Controller’s parameters tuning in presence of time-delay measurements: an application to vision-based quad-rotor navigation

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Abstract—The stability analysis of a quad-rotor rotorcraft which uses a combination of imaging, inertial and altitude sensors to measure its states is addressed. A hierarchical controller is designed in two steps. The Fast-time scale dynamics are stabilized using classical PD controllers. The slow time-scale dynamics are analyzed with delay frequency and time domain techniques. The proposal is validated by simulation and experimental results, showing the effectiveness of the method.

I. INTRODUCTION

The stabilization of a quad-rotor Unmanned Aerial Vehicle (UAV) requires knowledge of the aircraft linear and angular velocity and position. It is a common practice to separate the control strategy in two loops: the high-speed, inner loop that controls the attitude, and the slower outer loop that controls the position. While angular dynamics can be measured at high rates by means of an Inertial Measurement Unit (IMU) installed onboard the vehicle, a positioning system capable of providing the vehicle’s translational dynamics for any working environment is currently unavailable.

When evolving outdoors, a Global Positioning System can be used. However, in indoors environments interferences may exist, which impair the quality of satellite signals. A promising solution towards increasing the autonomy degree of UAVs consists on installing a video camera on-board and to develop computer vision algorithms to estimate the movement of the vehicle with respect to (w.r.t.) its surrounding environment [4], [5], [6]. However, image processing is computationally intensive and introduces latencies from the time of capture to the time measurements are available, limiting the controller gains required to ensure stability.

In [7] a filter-based position and velocity estimation technique is presented, which fuses inertial and delayed dropout-susceptible vision measurements without the a priori knowledge of the exact variable time delay. Using the presented filter and a simple PID controller a quad-rotor is stabilized and kept in the region of a specific set-point. Video time-stamping and a slightly modified Kalman filter are implemented in [8] for determining the time delay of the vision-based measurements. This method allows performing an autonomous flight of a quad-rotor over an artificial visual landmark. An adaptive visual odometer that can be used. However, in indoors environments interferences may exist, which impair the quality of satellite signals. A promising solution towards increasing the autonomy degree of UAVs consists on installing a video camera on-board and to develop computer vision algorithms to estimate the movement of the vehicle with respect to (w.r.t.) its surrounding environment [4], [5], [6]. However, image processing is computationally intensive and introduces latencies from the time of capture to the time measurements are available, limiting the controller gains required to ensure stability.

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vehicle’s translational dynamics. In the present application
the imaging system measures the \( \psi, \dot{x}, y, \) and \( \dot{y} \) states.

A road model is used for measuring the vehicle’s relative lateral position \( (y) \), while an optical flow computation in combination with the vehicle’s altitude \( (z) \) provides the forward \( (\dot{x}) \) and lateral \( (\dot{y}) \) velocities. In addition, processing
the camera’s image allows to compute the relative heading angle \( B_\psi \psi_d \) of the quad-rotor w.r.t. the road’s longitudinal axis. The image-based measurements are obtained at 100 Hz
only, for this reason they will be constrained by a time delay expressed by \( T_v \) [18]. Further details concerning the
algorithm and its implementation can be found in [19].

For an underactuated system like the quad-rotor, it is clear
that the forward translational dynamics depends on the pitch angle \( (x-\theta \) subsystem), while the lateral dynamics depends on
the roll angle \( (y-\phi \) subsystem). Each one of these subsystems
presents a time-scale separation due to different measurement
rates on the translational dynamics (slow time-scale at 100 Hz)
and on the orientation dynamics (fast time-scale at 1 kHz). This motivates the development of vision-based control strategies considering the latencies from the time of image
capture to the time when position and velocity measurements are available.

The road following mission can be detailed as follows. First, the vehicle performs an autonomous take-off, reaching a
desired altitude \( z_d \). The heading of the vehicle, expressed by \( \psi \), is driven to yield a parallel orientation between the
 UAV \( x \)-axis \( (B_x) \) and the longitudinal direction of the road \( (R_z) \). The UAV’s forward velocity \( (\dot{x}) \), is kept constant,
while the distance between \( B_x \) and \( R_z \), expressed by \( B_y \),
is regulated and kept to a minimum value.

Due to the fast-time measurements provided by the altitude
sensor and the IMU, classical PD controller can be designed
for stabilizing the states associated with these sensors. How-
ever, since the states provided by the imaging system are
obtained at lower rates they present latencies and time delays.
For these reasons, image based measurements require a more
detailed analysis based on delay frequency and time domain
techniques.

III. QUAD-ROTOR MODEL

Let \( \{I\} = \{I_x, I_y, I_z\}^T \) represent the inertial frame and
\( \{B\} = \{B_x, B_y, B_z\} \) represent the helicopter’s body-fixed
frame, both of them in North-East-Down (NED) formalism.
The position vector of the UAV’s center of mass relative
to the frame \( \{I\} \) is denoted by \( \xi = [x, y, z]^T \). The quad-
rotor’ orientation vector with respect to \( \{I\} \) is expressed by
\( \eta = [\psi, \theta, \phi]^T \), referred to as yaw, pitch and roll respectively
[20]. A simplified quad-rotor model that does not consider
all the effects acting on the helicopter but serves well for the
present research purposes can be written as:

\[
\begin{align*}
\dot{x} &= -u(c_\psi s_\phi s_\theta + s_\psi s_\theta) \\
\dot{y} &= -u(s_\psi s_\theta c_\phi - c_\psi s_\phi) \\
\dot{z} &= -u(s_\psi c_\phi) + mg \\
\dot{\psi} &= \tau_\psi \\
\dot{\theta} &= \tau_\theta \\
\dot{\phi} &= \tau_\phi
\end{align*}
\]

where \( c_\phi \) and \( s_\phi \) stand for \( \cos(*) \) and \( \sin(*) \), respectively.

IV. CONTROL STRATEGY

A. Altitude and yaw controllers

The \( z \) position of the quad-rotor can be stabilized by using the
following control input in equation (3):

\[
u = m(-k_{dz}\dot{z} - k_{pz}e_z + g)/\cos \theta \cos \phi\] (7)

with \( e_z = z_d - z \) as the \( z \) error position and \( z_d \) as
the quad-rotor desired altitude. \( k_{dz} \) and \( k_{pz} \) are positive constants of
a PD controller which should be carefully chosen to ensure
a stable well-damped response in the vertical axis [4].

For the yaw angle given by equation (4), one can apply

\[
\tau_\psi = k_{\psi \dot{\psi}}(\dot{\psi}_d - \psi(t)) + k_{\psi \psi}(\psi_d - \psi(t - T_v))\] (8)

where \( \psi_d \) represents the desired yaw angle given by the road
orientation, which is smooth enough in such a way that its
derivative exists. \( k_{\psi \dot{\psi}} \) and \( k_{\psi \psi} \) denote the positive constants
of a PD controller which are tuned using time-delay systems
methods presented in Section V.

Substituting (8) into (4) one obtains the block diagram of
the quad-rotor’s \( \psi \) dynamic shown in Fig. 2 whose transfer
function is expressed as:

\[
\frac{\Psi(s)}{\Psi_d(s)} = \frac{k_{\psi \dot{\psi}}s + k_{\psi \psi}}{\Delta \psi(s)}\] (9)
where

$$\Delta \psi(s) = s^2 + k_d \psi s + k_p \psi e^{-T_v s}$$  \hspace{1cm} (10)$$

is the characteristic quasipolynomial. Introducing equations (7) and (8) into the set of equations (1)-(4) and provided that $$\cos \theta \cos \phi \neq 0$$, one obtains

$$m \ddot{x} = -(r_1 + mg) (c_p \theta + s_p \phi) / c_\theta$$  \hspace{1cm} (11)$$

$$m \ddot{y} = -(r_1 + mg) (s_p \theta - c_p \phi) / c_\theta$$  \hspace{1cm} (12)$$

$$m \ddot{z} = -k_{dp} \dot{z} - k_{pz} \psi_e$$  \hspace{1cm} (13)$$

$$\ddot{\psi} = -k_{d\psi} \dot{\psi} - k_{p\psi} \psi_e$$  \hspace{1cm} (14)$$

where $$T_v$$ stands for $$\tan(\cdot)$$. From equations (13) and (14) it follows that $$z \rightarrow z_d$$ and $$\psi \rightarrow \psi_d$$. For a large enough time, $$e_z$$ and $$e_\psi$$ are arbitrarily small, therefore, equations (11) and (12) reduce to

$$\ddot{x} = -g \tan \theta$$  \hspace{1cm} (15)$$

$$\ddot{y} = g (\tan \phi) / (\cos \theta)$$  \hspace{1cm} (16)$$

B. Forward position and pitch angle control

Consider the subsystem given by equations (5) and (15). To further simplify the analysis, let’s impose a very small upper bound on $$|\theta|$$ in such a way that the difference $$\tan(\theta) - \theta$$ is arbitrarily small ($$\tan(\theta) \approx \theta$$). Therefore, the linearization of subsystem (5)-(15) is:

$$\dot{x} = -g \theta$$  \hspace{1cm} (17)$$

$$\dot{\theta} = \ddot{\theta}$$  \hspace{1cm} (18)$$

To stabilize the pitch angular position (18), one can apply

$$\ddot{\theta} = k_{d\theta} \dot{\phi} + k_{p\theta} \phi$$

where $$\phi = \dot{\phi} - \phi$$, and $$\dot{\phi}$$ results at constant velocity i.e., $$\ddot{x} = \dot{x}$$, let’s define $$\theta_d = k_{dx} (\dot{x} - x(t - T_v))$$. The parameter $$k_{dx}$$ will be obtained in Section V using time delay systems methods. Due to the requirement that $$\dot{x}_d$$ equals a constant (forward velocity) value, $$\theta_d$$ results as $$\dot{\theta}_d = k_{dx} (\dot{x}_d - \dot{x}(t - T_v))$$, whose derivative is given by

$$\ddot{\theta}_d = k_{dx} (\ddot{x}_d - \ddot{x}(t - T_v))$$. Now, $$\ddot{\theta}_d$$ can be written as:

$$\ddot{\theta}_d = k_{p\theta} \dot{k_{dx}} \dot{x}_d - k_{p\theta} \dot{k_{dx}} \dot{x}(t - T_v) - k_{p\theta} \theta_d$$

Thus, substituting $$\ddot{\theta}_d$$ in the subsystem (17)-(18) one obtains the block diagram of the quad-rotor’s $$x$$-$\theta$$ dynamic shown in Fig. 3 whose transfer function is expressed as:

$$\frac{S_X(s)}{S_X(d(s))} = \frac{g k_{dx} (k_{dx} \theta d + k_{p\theta})}{\Delta_x(s)}$$  \hspace{1cm} (19)$$

where

$$\Delta_x(s) = s^4 + k_{d\theta} s^2 + k_{p\theta} s - g k_{dx} (k_{d\theta} s + k_{p\theta}) e^{-T_v s}$$  \hspace{1cm} (20)$$

is the characteristic quasipolynomial. Using a hierarchical control approach, the dynamic of the fast-scale system is stabilized by a means of a PD controller with $$k_{p\theta} = 10$$ and $$k_{d\theta} = 100.1$$, replacing the system’s poles at [-100, -0.1]. This controller makes the fast-time scale subsystem behave as a first order system with a response time $$t_r = 0.05$$ s.

C. Lateral position and roll angle control

Consider the subsystem given by equation (6) and equation (16). Imposing a very small upper bound on $$|\phi|$$ in such a way that the difference $$\tan(\phi) - \phi$$ is arbitrarily small ($$\tan(\phi) \approx \phi$$), the subsystem given by equation (6) and equation (16) reduces to the linear system:

$$\ddot{y} = g \phi$$  \hspace{1cm} (21)$$

$$\ddot{\phi} = \ddot{\phi}$$  \hspace{1cm} (22)$$

To stabilize the roll angular position (22), one can apply

$$\ddot{\phi} = k_{d\phi} \dot{\phi} + k_{p\phi} \phi$$

where $$e_\phi = \phi_d - \phi$$. In order to stabilize the lateral position, let’s define $$\phi_y = k_{dy} (y_d - y(t - T_v)) + k_{p\phi} (y_d - y(t - T_v))$$.

The controller gain $$k_{p\phi}$$ and $$k_{dy}$$ will be obtained latter in Section V using time delay systems methods. Due to the requirement that $$y_d = y_d = 0$$, $$\phi_y$$ reduces to $$

$$\ddot{\phi} = -k_{d\phi} \dot{y}_d \dot{y}(t - T_v) - k_{p\phi} \dot{y}(t - T_v)$$

whose derivative is given by

$$\ddot{\phi} = -k_{d\phi} \dot{y}_d \dot{y}(t - T_v) - k_{p\phi} \dot{y}(t - T_v)$$

Thus, substituting $$\ddot{\phi}$$ in the subsystem (21)-(22) one obtains the block diagram of the $$y$$-$\phi$$ subsystem shown in Fig. 4 whose transfer function is expressed as:

$$\frac{Y(s)}{Y_d(s)} = \frac{g (k_{dy} s + k_{p\phi})(k_{d\phi} s + k_{p\phi})}{\Delta_y(s)}$$  \hspace{1cm} (23)$$

where

$$\Delta_y(s) = s^4 + k_{d\phi} s^3 + k_{p\phi} s^2 + g (k_{dy} + k_{p\phi})(k_{d\phi} s + k_{p\phi}) e^{-T_v s}$$  \hspace{1cm} (24)$$

is the characteristic quasipolynomial. Using the hierarchical control approach, the dynamics of the fast-time scale subsystem is stabilized by means of a PD controller with $$k_{p\phi} = 8$$ and $$k_{d\phi} = 80.1$$, placing the system’s poles at [-80, -0.1]. This controller makes the fast-time scale subsystem behave as a first order system with a time response $$t_r = 0.06$$ s.

V. STABILITY ANALYSIS FOR SLOW-TIME DYNAMICS: TIME AND FREQUENCY DOMAINS APPROACHES

The quasipolynomials shown in equations (10), (20) and (24) have an infinite number of roots, therefore it is not possible to analyze their stability using standard techniques for polynomials. Nevertheless, as the roots move continuously w.r.t parameters, it is possible to find the imaginary crossing.
boundaries that divides the space of parameters into regions with constant number of unstable roots.

A. D-partitions method for finding stability regions

As shown in [14], the D-partitions method allows to find the stability regions in the parametric space of the subsystems under analysis.

1) y-φ subsystem: For this subsystem, the D-partitions diagram is found by evaluating the characteristic quasi-polynomial (10) at \( s = 0 \), which yields \( k_{py} = 0 \) \( \forall \) \( k_{dy0} \in (-\infty, \infty) \). On the other hand, for \( s = j\omega \) one obtains

\[
k_{py}(\omega) = \omega^2 / \cos(\omega T_v) ; \quad k_{dy}(\omega) = \tan(\omega T_v) \omega
\]

These equations are depicted as the solid lines in Fig. 5(a).

2) x-θ subsystem: This D-partitions diagram is found by considering the derivative gain \( k_{dx} \) and the time delay \( T_v \) induced by the vision-based estimations. Evaluating the characteristic quasipolynomial (20) at \( s = 0 \), one obtains that \( k_{dx} = 0 \), \( T_v \in [0, \infty) \). For \( s = j\omega \) one obtains

\[
k_{dx}(\omega) = \pm \sqrt{T_x / \left( k_{py}^2 g + k_{dy}^2 g \omega^2 \right)}
\]

where

\[
T_x = k_{dy}^2 \omega^4 + k_{dy}^2 \omega^2 + 2k_{py}^2 \left( k_{dy}^2 - k_{py}^2 \right) \omega^4 + \left( k_{dy} - k_{py} \right)^2 \omega^6,
\]

and

\[
T_v(\omega) = \frac{1}{\omega} \tan^{-1} \left( \frac{k_{py}^2 + (k_{py}^2 - k_{py}^2) \omega^2}{k_{dy}^2 \omega^3} \right) + \frac{k\pi}{\omega}
\]

The corresponding crossing boundaries are depicted as the solid lines on Fig. 5(b).

3) y-φ subsystem: For the y-φ subsystem, notice that if \( \deg Q(s) > P(s) + 1 \), then the line \(-q_0/p_0, k_{dy} \) belongs to the stability crossing boundaries [15], which can be found by evaluating the characteristic quasipolynomial (24) at \( s = 0 \). On the other hand, this characteristic quasipolynomial can be rewritten as

\[
\Delta_q(s) = Q(s) + (k_{py} + k_{dy}s)P(s)e^{-T_v s} = 0
\]

by arranging terms one obtains

\[
\frac{Q(s)}{P(s)}e^{-T_v s} = -(k_{py} + k_{dy}s)
\]

Then, for \( s = j\omega \), and matching the real and imaginary parts of both sides of equation (28) yields

\[
k_{py}(\omega) = -\Re \left( \frac{Q(j\omega)}{P(j\omega)}e^{-T_v j\omega} \right)
\]

\[
k_{dy}(\omega) = -\frac{1}{\omega} \Im \left( \frac{Q(j\omega)}{P(j\omega)}e^{-T_v j\omega} \right)
\]

These equations are depicted as solid lines on Fig. 5(c). At this point the regions of stability are already obtained, however, it is not known yet which of those regions are stable and which ones are unstable. It is sufficient to find a stable point in the parametric space to conclude that the corresponding region is a region of stability. In order to detect such regions we propose here doing a sweeping of the parameter space, testing for each point the LMI type sufficient condition of the descriptor approach [17]. Clearly, if a point satisfying the condition is found, the whole region is a stable one.

B. State space representation for a delay system

The time domain methods for analyzing the stability of time-delay systems used in this work are based on a state space representation for a linear time-invariant, functional differential equation (RFDE) [12]. For systems having a single delay, such equation is given by

\[
\dot{x}(t) = A_0 x(t) + A_1 x(t - T_v) + B u
\]

The specific representation of each one of the subsystems considered in the present study is described next.

1) y-φ subsystem: From Fig. 2 and equation (8) one has

\[
\ddot{x}_y = k_{py}(\psi_d - \psi(t - T_v)) - k_{dy}\dot{\psi}
\]

which, substituting in equation (4), and choosing the state variables as \( x_1 = \psi \) and \( x_2 = \dot{\psi} - k_{dy}\dot{\psi}_d \), where \( u = \dot{\psi}_d \), it yields

\[
A_0 = \begin{bmatrix} 0 & 1 \\ 0 & -k_{dy} \end{bmatrix} \quad A_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} k_{dy} \\ k_{py} - k_{dy} \end{bmatrix}
\]

2) x-θ subsystem: Consider Fig. 3, and equations (15)-(16). Let’s define the state variables as \( x_1 = \dot{x} \), \( x_2 = \dot{\theta} \) and \( x_3 = \dot{k}_{dy}k_{dx}u \), and let \( u = \dot{x}_d \). Then

\[
A_0 = \begin{bmatrix} 0 & -g & 0 \\ 0 & 0 & 1 \\ 0 & -k_{dy} & -k_{dy} \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ k_{dx}k_{dy} \\ k_{dy}k_{dx} - k_{dy}k_{dx} \end{bmatrix}
\]

\[
A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -k_{dy}k_{dx} & k_{dy}k_{dx} & g \end{bmatrix}
\]

3) y-φ subsystem: From equations (21)-(22) and Fig. 4, and defining the state variables as \( x_1 = y \), \( x_2 = \dot{y} \), \( x_3 = \phi \) and \( x_4 = \phi \), it yields

\[
A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_{dy} & -k_{dy} \end{bmatrix}
\]

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C. Sufficient stability conditions

The descriptor method [17] allows to obtain sufficient stability conditions for time delay systems. This procedure is based in the solution of the linear matrix inequality

\[
M = \begin{bmatrix}
  m_{11} & m_{12} & m_{13} \\
  m_{21} & m_{22} & m_{23} \\
  m_{31} & m_{32} & m_{33}
\end{bmatrix} > 0
\] (33)

with \(m_{11} = P_2^T A_0 + A_0^T P_2 + S - R, m_{12} = P - P_2 + A_0^T P_3, m_{13} = R + P_2^T A_1, m_{21} = P - P_2^T + P_3^T A_0, m_{22} = T_0^2 R - P_3 - P_3^T, m_{23} = P_3^T A_1, m_{31} = R + A_0^T P_2, m_{32} = A_0^T P_3\) and \(m_{33} = -S - R.\) In the previous equations \(R > 0, P > 0\) and \(S > 0.\) This procedure ensures that, if there exist matrices \(P,Q,R > 0, P_2,P_3 \in R^{n \times n}\) such that \(M > 0,\) then the system is exponentially stable.

The Yalmip [21] and Sedumi [22] Matlab toolboxes have been used to check condition (33) in the space of parameters for the three subsystems analyzed in this paper. The isolated points in Fig. 5(a), Fig. 5(b) and Fig. 5(c), are points of the space of parameters that satisfy this condition, thus allowing to conclude on the stability of the corresponding regions.

VI. SIMULATION RESULTS

The simulations shown in Fig. 6, Fig. 7, and Fig. 8, were obtained using the model shown in equations (1)-(6). The gains of the external PD controllers were tuned using the results from the methods described above. The corresponding values are \(k_{\psi\psi} = 10, k_{p\psi} = 2, k_{d\psi} = -5, k_{d\psi} = 5,\) and \(k_{p\psi} = 2.\) Those values are depicted in Fig. 5(a), Fig. 5(b) and Fig. 5(c) as squared solid points. The initial conditions employed are \(x = 1m, y = 0.5m\) and \(z = 0.5m, \dot{x} = -0.5m/s, \dot{y} = 0.01m/s,\) and \(\dot{z} = 0.1m/s,\) and finally \(\theta = 1^\circ, \phi = 2^\circ\) and \(\psi = 1.5^\circ.\) The final value for the desired altitude is \(z_d = 0.7m,\) the desired forward velocity is \(\dot{x}_d = 0.3\) m/s, while \(\psi_d\) is computed in online from the difference between the UAV heading and the road orientation.

VII. REAL TIME EXPERIMENTS

The control strategies proposed were validated in real-time experiments, consisting on a vision-based road following mission. For details concerning the UAV platform see [19]. The quad-rotor starts on-ground over the road, in a position that is considered as the origin of the inertial reference frame \(I.\) An autonomous take-off is performed, where the goal is to achieve a desired altitude of 0.70 m over the road. At this height the helicopter can detect the road, which allows orientating its heading and stabilizing its lateral position. The vehicle is then required to navigate forward at a desired speed of 0.3 m/s, while regulating its heading angle and lateral position w.r.t. the road. Once the vehicle is near to the end of the road segment, the landing is performed autonomously.

Fig. 9 shows the quad-rotor’s \((x, y, z)\) position w.r.t. the road during the experiment. The upper graphic represents the translational displacement described by the quad-rotor along the longitudinal axis of the road \((x_\tau),\) which was computed by integrating the optical flow sensed in the forward direction. The middle graphic represents the quad-rotor lateral position w.r.t. the road. Finally, the lower graphic represents
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