s averaged periodogram, which computes the average ratio of the input/output waveform cross-correlation spectrum to the input waveform autocorrelation spectrum. These estimated transfer functions are plotted in the frequency domain in Figure 8. For the setup involving only the haptic wheel, we plotted the estimated frequency responses for three sets of data and they are almost indistinguishable from one another. We can see that the estimated frequency response looks like a rotational inertia. We also recorded three sets of data for the haptic wheel and human hand setup. The frequency response of this setup is like that of a second-order system that is comprised of a spring, a damper, and a mass component. The mass component includes both the mass of haptic wheel and the mass of human hand. We can also see from the frequency response estimates that there is no structural resonance below 100 Hz. We then fit the spring-damper-mass model of the human hand using the MATLAB System Identification Toolbox. The frequency response of the estimated transfer function and the theoretical frequency response of the fitted model are shown in Figure 9.

We can see that the fitted model produces a theoretical frequency response that matches the empirical transfer function estimate over a bandwidth of $0.1 - 30$ Hz. The transfer function of the fitted model is

$$\frac{\theta(z)}{T(z)} = \frac{1}{J_0 s^2 + \beta s + \alpha}$$

with

- a torsional spring component $\alpha$ of 2.1009 N-m/rad;
- a torsional damping component $\beta$ of 0.02721 N-m.s/rad;
- a torsional mass component $J_0$ of 0.0006811 N-m.s$^2$/rad.

**IV. EXPERIMENTAL VERIFICATION AND DISCUSSION**

We propose the model shown in Figure 7 as the simplest competent model that explains the dissipation observed in the haptic rendering of an undamped oscillator as described above in Section II. The values for parameters $\alpha$, $\beta$, and $J_0$ are set according to the system identification experiment described in Section III. The motion source of displacement $\theta_r$ is interpreted as an embodiment of neural command to muscle (reference command), while the spring of stiffness $\alpha$ in the human hand model is an elastic element in series with the motion source. The motion source is somewhat reminiscent of the contractile element and the spring of stiffness $\alpha$ as the series elastic element of the classical Hill Muscle model [11]. Thus the series elastic element accounts for the finite source impedance with which the hand imposes a position on the haptic wheel. It also allows us to account for the smoothing effect observed in the recorded motion of the haptic wheel.

We are now in a position to test the model of the user coupled through a haptic device to the virtual environment that includes the impedance of the human hand. We render a spring-mass virtual environment whose values are given by $k = 1.092$ N-m/rad and $J_0 = 0.00273$ N-m.s$^2$/rad. This virtual environment is implemented using the single-axis haptic device shown in Figure 1. The same human user whose hand impedance was identified in Section III turns the wheel to a desired position and attempts to hold it steady at that position. The angular displacements of the haptic wheel...
θz and the virtual wheel θw are recorded. In the simulation, θr is assumed to be

\[
\theta_r(t) = \begin{cases} 
\theta_f & 0 \leq t < t_f, \\
\theta_f & t \geq t_f,
\end{cases}
\]

where \( \theta_f \) is the desired final position of human hand and \( t_f \) is the time interval from the initial to final position. The experimental data and simulation results are compared in Figures 10 and 11.

![Fig. 10. Comparison results of \( \theta_z \).](image)

![Fig. 11. Comparison results of \( \theta_w \).](image)

From these figures, we can see that the simulation results show decay rates similar to the decay rates in the experimentally observed oscillations. We can also see from Figure 10 that the human hand is backdriven and cannot hold the haptic wheel perfectly steady.

There also exist significant differences between the experimental data and simulation results. For example, the impedance of the human user depends on the torque input supplied by the motor; in other words, the ability of the human to hold the wheel motionless depends on the motor torque, and this in turn affects the dissipativity of the human user. Other potential causes are the uncertainty in the estimated hand impedance determined in Section III, and the fact that the impedance values also depend on the rotational location of the human hand and the grip force. Another possible cause is nonlinearities and unmodeled effects in human muscle that are not captured in our current model. There is also a possibility that human users adopt more complex feedback or feedforward control strategies (internal models) when interacting with the virtual spring-mass system.

V. SUMMARY AND CONCLUSIONS

In this paper we have studied the dissipative phenomena observed when human users interact with a virtual environment that lacks dissipative elements. Through a frequency-domain system identification approach, we have shown that the impedance of the human hand consists of inertia, damping and stiffness, and we have identified the values of these components. It is the damping component that causes the energy to decay. We also built a coupled model that includes the human user and virtual spring-mass environment. Simulation studies based on the proposed coupled model have shown similar phenomena to the experimental data. In future work, we will investigate the reasons for the difference between simulation and experimental results, and further improve our coupled model.

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REFERENCES