Grid Integration of Distributed Renewables through Coordinated Demand Response

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Abstract—There is a growing interest in developing solutions to facilitate large scale integration of distributed renewable energy resources and, in particular, contain the adverse effects of their volatility. In this paper, we introduce a neighborhood-level demand response program that aims at coordinating the Home Energy Management Systems (HEMS) of residential customers in order to opportunistically consume spikes of locally generated renewable energy. We refer to this technique as Coordinated Home Energy Management (CoHEM). Our model predictive control technique modulates the aggregate load to follow a dynamically forecasted generation supply. Both centralized and decentralized deployments of CoHEM are considered. The decentralized version requires a more demanding communication backbone to connect individual HEMS but, it is more resilient to failures of individual computational units or communication links and, compared to the centralized model, it preserves consumers privacy. In our numerical results section, we compare the scenario where individual HEMS optimize their energy use selfishly, under a hypothetical dynamic pricing program, to the performance of the centralized and decentralized versions of our proposed CoHEM architecture. The results highlight the advantages of using the CoHEM model in absorbing the fluctuations in the generation output of distributed renewables.

I. INTRODUCTION

Extensive incentives, such as the federal residential renewable energy tax credit and net metering tariffs, have stimulated a considerable growth in the installation of residential and commercial renewable energy resources. That includes solar photovoltaics, small residential wind, and fuel cells and micro-turbine systems. They operate in three different modes:

- Grid-tied or grid-connected generators produce electricity for their owner and sell excessive electricity back to the grid, currently under plans known as Net Metering.
- Grid-tied generators with a battery bank back-up act the same as grid-tied units except for the fact that they have an on-site battery to compensate for their intermittency.
- Off-grid stand alone systems operate independent of the electrical grid, mainly by depending on a battery bank. These systems are most common in remote locations.

In this paper, we will mainly focus on introducing measures that can facilitate the grid integration of resources of the first type. The owner of these resources will have to buy power from the grid when the generator is not producing enough electricity. Also, generation surpluses are injected back into the grid. Thus, the grid is equivalent to a large battery that compensates for the intermittency and uncontrollability of the power output of local renewables. While this will not be an issue at small integration levels, this model is not sustainable if future 30–50% renewable integration goals are met [1].

To resolve this issue, many advocate solutions that can make electricity demand more elastic so that it can respond to the volatilities on the supply side and help ensure that demand and supply are balanced at all times, which is one of the most important operational requirements of the power grid. To address this problem, we introduce a new model for coordinated home energy management (CoHEM) so that the demand can follow volatile generation. Thus, an important aspect of our problem formulation is the specific network utility we define, with a cost that is lowered when the aggregate load approaches a desired Zero Incremental Cost (ZIC) profile over a certain horizon. The ZIC profile is the sum of the day-ahead bid of the neighborhood load serving entity/aggregator, plus the locally available renewables. Rather than delegating the task of managing volatility to the grid, with this utility, customers use their responsive loads to shape the aggregate demand to follow the renewable generation profile. To alleviate computational complexities associated with a centralized implementation of the CoHEM scheme, we also present a decentralized method which can reduce the required resources to implement such a program and protect the privacy of the participating customers. This work is based on a coordinated balancing technique we introduced in [4]. Here, we add renewables, storage and handle a broader class of controllable appliances.

Background: The idea of coordinated energy management systems has been advocated by several other authors, though with different goals and models for demand flexibility. In [3], a heuristic neighborhood-level energy management algorithm is proposed for scheduling the load of residential units such that the aggregate load has a peak specified by the retailer. In [5], a distributed energy management algorithm, based on a game-theoretic approach, was proposed to minimize the cost of the retailer as well as the peak-to-average ratio of the aggregate load. The work in [6] takes into account user dissatisfaction and proposes a distributed energy management algorithm that minimizes the cost of retailer and a cost that reflects the degree of user dissatisfaction. Other examples include the work presented in [7], which considers a general convex cost for the utility company and proposes a method to decentralize the load management problem, the work in [8], which assumes a cost function for the utility company that increases linearly if the load gets above a constant threshold.
and [9], which uses particle swarm optimization to coordinate DR resources. All the works in [5], [6] and [7] allowed the HEMS to optimize the load injection of appliances. However, in many cases, the appliances have fixed load profiles that cannot be altered.

II. INDIVIDUAL HEMS UNITS UNDER RTP

We assume that the appliances controllable by the HEMS can belong to two categories: deferrable, and heating and cooling loads. They are modeled next.

A. Deferrable Uninterruptible Appliances

We denote this group of appliances by the set $\mathcal{U}$. We will assume that the HEMS will have the authority to change the time at which appliances in $\mathcal{U}$ turn on but it will not modify their loads. This assumption is consistent with the operations of various high-wattage devices such as washing machines, dryers, etc. and it is a preferable choice for electric vehicles (EV). To model the stochastic nature of the electricity demand of these type of household appliances, we use a probabilistic request arrival model. We assume that the $i$-th appliance in the residence has $J_i$ different operating modes. A simple example of different operating modes is that of EVs, for which the mode can represent the amount of charge needed by the car upon its arrival. The customer will request to use the $i$-th appliance in $\mathcal{U}$ under operating mode $j$ at time $t_{i,j,k}, t_{i,j,2}, \ldots \in \{0, \ldots, L\}$, where $0$ denotes the current time and $L > 0$ is the look-ahead horizon of the optimization.\(^1\) Thus, we can describe the requests for appliance $i$ to be operated at mode $j$ as an arrival process, increased by one step with each arrival:

\[
a_{i,j}(\ell) = \sum_{k=1}^{\infty} u(\ell - t_{i,j,k}), \quad \ell = 0, \ldots, L, \tag{1}
\]

where $u(\cdot)$ is the unit step function. To model the customer’s behavior in requesting service from appliance $i$, we assume that the arrival process $a_{i,j}(\ell)$ is a non-stationary random process with the average number of new arrivals at time $\ell$ being $\alpha_{i,j,\ell}$, i.e., $E\{a_{i,j}(\ell) - a_{i,j}(\ell - 1)\} = \alpha_{i,j,\ell}$. For example, one may model $a_{i,j}(\ell) - a_{i,j}(\ell - 1)$ as a binary r.v., with $\alpha_{i,j,\ell}$ being the probability that appliance $i$ will be requested at time $\ell$ under operating mode $j$. $\alpha_{i,j,\ell}$’s are updated dynamically as new information becomes available about the statistics of arrival.

Remark 2.1: As an example for calculating the distribution of the $\alpha_{i,j,\ell}$’s, we look at the case of an electric vehicle returning home and requiring charge. Before a charge request is sent to the HEMS, the statistics of arrivals can be estimated based on the historical behavior of the customer, i.e., if we denote the probability of a customer returning home and requesting charge at time $\ell$ as $P_r(t^n = \ell)$, we can write

\[
Pr(a_{i,j,\ell} - a_{i,j,\ell-1} = 1) = Pr(t^n = \ell)Pr(\text{charge mode} = j).
\]

After a charge request happens, these probabilities will be updated accordingly. For example, on a normal day, the probability that another charge will be requested can be safely set to zero for all modes.

When the $i$-th type of appliance belonging to this group arrives at time $t_{i,j,k}$, under mode $j$, we denote its associated discrete time load profile by a pulse $g_{i,j}(\ell), \ell = 0, \ldots, G_{i,j}$ if the appliance is turned on at time $0$. $G_{i,j} > 0$ is the duration of an energy request by appliance $i$ in mode $j$. For example, in the simple case of EVs, the $g_{i,j}(\ell)$’s would be rectangular pulses with different durations and amplitudes corresponding to different charge lengths and levels. If energy is provided to appliances immediately following their request, the load due to the $i$-th appliance in mode $j$ is:

\[
L_{i,j}(\ell) = \sum_{k=1}^{\infty} g_{i,j}(\ell - t_{i,j,k}), \quad \ell = 0, \ldots, L. \tag{2}
\]

Suppose that $s_{i,j,1}, s_{i,j,2}, \ldots \in \{0, \ldots, L\}$ are the scheduled times at which appliance $i$ will turn on. Here, we assume that the customer assigns a maximum tolerable delay for each appliance, which we denote by $\zeta_i \geq 0$. So, we need to have $t_{i,j,k} \leq s_{i,j,k} \leq t_{i,j,k} + \zeta_i$ for all $k$. The scheduled load of appliance $i$ in mode $j$ is given by:

\[
S_{i,j}(\ell) = \sum_{k=1}^{s_{i,j}(\ell)} g_{i,j}(\ell - s_{i,j,k}), \quad \ell = 0, \ldots, L. \tag{3}
\]

Similarly, the operating times of appliance $i$ can also be described by a task departure (launching) process as

\[
d_{i,j}(\ell) = \sum_{k=1}^{s_{i,j}(\ell) - 1} u(\ell - s_{i,j,k}), \quad \ell = 0, \ldots, L. \tag{4}
\]

Alternatively, we can write (3) as the convolution of the launching process $d_{i,j}$ with the load pulse $g_{i,j}$:

\[
S_{i,j}(\ell) = \sum_{t=0}^{\ell} [d_{i,j}(t) - d_{i,j}(\ell - 1)]g_{i,j}(\ell - t). \tag{5}
\]

In this case, the optimization variable changes from the launching times $s_{i,j}$ (the load is a non-linear function of which) to the departure (launching) process $d_{i,j}(\ell)$, which describes the load through the linear mapping (5).

B. Heating and Cooling Devices

We denote this group of appliances by the set $\mathcal{I}$. To optimize the energy use of these devices, we need an accurate enough model with learning abilities to forecast their future energy use. Here, we adopt a model proposed in [10] to explain the state dynamics of these appliances. Let $x_i(\ell)$ (a representative temperature) characterize the state of the $i$-th device; $x_i(\ell)$ evolves according to the linear stochastic difference equation

\[
x_i(\ell) - x_i(\ell - 1) = -k_i(x_i(\ell - 1) - x_0(\ell - 1)) + W_i b_i(\ell) + \nu_i(\ell),
\]

where we denote by

- $k_i$, the average loss rate for the building;
- $x_0(\ell)$ the ambient temperature;
- $W_i$ the rate of heat/cold gain supplied by the device;
- $b_i(\ell)$ the operating state of the device at time $\ell$ (1 for "on" or 0 for "off");
\[ \nu_i(\ell) \text{ a zero mean Wiener noise process.} \]

If we know the current value of state, i.e., \( x_i(0) \), we can write the temporal evolution of \( x_i(\ell) \) as

\[ x_i(\ell) = c_i(\ell) + \sum_{t=0}^{\ell-1} (1 - k_i)^{\ell-t-1} W_i b_i(t) \]  

(6)

with \( c_i(\ell) = \sum_{t=0}^{\ell-1} (1 - k_i)^{\ell-t-1} (x_i(0) (1 - k_i) + k_i x_a(t) + \nu_i(t)) \). The \( b_i(\ell) \)'s are the decision variables for the HEMS.

The goal of the HEMS is to keep the temperature inside a comfort band \([x_i^{\min}, x_i^{\max}]\) set by the customer.

### C. Online Job Scheduling Using Discrete Time MPC

The total demand of a residence is the summation of the controllable load due to appliances in \( \mathcal{U} \) and \( \mathcal{I} \) and the uncontrollable load (e.g., hairdryers):

\[ D_{\text{total}}(\ell) = U(\ell) + \sum_{i \in \mathcal{U}} \sum_{j=1}^{I_i} S_{i,j}(\ell) + \sum_{i \in \mathcal{I}} W_i b_i(\ell), \]  

(7)

where \( U(\ell) \) is the load due to uncontrollable appliances. Note that since we choose not to interrupt appliance’s jobs once they are scheduled to start it, the load due to previously scheduled appliances is added to \( U(\ell) \) after each epoch.

We will now look at the optimization that individual HEMS units need to solve under RTP. Here, we assume that every agent is selfish and only cares about minimizing its own cost. We assume that every household can own a distributed energy resource that is grid-connected under a net-metering tariff. Currently, policies to compensate customers for their net excess generation varies from state to state. To account for a general case, we assume the customer is paid at a rate \( i(\ell) \) per kWh of energy injected into the grid at time \( \ell \). For example, California credits the excess kWhs to customer’s next bill at a rate equal to the average spot market price for the hours of 7 am to 5 pm. Thus, \( i(\ell) \) will be constant in this case. Also, under dynamic pricing, the customer pays a price \( p(\ell) \) for each kWh consumed from the grid at time \( \ell \).

The HEMS unit at each residence aims at minimizing the average electricity cost of the household by optimizing the time at which appliances turn on. The scheduling is subject to deadline constraint that reflects the customer’s flexibility. The available amount of power that is generated at time \( \ell \) by the distributed energy resource is denoted by \( R(\ell) \). However, only rough predictions of the future values of \( R(\ell) \) are available to the decision unit. Also, as discussed previously, the decision unit is uncertain about future energy requests and only has access to their statistics. Since the cost incurred by the residence in future epochs is tied to the scheduling decisions made at the current epoch, a myopic decision making strategy can lead to great losses for the customer. Thus, we choose to use a model predictive control (MPC) model and include the effects of future decisions in our cost. Consequently, at each epoch, the HEMS solves a new stochastic optimization, with a look-ahead horizon of \( L \) epochs in a rolling window:

\[
\min_{d_{i,j}(\ell), b_i(\ell)} \sum_{\ell=0}^{L} \mathbb{E}\{p(\ell) (D_{\text{total}}(\ell) - R(\ell))^+ - i(\ell) (R(\ell) - D_{\text{total}}(\ell))^+) \}
\]

(8a)

s.t. \( D_{\text{total}}(\ell) \) in (7), \( S_{i,j}(\ell) \) in (5),

\[
d_{i,j}(\ell) \in \mathcal{D}, \quad b_i(\ell) \in \mathcal{B}, \quad \forall \ i,j, \ell
\]

(8b)

where \( (D_{\text{total}}(\ell) - R(\ell))^+ = \max\{D_{\text{total}}(\ell) - R(\ell), 0\} \); \( \mathcal{D} \) describes the set of departure processes that satisfy the following constraints:

\[
\mathcal{D} = \left\{ d_{i,j}(\ell) \mid i \in \mathcal{U}, \ a_{i,j}(\ell - \zeta_i) \leq d_{i,j}(\ell), \ d_{i,j}(\ell) = a_{i,j}(\ell) \quad \forall \ \ell \geq L - G_{i,j}, \right\}
\]

(9)

and \( \mathcal{B} \) describes the set of possible control actions for the heating and cooling devices such that

\[
\mathcal{B} = \left\{ b_i(\ell) \mid i \in \mathcal{I}, \ x_i^{\min} - c_i(\ell) \leq \sum_{t=0}^{\ell-1} (1 - k_i)^{\ell-t-1} W_i b_i(t), \right. \\
\left. \sum_{t=0}^{\ell-1} (1 - k_i)^{\ell-t-1} W_i b_i(t) \leq x_i^{\max} - c_i(\ell), \right. \\
\left. b_i(\ell) \in \{0, 1\} \right\}
\]

(10)

The second constraint in \( \mathcal{D} \) ensures that the appliance will be turned on no later than the specified tolerable delay \( \zeta_i \); the third constraint implies that all the appliances have to finish their jobs before \( L \) if possible. The reason for this is twofold: that the algorithm does not delay appliances beyond its look-ahead horizon with the false hope of having a zero cost for their electricity consumption and, that the comparisons made in the simulation results are fair.

Note that, under MPC, we will only commit to the optimal decision variable obtained for the current time (epoch 0) and we will discard the rest of them, i.e., \( d_{i,j}(\ell) \) and \( b(\ell) \) for \( \ell > 1 \). After determining these optimal values, the deferrable load scheduled at the current epoch due to the appliances in \( \mathcal{U} \) is added to the uncontrollable part of the load for the future, i.e. \( U(\ell) \) is updated for \( \ell \geq 1 \). (8) is solved again at the next epoch with dynamic updates on \( a_{i,j}(\ell) \) and \( x_i(\ell) \).

### D. Certainty Equivalence

The optimization problem in (8) can be solved by a standard DP approach, i.e., using the principle of optimality [11]. However, due to the non-stationary and high-dimensional nature of the problem, this approach will be computationally intensive. Instead, here we choose to apply a certainty equivalent control model, for which we replace the stochastic values in (8) with their expected future values, i.e., we respectively replace \( R(\ell), U(\ell), a_{i,j}(\ell) \) and \( \nu_i(\ell) \) with \( \bar{R}(\ell) = \mathbb{E}\{R(\ell)|R(0)\}, \ \bar{U}(\ell) = \mathbb{E}\{U(\ell)|U(0)\}, \ a_{i,j}(\ell), \) and zero. Thus, at each epoch, we use dynamically updated predictions of future stochastic values. This approximation often works quite well for solving stochastic decision making problems.

Define the set of slack variables \( z_\ell = (D_{\text{total}}(\ell) - R(\ell))^+ \).
If we assume that \( p(\ell) - i(\ell) \geq 0 \), which seems logical due to the common bid-ask spread in many existing markets, we can rewrite the certainty equivalent version of (8) as an MILP:

\[
\begin{align*}
\min_{d_{i,j}(\ell), b_{i,j}(\ell), z_\ell} & \quad \sum_{\ell=0}^L (p(\ell) - i(\ell)) z_\ell + i(\ell) \left( D_{\text{total}}(\ell) - \bar{R}(\ell) \right) \\
\text{s.t.} & \quad z_\ell \geq 0, \quad D_{\text{total}}(\ell) - \bar{R}(\ell) \leq z_\ell, \quad \forall \ell \tag{11a} \\
\end{align*}
\]

\[
\begin{align*}
& \text{constraints (8b) - (8c).} \tag{11b}
\end{align*}
\]

### III. COORDINATED HOME ENERGY MANAGEMENT

Under pre-determined net metering tariffs, the grid can only support a small level of renewable penetration without considerably increasing reserve requirements [12]. It can be argued that this can be improved by dynamic pricing tariffs. However, this is a highly complicated problem due to many factors such as the highly non-stationary nature of renewables; the fact that customers are much less predictable on dynamic tariffs; and the legitimate expectation that prices should be provided a few hours beforehand to allow for planning. As shown in the numerical results section later, simplistically designed price signals will worsen the supply/demand imbalance due to rebound peaks [3].

In this section, we will propose a centralized solution that can coordinate the HEMS units of a neighborhood in order for the aggregate demand to follow a certain non-stationary power profile (which constitutes of the locally generated grid-connected renewables + the day-ahead bid). In our scheme, we assume that customers’ comfort settings will be honored by the central control unit. Thus, given appropriate financial incentives, the HEMS units will act as unselshful agents that allocate their resources to maximize the social welfare.

To look at a general case, here we will assume that the neighborhood control hub also has control over \( C \) community-owned energy storage (CES) units. We will denote by \( h^{(i)}(\ell), q^{(j)}(\ell), c^{(j)}, r_c^{(j)} \) and \( r_d^{(j)} \), the state of charge, the power drawn by (\( < 0 \) for discharging), the capacity and the charge and discharge rate of the \( j \)-th storage unit. Also, the demand parameters defined previously will each have a superscript \( (m) \) to denote that of the \( m \)-th residence.

There is a major difference between the optimization solved in this case with the one solved by the individual HEMS units. The control hub here is a wholesale market participant and has access to real-time data about the actual price of electricity that is being consumed in the neighborhood. Accordingly, there is a forward bid that the control hub has placed in the day-ahead market for each hour of the next day and has already paid for it. We denote the power purchased for decision epoch \( \ell \) on a forward basis by \( F(\ell) \).

Also, for simplicity of notation, and since we will not take flow constraints into account for now, we will denote by the same variable \( \bar{R}(\ell) \) the adaptively updated forecast of the aggregated amount of renewable generation by all the locally owned distributed units. The goal of the control hub will be to minimize its real-time balancing cost. Consequently, if we denote by \( \pi_p(\ell) \) and \( \pi_s(\ell) \) the forecasted wholesale upward and downward\(^2\) balancing prices, the optimization solved online by the control hub that covers \( M \) residences is

\[
\begin{align*}
& \min_{d_{i,j}(\ell), b_{i,j}(\ell), \pi_p(\ell), \pi_s(\ell)} \quad \sum_{\ell=0}^L (\pi_p(\ell) - \pi_s(\ell)) V(\ell) \\
& \text{s.t.} \quad 0 \leq \lambda_\ell \leq \pi_p(\ell) + \pi_s(\ell) \quad \forall \ell. \tag{12a}
\end{align*}
\]

\[
\begin{align*}
& \text{for each residence } m, \tag{12b} \\
& q^{(j)}(\ell) \in Q^j, \quad j = 1, \ldots, C. \tag{12c}
\end{align*}
\]

In (12), \( V(\ell) \) denotes the deviation of demand from the already available (ZIC) energy supply \( \bar{R}(\ell) + F(\ell) \), and

\[
Q^j = \left\{ q^{(j)}(\ell) \left| q^{(j)}(\ell) \leq \min\{c^{(j)}(\ell) - h^{(j)}(\ell), r_d^{(j)}(\ell)\}, \quad -q^{(j)}(\ell) \leq \min\{h^{(j)}(\ell), r_c^{(j)}(\ell)\}, \quad h^{(j)}(\ell) + 1 = h^{(j)}(\ell) + q^{(j)}(\ell) \right\}. \right.
\]

Note that, by certainty equivalence, (12) can be approximated by a MILP using the same trick used for problem (11), i.e., by defining \( z_\ell = [V(\ell)]^+ \), we can write

\[
\begin{align*}
& \min_{d_{i,j}(\ell), b_{i,j}(\ell), \pi_p(\ell), \pi_s(\ell)} \quad \sum_{\ell=0}^L (\pi_p(\ell) + \pi_s(\ell)) z_\ell - \pi_s(\ell) V(\ell) \\
& \text{s.t.} \quad z_\ell \geq 0, \quad V(\ell) \leq z_\ell \forall \ell, \tag{13a} \\
& \text{constraints (12b) - (12c).} \tag{13b}
\end{align*}
\]

### IV. DISTRIBUTED COHEM

A centralized solution of (12) requires the following to be communicated to the control center and back to the residences and storage devices: all the appliance arrival data; predictions of \( \alpha^{(m)} \); individual comfort settings; and real time energy storage status updates from CES units. This will be faced with challenges in large-scale scenarios. Consequently, here we look at an alternative distributed implementation method based on dual decomposition [13].

Let the set of variables \( \mu_\ell \geq 0 \) and \( \lambda_\ell \geq 0 \) be the dual variables associated with the inequality constraints at time \( \ell \) in (13a). Then, the associated dual problem of (13) is [14],

\[
\max_{\mu_\ell \geq 0, \lambda_\ell \geq 0} \phi(\mu_\ell, \lambda_\ell) \tag{14}
\]

where \( \phi(\mu_\ell, \lambda_\ell) \), known as the Lagrange dual function, is given by the minimum value of the Lagrangian and can be easily shown to be:

\[
\min_{\substack{d_{i,j}(\ell) \in D^{(m)}, b_{i,j}(\ell) \in B^{(m)}, q^{(j)}(\ell) \in Q^{(j)}}} \left\{ \sum_{\ell=0}^L (\lambda_\ell - \pi_s(\ell)) V(\ell), \right. \tag{15} \]

- \( \infty \), if \( \pi_p(\ell) + \pi_s(\ell) - \lambda_\ell = \mu_\ell \forall \ell \),
- elsewhere.

Hence the dual problem (14) is given by

\[
\max_{\lambda_\ell \forall \ell} \left\{ \min_{\substack{d_{i,j}(\ell) \in D^{(m)}, b_{i,j}(\ell) \in B^{(m)}, q^{(j)}(\ell) \in Q^{(j)}}} \sum_{\ell=0}^L (\lambda_\ell - \pi_s(\ell)) V(\ell) \right\} \tag{16}
\]

\[
\text{s.t. } 0 \leq \lambda_\ell \leq \pi_p(\ell) + \pi_s(\ell) \forall \ell.
\]

\(^2\)prices for buying or selling electricity in the real-time market.
If we relax the integrality constraints, (13) is a convex problem for which Slater’s condition [14] holds (since scheduling appliances right at the time they arrive is always a feasible solution). Thus, there is no duality gap between (16) and the relaxed version of (13). As an alternative, we can apply the standard subgradient method by iteratively solving the inner and outer problems of (16).

We use the above formulation to provide a distributed scheme to solve (16). One can see from (12a) that the inner part of (16) can be separated into two parts: optimization of \( \sum_{m=1}^{M} D_{\text{total}}^{(m)} \) and optimization of \( \sum_{j=1}^{C} q^{(j)}(\ell) \), each of which can be further decomposed into \( M \) and \( C \) subproblems. Thus, given \( \lambda_{\ell}(n) \) during the \( n \)-th iteration, each household \( m \) can solve for its optimal consumption by

\[
\begin{align*}
\min_{d_{(\ell),j}^{(m)} \in D_{(\ell),j}^{(m)}} & \sum_{\ell=0}^{L} (\lambda_{\ell}(n) - \pi_{s}(\ell)) D_{\text{total}}^{(m)}(\ell), \\
\min_{b_{(\ell),j}^{(m)} \in B_{(\ell),j}^{(m)}} & \sum_{\ell=0}^{L} (\lambda_{\ell}(n) - \pi_{s}(\ell)) q^{(j)}(\ell).
\end{align*}
\]

Similarly, given \( \lambda_{\ell}(n) \), the \( j \)-th storage device can find its optimal charge and discharge strategy by

\[
\begin{align*}
\min_{q^{(j)}(\ell) \in Q^{(j)}} & \sum_{\ell=0}^{L} (\lambda_{\ell}(n) - \pi_{s}(\ell)) q^{(j)}(\ell).
\end{align*}
\]

Let \( D_{\text{total}}^{(m)}(\ell; n) \) and \( q^{(j)}(\ell; n) \) be the optimal counterparts of \( D_{\text{total}}^{(m)}(\ell) \) and \( q^{(j)}(\ell) \) in (17) and (18) during the \( n \)-th iteration. Then given \( R(\ell) + F(\ell) \), the dual variable \( \lambda_{\ell}(n) \) can be updated as

\[
\lambda_{\ell}(n+1) = P \left( \lambda_{\ell}(n) + c_{n} \left( \sum_{m=1}^{M} D_{\text{total}}^{(m)}(\ell; n) + \sum_{j=1}^{C} q^{(j)}(\ell; n) - R(\ell) - F(\ell) \right) \right),
\]

for \( \ell = 0, \ldots, L \), where \( c_{n} > 0 \) is the step size and \( P(\cdot) \) denotes the operation of projection onto the set \([0, \pi_{s}(\ell) + \pi_{s}(\ell)]\). These iterations are repeated until we reach convergence or the preset stopping criterion is satisfied. Suppose that the algorithm stops at iteration \( n^{*} \). Instead of using \( \{d_{(\ell),j}^{(m)}(\ell; n^{*}), b_{(\ell),j}^{(m)}(\ell; n^{*}), q^{(j)}(\ell; n^{*})\} \) as the primal solution, we can use the running-averaged version, which, by taking \( q^{(j)}(\ell) \) as an example, is given by

\[
\hat{q}^{(j)}(\ell) = \frac{1}{n^{*}} + 1 \sum_{n=1}^{n^{*}} q^{(j)}(\ell; n).
\]

It is shown [15] that the averaged version is more numerically stable, especially for non-strictly convex problems like ours.

The dual updates in (19) can be carried out centrally. A centralized update will need to gather information about the scheduled demand of all the households at every step. Alternatively, we can apply consensus averaging or gossiping algorithms [16], [17] among the households to achieve consensus on the aggregate load \( \sum_{m=1}^{M} D_{\text{total}}^{(m)}(\ell; n) \) enabling the subgradient update step (19) to be independently performed by each household in a fully decentralized fashion.

V. Numerical Results

In order to simulate the output of distributed renewable resources, we used NREL’s PVWatts calculator [18], which simulates the energy production of grid-connected photovoltaic (PV) systems. We used several PV outputs in the city of Charleston, WV on 09/01/88. Since our MPC schemes all require predictions of the available renewable generation for the look-ahead horizon, we had to choose a time-series model to generate these forecasts. Thus, after removing the sample mean of the hourly series \( R(\ell) \), we applied a period autoregressive PAR(3) model with a period of 24 hours (representing daily seasonabilities) to the series [19]. A PAR(\( p \)) model with period \( M \), applied to a series \( R(\ell) \), is given by:

\[
\sum_{i=0}^{P} \phi_{i}^{(m)} R(kM + m - i) = \epsilon(kM + m), \quad 1 \leq m \leq M,
\]

where \( \epsilon(\cdot) \) is a white process. Here, the coefficients \( \phi_{i}^{(m)} \) were calculated using the periodic Yule Walker equation.

We chose to run the optimization every 15 minutes, with a look-ahead horizon of 5 hours. The deferrable appliances controlled by the algorithm are EVs owned by 150 residences. Vehicles follow a random non-homogeneous arrival process and, upon arrival, they request a random amount of charge with a maximum length of 4 hours (independent, uniformly distributed random variables). While on, the appliances had a constant consumption of 3 kW. The vehicles were assumed to be flexible in their charging deadlines by a random amount (assumed to be independent and uniformly distributed between 1 to 2.5 hours). To simulate the effect of locally owned renewables, we equipped 30 of the houses with PV systems (fixed tilt arrays), all with a DC rating of 4kW and DC to AC derate factor of 0.77. The day-ahead bids were generated by averaging various realizations of the uncontrollable load, minus the predicted value of the solar generation (which can have a 30 – 40% error the day before). Lastly, wholesale balancing prices were modeled as homogeneous so as to make the results easily interpretable (upward prices were four times greater than downward prices).

In the first part of our simulations, we tested the performance of the algorithm presented in Section II. Thus, houses were provided with real-time price signals that mirrored the congestion status of the grid. We chose to model these prices as inversely proportional to the amount of generation available at no cost to the aggregator (the ZIC profile). Also, owners of renewables were compensated at 10 cents per kWh of excess energy that they produced. Figure 1.a compares the unscheduled load to the modified load. As seen, these simplistic price signals merely worsen the situation by creating rebound peaks during the lower priced epochs.

The second simulation looks at the performance of the COHEM algorithm. Here, the aggregator is assumed to have access to dynamically updated forecasts of the renewable generation, provided through (21) (which reduces the prediction error to 10 – 15%). Figure 1.b compares the load profile of the aggregator in the following two cases: 1) charging controlled by COHEM 2) charging controlled by
COHEM with 50 kWhs of storage capacity available to the scheduler. As expected, the load follows the ZIC profile much closely under COHEM. However, an interesting observation is that the presence of storage does not lead to a considerable improvement and deferrable loads alone can act as assets that can compensate for the variability present in the supply and help the grid balance.

Fig.1.c looks at the performance of the decentralized implementation presented in Section IV and compares it to that of the centralized version. The iterations for each decision epoch were stopped under the following criteria: either the duality gap is less than 0.1 or the number of iterations reached a maximum number of 150. The dual variables were updated in each iteration using average consensus.

It is noteworthy to mention that the main difficulty arising in the simulations is due to the integrality constraints $d_{i,j}(t) \in \mathbb{Z}_+$ and $b_i(t) \in \{0,1\}$. Here, we simply ignored the former and relax the latter to the box constraint $0 \leq b_i(t) \leq 1$. The solutions were then projected into the original integer sets as an approximate solution to (11). However, the resulting error accumulates with more appliances added. This can be improved by using more complicated discrete optimization techniques. But, remember that with the integrality constraints imposed, the argument that there is no duality gap between (16) and (13) is no longer valid and, the level of sub-optimality of the distributed solution should be investigated.

VI. CONCLUSIONS AND FUTURE WORK

As we saw, implementing centralized demand control strategies requires a strong communication backbone and computational resources. Thus, our next step would be to find efficient implementation techniques for the proposed decentralized version of the CoHEM algorithm and assess convergence rates.

REFERENCES


