Risk-Averse Shortest Path Problems

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Abstract—We investigate routing policies for shortest path problems with uncertain arc lengths. The objective is to minimize a risk measure of the total travel time. We use the conditional value-at-risk (CVaR) for when the arc lengths (durations) have known distributions and the worst-case CVaR for when these distributions are only partially described. Policies which minimize the expected travel time (average-optimal policies) are desirable for experiments that are repeated several times, but the fact that they take no account of risk makes them unsuitable for decisions that need to be taken only once. In these circumstances, policies that minimize a risk measure provide protection against rare events with high cost.

We show that shortest path problems formulated using dynamic risk measures can, typically, be cast as dynamic programs and can therefore be solved efficiently.

A comparison between various routing policies demonstrates the trade-off between average-optimality and risk-aversion. We also illustrate that access to lookahead information yields adaptive routing policies which can significantly reduce cost.

I. INTRODUCTION

Many practical problems that arise in engineering, such as road network planning or data package routing in communications can be naturally formulated as shortest path problems. Unsurprisingly then, shortest path problems are well-studied and have a long history dating back to the works of Bellman [1].

While technological advances allow for exhaustive evaluation of costs for larger shortest path problems (SPP) than before, the size of real world network problems is such that the need for computationally tractable problems still remains.

In the absence of full information, shortest path problems have been looked at in the past in the context of robust linear optimization [2]. However, computationally tractable problems for their robust counterparts have only been obtained in special cases of the uncertainty set. In [3] box uncertainty is considered, i.e. arc durations belonging to an interval, while in [4] a tractable problem is obtained for box and ellipsoidal uncertainty sets.

On the other hand, stochastic shortest path problems (SSPP) have mainly been studied as Markovian decision problems, see e.g. [5], [6]. SSPP have also been studied in the context of information theory, see e.g. [7]–[9].

In this paper we are mainly interested in obtaining risk-averse problem formulations for the SSPP, the solutions of which are amenable to dynamic programming algorithms. Immunization against uncertainties in path lengths is achieved by means of risk measures. This approach offers the advantage that no restrictions are placed on the type of distributions allowed and, at the same time, the level of protection required is a parameter of choice.

We also consider the case when the distributions are unknown but are governed by known means and variances. The overall path length here is optimized with regards to a worst case distribution for each arc which is obtained by solving a tractable optimization problem [10]. This discussion is a matter of Section III.

Finally, in Section IV we evaluate the performance of risk-averse routing policies by comparing them against the standard average-optimal paths in two scenarios: the first assumes that all upcoming arc lengths of paths starting from any given node are unknown while in the second we assume that from any node we can observe the realized lengths of arcs that follow immediately (the rest are still unknown). The latter case will be referred to as the one-step-lookahead.

In summary, the main contributions of this paper are:

- The use of risk measures to hedge against riskiness of solutions, and the formulation of the stochastic shortest path problem with risk measures as a dynamic program. We examine the cases of fully and partially known distributions for the arc lengths.
- We present a problem formulation for which the adaptive optimal strategy (one-step-lookahead) can be computed as the solution of a dynamic program.
- We compare the performance of risk-averse strategies and average-optimal policies.
- We implement the policies obtained using one-step-lookahead and demonstrate that additional information is valuable.

Notation

Random variables are defined on the abstract probability space \((\Omega, \Sigma, \mathbb{P})\), where \(\Omega\) is the sample space, \(\Sigma\) is the \(\sigma\)-algebra of events and \(\mathbb{P}\) is a probability measure on \(\Sigma\). More precisely, a random variable is defined as a \(\Sigma\)-measurable function \(\xi : \Omega \rightarrow \mathbb{R}\). Random variables are denoted in bold face, while their realizations are denoted by the same symbols in normal font. The expectation operator with respect to \(\mathbb{P}\) is denoted by \(\mathbb{E}\), and for a random variable \(\xi\) we let \(\text{var}[\xi] = \mathbb{E}[(\xi - \mathbb{E}[\xi])^2]\) be the variance of \(\xi\).

A directed graph \(G = (\mathcal{X}, \mathcal{E})\) consists of a set of nodes \(\mathcal{X} := \{0, 1, 2, \ldots, N\}\) and a set of directed arcs \(\mathcal{E} \subseteq \mathcal{X} \times \mathcal{X}\). A sequence of nodes \((x_i)_{i=0}^k\) is called a path if \((x_i, x_{i+1}) \in \mathcal{E}\) for all \(i = 0, \ldots, k - 1\). We call a path \((x_i)_{i=0}^k\) acyclic if \(x_i = x_j\) implies \(|i - j| \leq 1\). A directed acyclic graph (DAG)
is a graph for which all paths are acyclic. Finally, we denote by \(|A|\) the cardinality of a finite set \(A\) set.

II. PROBLEM STATEMENT

Consider a directed acyclic graph \(G = (X, E)\) with a unique source node 0 and sink node \(N\). We assume that \(G\) is connected in the sense that there exists a path from the source to any node \(i \in X - \{0\}\) as well as a path from any node \(j \in X - \{N\}\) to the sink. By the acyclicity, no path in the graph can accommodate more than \(N+1\) nodes. By introducing a self-loop at the sink, if necessary, we can thus assume without loss of generality that every path from 0 to \(N\) comprises exactly \(N+1\) nodes. For ease of exposition, we define the set of all source-sink paths as

\[
X := \{(x_i)_{i=0}^N | x_0 = 0, x_N = N \text{ and } (x_i, x_{i+1}) \in E \text{ for all } i = 0, \ldots, N - 1\}.
\]

We further assume that the graph has uncertain arc weights \(\xi_{ij}, (i,j) \in E\), which are mutually independent and almost surely nonnegative. It is convenient to interpret these weights as the durations needed to travel along the respective arcs.

The aim of this paper is to describe efficient dynamic programming-based algorithms for finding short (to be defined precisely below) paths traversing uncertainty-affected graphs of the above type. As we move across the graph, we successively define a path \((x_i)_{i=0}^N \in X\) in the following manner. We start at node \(x_0 = 0\). Then, after having visited the nodes \((x_i)_{i=0}^k \in X\) for some \(k = 0, \ldots, N - 1\), we must select a routing decision. This entails choosing an immediate successor node to be visited next. The set of candidate successor nodes is given by \(X(x_k) = \{x_{k+1} \in X | (x_k, x_{k+1}) \in E\}\). In this sense we have to select \(N\) sequential routing decisions. To simplify terminology, we will say that the \(k\)th routing decision (which is due at node \(x_k\)) is selected at the \(k\)th decision stage.

Sometimes it is reasonable to assume that some of the uncertain arc durations are observable at the time when a routing decision needs to be taken. For instance, we may have access to accurate traffic forecasts within certain parts of a street network. The information that is available at stage \(k\) can conveniently be captured by a \(\sigma\)-algebra \(\mathcal{F}_k \subseteq \Sigma\). Hence, the successor node selected a stage \(k\) can be different for events that are distinguishable under \(\mathcal{F}_k\). In fact, this successor node can be modeled as an \(\mathcal{F}_k\)-measurable random variable \(x_{k+1}\) valued in \(X\). Unless \(\mathcal{F}_k\) coincides with the trivial \(\sigma\)-algebra \(\{\emptyset, \Omega\}\) for all \(k = 0, \ldots, N - 1\), we therefore end up with a random path \(x = (x_i)_{i=0}^N\) valued in \(X\).

The overall duration of a random path \(x\) is denoted by

\[
d(x, \xi) = \sum_{k=0}^{N-1} \xi_{x_k x_{k+1}},
\]

where \(\xi = (\xi_{ij})_{(i,j) \in E}\) represents the vector of uncertain arc durations. We are now in a position to formulate the class of shortest path problems under uncertainty to be studied in this paper:

\[
J = \min_{x \in X} \mathbb{F}[d(x, \xi)]
\]

s.t. \(x \in X\) \(\mathbb{F}\)-a.s.

\[
x_{k+1} \mathcal{F}_k\text{-measurable } \forall k = 0, \ldots, N - 1.
\]

Here, \(\mathbb{F}(\cdot)\) represents a probability functional that maps (suitable classes of) random variables to real numbers. If \(d(x, \xi)\) is guaranteed to be integrable, \(\mathbb{F}(\cdot)\) is often chosen to be the expectation operator. The resulting average-optimal solutions may be desirable in some situations. However, they may be too risky when the problem is solved only once. Indeed, it is quite possible that the solution with least expected cost has a high probability of realizing very long travel times. While optimizing the expectation is reasonable when an experiment is repeated a large number of times (since the average of the outcomes will converge to the mean by the Law of Large Numbers), in many practical applications this is not the case. In fact, there are many situations where a routing policy needs to be computed only once. For these cases it may be preferable to choose a policy with a slightly higher expected cost but with small probability of realizing very high costs. Such policies arise if \(\mathbb{F}(\cdot)\) is chosen to be a proper risk measure.

Unfortunately, the use of popular risk measures such as a mean-variance functional, the expected shortfall, the value-at-risk or an expected utility function hampers the dynamic decomposability of problem (1), which makes it difficult to solve it via efficient dynamic programming algorithms.

If no information is available at the time when the routing decisions are taken, that is, if \(\mathcal{F}_k = \{\emptyset, \Omega\}\) for all \(k = 0, \ldots, N - 1\), then the optimization in (1) is over deterministic paths \(x \in X\), in which case problem (1) reduces to

\[
\bar{J} = \min_{x \in X} \mathbb{F}[d(x, \xi)].
\]

Moreover, if the decision maker is risk-neutral, thereby optimizing in view of the expected total travel time, the objective function of problem (2) can be re-expressed in terms of the expected arc durations as

\[
\mathbb{E}[d(x, \xi)] = \sum_{k=0}^{N-1} \mathbb{E}[^{\xi_{x_k x_{k+1}}}].
\]

Thus, problem (2) simplifies to a classical deterministic shortest path problem [11]–[13]. It is well known that a solution to this problem can be obtained using dynamic programming, and there is vast literature on algorithms that can do so efficiently, see e.g., [1], [14], [15]. Naturally, this solution only utilizes information about the mean arc durations; other types of distributional information, such as the variances of the arc durations, are disregarded.

Another extreme case arises if full information about the arc durations is available when the routing decisions are taken, that is, if \(\mathcal{F}_k = \Sigma\) for all \(k = 0, \ldots, N - 1\). In this situation the decision maker knows the realizations of all arc durations at the planning stage and can solve a deterministic shortest path problem for every possible scenario. If the
risk functional $F(\cdot)$ satisfies the monotonicity property of coherent risk measures, see [16], then (1) reduces to
\[ J = F \left( \min_{x \in X} d(x, \xi) \right). \]
See also [8], [9]. As more information is always desirable, we may conclude that $J \leq J \leq \bar{J}$.

Several authors have considered performance measures other than the expectation for choosing the best path. For example in [17] an optimal deterministic policy is found by solving
\[ \min_{x \in X} P[d(x, \xi) \geq c] \]
for some given positive constant $c$. In [5] an alternative optimization problem is developed in which the optimality criterion, called the path optimality index, is the probability that a path gives lower cost than all other paths. Even though an analytical expression for the optimal policy exists, both approaches require an exhaustive search and therefore do not scale well with network size if a globally optimal solution is desired. Other alternatives, such as the expected utility maximization criterion of von Neumann and Morgenstern, which explicitly account for the riskiness of a solution, are summarized in [18]. It has been shown, however, that these criteria give rise to dynamic programming formulations only in very special cases (e.g., when the utility function is linear or exponential) and hence in general admit no efficient solution.

III. MAIN RESULTS

In this section we present problem formulations which take into account the riskiness of random paths but can still exploit Bellman’s optimality principle [19]; they are therefore amenable to dynamic programming implementations.

The first measure of risk we consider is the conditional value-at-risk (CVaR) which was introduced in [20], [21] and predominantly appears in financial applications. For a given random variable $\xi$ representing loss and a tolerance level $\epsilon \in (0, 1]$, the CVaR of $\xi$ is essentially the conditional expectation of $\xi$ above its $(1-\epsilon)$-quantile. Consequently, the resulting solutions favor paths with a small probability to incur long delays. We discuss efficient methods to minimize the CVaR of the path duration over all deterministic paths.

Next, we consider cases where the distribution $P$ of the arc durations is not precisely known. Instead, only the first and second moments of the arc durations are assumed to be accessible. In this setting we formulate the optimization problem over a family of distributions and compute distributionally robust deterministic paths from source to sink.

Finally, we discuss a dynamic programming-based approach for optimizing over a class of adaptive (i.e., non-deterministic) routing decisions. Here it is assumed that we have access to lookahead information, that is, we have the ability to observe the durations of some of the successor nodes when routing decisions are due to be taken. It will turn out that this type of information can be very valuable.

A. Conditional value-at-risk

Formally, for any integrable random variable $\xi$ and tolerance $\epsilon \in (0, 1]$, the CVaR of $\xi$ at level $\epsilon$ is defined as
\[ \mathbb{P}\text{-CVaR}_\epsilon[\xi] = \inf_{\beta} \left\{ \beta + \frac{1}{\epsilon} \mathbb{E}[(\xi - \beta)^+] \right\}, \]
where $[x]^+ \equiv \max\{x, 0\}$, see [21]. This definition is popular as the above minimization problem can be formulated as a linear program.

We will now study risk-averse shortest path problems over deterministic paths and using a CVaR objective,
\[ \min_{x \in X} \mathbb{P}\text{-CVaR}_\epsilon[d(x, \xi)]. \]
Unfortunately, this problem does not admit a dynamic decomposition. By the sub-additivity property of CVaR, however, an upper bound can be obtained by solving
\[ J^c = \min_{x \in X} \sum_{k=0}^{N-1} \mathbb{P}\text{-CVaR}_\epsilon[\xi_{x_kx_{k+1}}] \quad \text{s.t.} \quad x \in X, \]
see, e.g., [16]. This modified risk-averse model does admit a dynamic programming-based solution.

Consider the dynamic programming recursion in which
\[ J_N(x_N) = 0, \]
and
\[ J_k(x_k) = \min_{x_{k+1} \in X(x_k)} \left\{ \mathbb{P}\text{-CVaR}_\epsilon[\xi_{x_kx_{k+1}}] + J_{k+1}(x_{k+1}) \right\} \]
for $k = N - 1, \ldots, 0$. Recall that $X(x_k)$ denotes the set of successor nodes of node $x_k$.

Proposition 3.1: $J_0(0) = J^c$.

The proof is by induction and is based on the principle of optimality, see e.g., [22]. For completeness, a brief sketch is included below.

Proof: For all $k \in \{0, \ldots, N - 1\}$ let $\phi_k : X' \to X'$ be a mapping that satisfies $\phi_k(x_k) \in X(x_k)$ for all $x_k \in X$. Then, $\pi = \{\phi_0, \ldots, \phi_{N-1}\}$ is referred to as an admissible policy and $\pi^k = \{\phi_k, \ldots, \phi_{N-1}\}$ denotes the corresponding sub-policy truncated before stage $k$. Define $J^c_N(x_N) = J_N(x_N) = 0$ and set
\[ J^c_k(x_k) = \min_{\pi^k} \sum_{i=1}^{N-1} \mathbb{P}\text{-CVaR}_\epsilon[\xi_{x_k\phi_i(x_i)}] \]
for $k = 0, \ldots, N - 1$.

Now suppose that $J^c_{k+1}(x_{k+1}) = J_{k+1}(x_{k+1})$ and that the principle of optimality holds. Then, the dynamic program-
ming recursions for $J_k(x_k)$ imply

$$J_k(x_k) = \min_{x_{k+1} \in X(x_k)} \left\{ \mathbb{P}\text{-CVaR}_\epsilon \left[ \xi_{x_k x_{k+1}} \right] + \ldots \right\}$$

follows if we can show that $J_{\text{la}}^k = J_{\text{la}}^{k+1}$ for all $k = 0, \ldots, N - 1$. By the recursion for $J_{\text{la}}^k (x_k)$, $J_{\text{la}}^k$ can be

Since $k$ was arbitrary, it holds that $J_0(0) = J_0^c(0) = J^c$, and thus the claim follows.

**B. Worst case conditional value-at-risk**

In practice the distributions of the arc durations may only be partially known. In fact, it is often more realistic to assume that only estimates of the first and second moments of the arc durations are available. These could have been obtained via sampling. In the remainder of this section we will therefore assume that the arc durations $\xi_{ij}$, $(i, j) \in E$, are independent, square integrable and almost surely nonnegative random variables and that their joint probability distribution is unknown beyond the marginal means and variances.

Formally, let $\mathcal{P}$ be the set of all probability distributions on $(\Omega, \Sigma)$ that are consistent with this distributional information. Given a tolerance $\epsilon \in (0, 1]$, the worst case CVaR ($\text{WCVaR}$) of a uniformly integrable random variable $\zeta$ is defined as

$$\text{WCVaR}_\epsilon [\zeta] = \sup_{p \in \mathcal{P}} \mathbb{P}\text{-CVaR}_\epsilon [\zeta], \quad (3)$$

see e.g. [23]. It has in fact been shown [24] that (3) can be expressed as

$$\text{WCVaR}_\epsilon [\zeta] = \inf_{\beta} \left\{ \beta + \frac{1}{\epsilon} \sup_{p \in \mathcal{P}} \mathbb{E}[\zeta - \beta]^{+} \right\},$$

giving rise to a tight semi-definite programming approximation if $\zeta$ exhibits a linear or quadratic dependence on the vector $\xi$ of primitive random variables. We can thus formulate the following risk-averse shortest path problem which also offers protection against ambiguity in the probability distribution of the arc durations,

$$J^w = \min_{x \in X} \sum_{k=0}^{N-1} \sup_{p \in \mathcal{P}} \mathbb{P}\text{-CVaR}_\epsilon \left[ \xi_{x_k x_{k+1}} \right]$$

s.t. $x \in X.$

**Proposition 3.2:** The optimal value $J^w$ can be computed via an efficient dynamic program.

The proof of Proposition 3.2 parallels the one of Proposition 3.1 and is therefore omitted.

**C. Lookahead information**

The shortest path problem with lookahead information is significantly more complex because the routing decisions are themselves random variables. For ease of exposition, we will assume that the decision at stage $k$ may depend on the durations of all arcs emanating from the current node.

To derive a recursive formulation for this problem, we apply a mathematical trick: we assume that there are $N$ independent and identically distributed copies $\{\xi^k\}_{k=1}^{N}$ of the vector of arc durations, and we let the duration of the arc traversed after stage $k$ be determined by the respective component of $\xi^k$. Thus, we modify the formula for the path duration as follows

$$d(x, \xi) = \sum_{k=0}^{N-1} \xi_{x_k x_{k+1}}^k,$$ (4)

where $\xi = \{\xi^k\}_{k=1}^{N}$. To model one-step-lookahead information, we can now define the $\sigma$-algebras in (1) as $\mathcal{F}_k = \sigma(\{\xi^i\}_{i=1}^{k+1})$ for $k = 0, \ldots, N - 1$. This definition ensures that $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \cdots \subseteq \mathcal{F}_{N-1} = \Sigma$ represents a filtration (i.e., information is never forgotten), and, more importantly, that the information at any stage $k$ is independent of the particular sub-path traversed in the past.

Using the modified path duration (4) and the one-step-lookahead information sets $\{\mathcal{F}_k\}_{k=0}^{N-1}$ defined above, we aim at solving the dynamic shortest path problem

$$J^\text{la} = \min_{x \in X} \mathbb{E}[d(x, \xi)]$$

s.t. $x \in X$ $\mathbb{P}$-a.s.

$$x_{k+1} \mathcal{F}_k$-measurable $\forall k = 0, \ldots, N - 1.$$

Next, set $J_N^\text{la}(x_N) = 0$, and define the (random $\mathcal{F}_k$-measurable) cost-to-go functions

$$J_k^\text{la}(x_k) = \min_{x_{k+1} \in X(x_k)} \mathbb{E}\left[ \xi_{x_k x_{k+1}}^{k+1} + J_{k+1}^\text{la}(x_{k+1}) \bigg| \mathcal{F}_k \right]$$

recursively for $k = N - 1, \ldots, 0$.

**Proposition 3.3:** $J_0^\text{la}(0) = J^\text{la} \mathbb{P}$-a.s.

**Proof:** For $k = 0, \ldots, N$ define

$$J_k^\text{la} = \min_{x \in X} \mathbb{E}\left[ \sum_{i=0}^{k} \xi_{x_i x_{i+1}}^{i+1} + J_i^\text{la}(x_i) \right]$$

s.t. $x \in X$ $\mathbb{P}$-a.s.

$$x_{i+1} \mathcal{F}_i$-measurable $\forall i = 0, \ldots, k - 1.$$

The convention is that the summation term is zero for $k = 0$. It is clear that $J_0^\text{la} = J_0^\text{la}$, while $J_0^\text{la}(0) = J_0^\text{la} \mathbb{P}$-a.s. Thus, the claim follows if we can show that $J_k^\text{la} = J_{k+1}^\text{la}$ for all $k = 0, \ldots, N - 1$. By the recursion for $J_k^\text{la}(x_k)$, $J_k^\text{la}$ can be
expressed as
\[
\min \mathbb{E} \left[ \sum_{i=0}^{k-1} \xi_{k+1}^{i+1} \right] \\
+ \min_{x' \in \mathcal{X}(x_k)} \mathbb{E} \left[ \xi_{k+1}^{i+1} + J^a_{k+1}(x') \mathcal{F}_k \right] \\
\text{s.t.} \ x \in X, \ \mathbb{P}\text{-a.s.} \\
x_{i+1} \mathcal{F}_i\text{-measurable} \ \forall i = 0, \ldots, k-1.
\]
By the law of iterated conditional expectations and the principle of interchangeability [25, Thm. 14.60], the above optimization problem is equivalent to
\[
\min \mathbb{E} \left[ \sum_{i=0}^{k} \xi_{k+1}^{i+1} + J^a_{k+1}(x_{k+1}) \right] \\
\text{s.t.} \ x \in X, \ \mathbb{P}\text{-a.s.} \\
x_{i+1} \mathcal{F}_i\text{-measurable} \ \forall i = 0, \ldots, k.
\]
The optimal value of this problem, in turn, equals \(J^a_{k+1}\) by definition. Thus, the claim follows.

We remark that, due to the independence of the arc durations, the dynamic programming reformulation of the shortest path problem with lookahead information can be solved efficiently if the nodes in the graph have uniformly bounded degrees.

IV. NUMERICAL RESULTS

In this section we evaluate the performance of the policies outlined in Section III and assess their suitability as real-world routing policies.

As proving ground we consider a DAG \(\mathcal{G}\) with 30 nodes \((N = 29)\) where each node \(i \in \mathcal{X}\) is connected to \(\min\{N-i, \lfloor 0.3 \times (N+1) \rfloor -1\}\) nodes.

The distributions of the uncertain arc durations are assumed to have first and second moments generated according to the rule \(\mathbb{E}[\xi_{ij}] = j - i + 1\) and \(\text{var}[\xi_{ij}] = \frac{1}{12} (2 \times \xi \times \mathbb{E}[\xi_{ij}])^2\), respectively, where \(j > i\) and \(\xi \sim U[0, 1]\) is a uniformly distributed RV supported on \([0, 1]\). The tolerance level is set to \(\epsilon = 0.1\).

A. Deterministic policies

The objective of the first experiment is to compute deterministic optimal policies as solutions to dynamic programs with risk measures, \(E\), \(CVaR\) and \(WCVaR\), and implement them in a dynamic environment. The cost of the optimal policies are labelled “Min-E”, “Min-CVaR”, and “Min-WCVaR”, respectively. Given a sequence of \(M\) (say historical) observations for the duration each arc, the deterministic average-optimal (E) and risk-averse (CVaR and WCVaR) policies are computed from the empirical means and variances.

For each arc we are given \(M = 100\) samples from a uniform distribution with the prescribed mean and variance. Once the deterministic policies have been computed, they are implemented in an experiment of 10,000 randomly generated trials with each trial corresponding to \(\mathcal{G}\) with deterministic arc durations. Arc durations are sampled from the same uniform distributions as before. Each policy dictates a path with total duration (or cost) equal to the sum of all realized durations of the traversed arcs.

Table I reports the mean and 10% CVaR of the total path durations over the 10,000 trials, for each policy. Figure 2 shows the corresponding cumulative densities. As expected, the average-optimal policy results in paths with least average total duration but which can potentially realize very high costs. It is also noted that the WCVaR optimal policy seems to offer similar protection against high cost as the CVaR one. In the sequel, the computed optimal policies are again implemented in 10,000 instances, but this time with arc durations sampled from a mixture of a uniform and a worst-case (2 point) distribution, see e.g. [10], with equal probabilities. As before, these distributions satisfy the prescribed first and second moments.

The results are shown in Figure 3 and Table II. All three policies yield similar means to the previous experiment, which is to be expected. The distributionally robust policy, however, now outperforms the optimal CVaR one and is by far superior to the average-optimal as far as protection against risk is concerned. In fact, the 10% CVaR of Min-E is now more than 26% higher than that of Min-WCVaR.

B. Adaptive policies

We consider the case when one-step-lookahead information is available. At first, the optimal cost-to-go function \(J^a_{k+1}(x_k)\) is computed from 100 samples as in Subsection IV-A. The optimal policy, which is now adaptive, is once more implemented for 10,000 instances of the durations. At each
We have also shown that adaptive strategies with one-step information can be obtained as solutions to efficient dynamic programs and have illustrated the value of this information.

B. Future Work

We have assumed that the arc durations are independent random variables. A more realistic hypothesis is that the arc durations are correlated, which is the case in many real-life networks such as road networks for example. Obtaining easily computable policies using other measures is also the subject of future work.

REFERENCES