Finite-Time Convergent Observer Design and Adaptive Control of a Nonlinear Boiler System

Johnathan Votion¹, Chuanlin Zhang²,¹, Chunjiang Qian¹, and Shihua Li²

Abstract—In this paper, based on recently developed finite-time convergent observer designs with time-varying coefficients, an observer is first designed to yield the explicit values of unknown parameters of a nonlinear boiler system. Then an adaptive controller is designed to regulate the states of the boiler system. Compared to existing results, the contribution of the paper is two-fold: (i) the new approach is able to handle the case when all parameters are unknown; (ii) faster convergence rates can be achieved to identify the unknown parameters and regulate the boiler system. Computer simulations are conducted to show the effectiveness of the proposed adaptive control schemes.

I. INTRODUCTION

In the nonlinear control community, the finite-time stabilization problem has attracted increasing attention since the 1990s. As presently known, finite-time convergence means that the states of system converge to their equilibrium in a finite time. As pointed out in [3], finite-time convergence implies that the dynamic system does not satisfy Lipschitz continuity. So continuous but non-smooth control via state or dynamic output feedback can provide a finite-time convergence rate for the equilibrium of the system. These closed-loop systems are of interest not only because of faster convergence rates around the equilibrium, but also because they can perform better disturbance rejection capabilities as shown from papers [1], [2], [3].

Due to the aforementioned benefits, abundant theoretical results and various design tools for finite-time control have been developed in recent years. First, for double integrator systems, the work [2] proposes a homogeneous state feedback control method which has further inspired many works in this direction. For high order systems, because the non-Lipschitz feature of the control system, the well-known backstepping (also called adding a linear integrator) strategy is not applicable. A new tool called adding a power integrator technique was first presented in [11], [4] and later extended in [8], [5], [6], etc. More significantly, the development of a homogenous approach plays an important role for the finite-time convergent controller design. In [7], a generalized homogeneous controller which expands many former works, including those in the area of finite-time stabilization, is proposed.

When compared to the various finite-time stabilization results using state feedback, the results of finite-time output feedback control are relatively fewer. Some interesting results about finite-time stabilization using output feedback were proposed based on homogeneous systems theory. In particular, second dimensional case was studied in [12], [6]. For the problem of designing an n-dimensional finite-time convergent observer, some design methods based on homogeneous systems theory can be found in [14], [15], [13], [16] and the references therein.

It is of practical value to use finite-time convergent observers since finite-time convergence enables us to obtain the exact values of the unmeasurable states after a finite time. This is in sharp contrast to the conventional linear based observers whose estimated states converge to the real states asymptotically (in infinite time). Also, finite-time convergent observers have been applied to some practical systems. For example, the work performed in [9] applies the finite-time convergent observers to a planar vertical take-off and landing aircraft (PVTOL), and a finite-time convergent output feedback controller is designed for a novel quad-rotor mini-aircraft in [10].

However, these continuous finite-time convergent observers mentioned above are only applicable to those systems with constant coefficients, and thus pose a limitation for their applications in practice where many nonlinear systems contain time-varying coefficients. For instance, time-varying functions are needed to be addressed in the nonlinear drum boiler model [18]. This paper intends to fill the gap in research and results surrounding finite-time convergent observers for nonlinear systems with time-varying coefficients. In this paper, a finite-time convergent observer for a class of nonlinear systems with time-varying constants and bounded states is first developed. Then a drum-boiler system will be considered as an application of the finite-time convergent observer.

The main objective of the boiler-turbine system control is to meet the load demand of electric power while maintaining the pressure and water level in the drum within tolerances. As is already known in real practices, some or even all of the parameters for a boiler turbine model are subject to change. Firstly, it is difficult to construct an exact model for the boiler system due to its complexities. Moreover, even if an exact model is acquired at a certain operating point, its parameters will still vary while the operating
point is changing. The variation of the system parameters would reduce the performance of pre-designed controllers. So an adaptive controller should naturally be considered in order to meet the control performance requirements with unknown system parameters [19]. To design a controller, the boiler-turbine system is usually modeled as a multi-input, multi-output (MIMO) nonlinear system, in which PID controls have been conventionally used. In recent years, the development of nonlinear control theory has made the applications to power plant systems possible and many nonlinear control techniques have been adopted (see [20], [26], [22], [23], [24], [25], and other references therein).

In this paper, we will further take advantage of finite-time convergent observers by applying them in the application of boiler-turbine systems. We will design finite-time convergent observers to estimate the parameters of the drum boiler system with all the parameters being unknown. In addition, an adaptive control scheme will be developed to show that we can regulate the pressures of the boiler systems to the desired pressures. Computer simulations will be conducted to show the performance of the proposed finite-time observer and adaptive controller.

II. PROBLEM STATEMENT

As depicted in Fig.1, the boiler model considered in this paper is a relatively simple nonlinear drum-boiler turbine model, which is capable of capturing the key dynamic features of the power plant. The mathematical model is described as follows [18]

\[
\begin{bmatrix}
\dot{p}_1 \\
\dot{p}_2
\end{bmatrix} =
\begin{bmatrix}
c_1 u_1(t) - c_2 p_1(t) 1.125 u_2(t) \\
c_3 p_1(t) 1.125 u_2(t) - c_4 p_2(t)
\end{bmatrix}
\]

(1)

where \( p_1(t) \) is the drum pressure in kg/cm\(^2\), \( p_2(t) \) is the reheater pressure in kg/cm\(^2\), \( u_1(t) \) is the normalized fuel mass flow rate with the range \([0,1]\) corresponding to \([0,70] \) tons/hour, and \( u_2(t) \) is the control valve position with a range \([0,1]\), 0 being that the valve is completely closed and 1 being that the valve is completely open.

The objective of this paper is to design controllers which can drive pressures \( p_1 \) and \( p_2 \) to their corresponding desired values. In practice, it is very common that the parameters \( c_i \), \( i = 1,2,3,4 \) might not be exactly known. It is also possible that these parameters might shift from one value to another due to changes in the operation conditions. Therefore, it is of paramount importance to design a controller adaptive to different parameters and regulating the states without off-line adjustment.

In a previous paper [22], an adaptive controller has been designed when partial parameters, e.g., two out of the four parameters are unknown. But the approach in [22] is not applicable when all four parameters are unknown. Later work presented in [26] provides a solution to identify all the unknown parameters using the least square method. However, the least square approach in [26] is an off-line approach which cannot provide a fast solution to identify the changing parameters.

In this paper, we will develop a new scheme based on the recently developed finite-time convergent observer with time-varying coefficients to identify the unknown parameters in real-time. Moreover, the estimates converge to the real parameters in a finite time. Using the identified parameters, an adaptive controller will be developed to regulate the states \( p_1 \) and \( p_2 \) to their desired pressure. The fast convergence rate of the adaptive controller will help improve the response speed of the power generation system.

III. FINITE-TIME CONVERGENT OBSERVER WITH TIME VARYING COEFFICIENTS

Consider the following system

\[
\begin{align*}
\dot{x}_1 &= \varphi_1(t,x_1)x_2 + f_1(t,x_1,u) \\
&\quad \vdots \\
\dot{x}_{n-1} &= \varphi_{n-1}(t,x_1)x_n + f_{n-1}(t,x_1,\cdots,x_{n-1},u) \\
\dot{x}_n &= \varphi_n(t,x_1)u + f_n(t,x_1,\cdots,x_n,u) \\
y &= x_1,
\end{align*}
\]

(2)

where \( x = (x_1,\cdots,x_n)^T \in \mathbb{R}^n \) is system state, \( u \in \mathbb{R} \) is control input, and \( y \in \mathbb{R} \) is system output. The functions \( \varphi_i(t,x_1) \), \( i = 1,\cdots,n \), are continuous positive functions in the operating region. Moreover, for \( i = 1,2,\cdots,n \), the nonlinear \( f_i(t,x_1,\cdots,x_n,u) \) are continuously differentiable with respect to \( x_2,\cdots,x_i \) in the operating region.

Assumption 3.1: There exist a set of bounds \( l_i \geq 0 \) and \( h_i \geq 0, i = 1,\cdots,n \), such that the solution trajectory \( x(t,x_0) \) of the nonlinear system (2) with initial state \( x(0) = x_0 \) is well-defined over the interval \([0,\infty)\) and

\[
l_i \leq x_i(x_0,t) \leq h_i, \forall t \in [0,\infty).
\]

(3)

Remark 3.1: It is natural to assume Assumption 3.1 since many systems in the real world are operating in a region whose bounds are known and often positive. System (1) is one of such examples whose states and parameters have known lower and upper bounds.

Theorem 3.1: Under Assumption 3.1, for any homogeneous degree \( \tau \in (-1/n,0) \), there exist constants \( k_i \)'s and
\( L \geq 1 \) such that the states \((\hat{x}_1, \ldots, \hat{x}_n)\) of the following observer
\[
\dot{\hat{x}}_i = \phi_1(t, x_1) \hat{x}_2 + f_1(t, x_1, u) + \phi_1(t, x_1) L_{\hat{x}_i} \text{sig}^{m_{i+1}}(x_1 - \hat{x}_1), \quad i = 2, \ldots, n - 1,
\]
\[
\dot{\hat{x}}_n = \phi_n(t, x_1) u + f_n(t, x_1, \hat{x}_1, \ldots, \hat{x}_n, u) + \phi_n(t, x_1) L_{\hat{x}_6} \text{sig}^{m_{n+1}}(x_1 - \hat{x}_1),
\]
where \( \text{sat}(s) = \begin{cases} l_i, & s \leq l_i \\ h_i, & s \geq h_i \\ s, & l_i < s < h_i \end{cases} \), \( i = 1, \ldots, n \), will converge to the real states \((x_1, \ldots, x_n)\) of system (2) in a finite time.

**Proof.** First, by the continuity and positiveness of the functions \( \phi_i(t, y), i = 1, \ldots, n \) in the bound region \([l_i, h_i] \), we know that there are positive constants \( \alpha \) and \( \tilde{\alpha} \), such that
\[
0 < \alpha \leq \phi_i(t, y) \leq \tilde{\alpha}, \quad \forall t \geq 0, \quad \forall y \in [l_i, h_i] .
\]
In addition, with the new change of coordinates
\[
y_i = 2 \frac{x_i - h_i + l_i}{h_i - l_i},
\]
we know that
\[
|y_i| \leq 1.
\]
The relations (5)-(6) implies that the \( y \)-dynamical system satisfies the conditions of the main result in [17, 26] where a finite-time convergent observer can be constructed. Under the original coordinates, the observer is constructed in the form of (14) whose states \( \hat{x}_i \)’s will converge to the real values \( x_i \) in a finite time. The detailed proof is omitted here.

IV. PARAMETER IDENTIFICATION USING A FINITE-TIME CONVERGENT OBSERVER

In this section, we show how the finite-time convergent observer can be used for the parameter identification of (1).

From (1), there can easily be designed constant controllers \( u_1 \) and \( u_2 \) that will stabilize the boiler system in the desired states \( p_{d1} \) and \( p_{d2} \). The inputs from the constant controller can be written as
\[
u_2 = \tilde{c}_4 * p_{d2} / (\tilde{c}_3 * p_{d1}^{1.125})
\]
\[
u_1 = \tilde{c}_2 * u_2 * p_{d1}^{1.125} / \tilde{c}_1
\]
where \( \tilde{c}_i \) are the initial estimates of parameters \( c_i, \quad i = 1, 2, 3, 4 \).

To implement the finite-time convergent observer designed in section three, the system is redefined against a new set of variables \( x_i, \quad i = 1, 2, 3, 4, 5, 6 \). The reformulation of the system involves six substitutions that can be evenly divided among the two dynamic equations in system (1).

Substitutions made for the first dynamic of boiler system (1) are
\[
x_1 = p_1
\]
\[
x_2 = c_1 - c_2 x_1^{1.125} u_2^{2.125} / u_1
\]
\[
x_3 = -c_2 x_2.
\]
The substitutions in (9) permit the presence of a \( p_1 \)-subsystem for system (1) corresponding to \( p_1 \) and can be rewritten as the following
\[
\dot{x}_1 = u_1 x_2
\]
\[
\dot{x}_2 = 1.125 * u_2 x_1^{0.125} x_3
\]
\[
\dot{x}_3 = 1.125 * u_2 x_1^{0.125} x_3 / x_2.
\]
Similarly, a \( p_2 \)-subsystem can be produced for the second dynamic through the substitutions
\[
x_4 = p_2
\]
\[
x_5 = \theta_1 + \theta_2 x_2^{1.125} x_4
\]
\[
x_6 = 1.125 * \theta_2 x_1 x_2 - \theta_2 x_5 x_1
\]
The \( p_2 \)-subsystem is written as
\[
\dot{x}_4 = x_4 x_5
\]
\[
\dot{x}_5 = u_2 x_1^{0.125} x_4
\]
\[
\dot{x}_6 = T + U + V
\]
The variables used for simplification are defined as
\[
\theta_1 = -c_4
\]
\[
\theta_2 = c_3
\]
\[
S = 1.125 * u_2 x_1^{0.125}
\]
\[
T = -\theta_2 x_1 x_2
\]
\[
U = -\theta_2 x_1 x_2
\]
\[
V = -\theta_2 x_1 x_2
\]
Under the assumption that all the states \( x_i, \quad i = 1, 2, 3, 4, 5, 6 \) are bounded with known upper and lower bounds, we have two constants \( \bar{b}_i \) and \( \underline{b}_i \) such that \( \underline{b}_i \leq x_i \leq \bar{b}_i \). In addition, it is easy to verify that if all the states are positive, which can be ensured in the operation region, by Theorem 3.1, we can construct an observer having the following form
\[
\dot{x}_1 = u_1 \dot{x}_2 + u_1 L_{\hat{x}} \text{sig}^{m_{i+1}}(x_1 - \hat{x}_1)
\]
\[
\dot{x}_2 = \text{sat}(\hat{x}_3) \dot{x}_3 + S \dot{x}_3 + L_{\hat{x}} \text{sig}^{m_{i+1}}(x_1 - \hat{x}_1)
\]
\[
\dot{x}_3 = \text{sat}(\hat{x}_2) \dot{x}_2 + L_{\hat{x}} \text{sig}^{m_{i+1}}(x_1 - \hat{x}_1).
\]
\[
\dot{x}_4 = x_4 \dot{x}_5 + x_4 L_{\hat{x}} \text{sig}^{m_{i+1}}(x_4 - \hat{x}_4)
\]
\[
\dot{x}_5 = u_2 x_1^{0.125} x_4 + u_2 x_1^{0.125} L_{\hat{x}} \text{sig}^{m_{i+1}}(x_4 - \hat{x}_4)
\]
\[
\dot{x}_6 = T + U + V.
\]
where sat denotes that the function is using a saturated version of the observer states as described in (14). With appropriately chosen gains, the states in the observer (14) will converge to the real states \((x_1 - x_6)\) of system (12) in a finite time.

V. Simulation

In what follows, computer simulation is used to show how unknown parameters in the system can be estimated.

For simplicity, the real parameters of the system are assumed to be

\[
[c_1, c_2, c_3, c_4] = [0.98, 0.0033, 0.01408, 0.064],
\]

and initial estimates are \(\hat{c}_1 = 1.5, \hat{c}_2 = 0.08, \hat{c}_3 = 0.06,\) and \(\hat{c}_4 = 0.1.\) The desired drum pressure and reheater pressure are set to \(x_d1 = 150\) and \(x_d2 = 30\) respectively. Due to the substitutions and constant controller, initial conditions for the observer are determined dependent from the initial pressures \(p_1\) and \(p_2.\) Therefore by choosing initial states for the system to be \(p_1 = 145\) and \(p_2 = 35\) and assuming that the initial estimated states have a 10% difference from the actual states, the initial conditions for the observer can be calculated to be

\[
\begin{align*}
x_1(0) &= 145, \hat{x}_1(4) = 146.45, \hat{x}_2(0) = 0.0399, \hat{x}_3(0) = -0.0032, \\
x_4(0) &= 35, \hat{x}_4(4) = 35.35, \hat{x}_5 = 0.0174, \hat{x}_6(0) = -0.16.
\end{align*}
\]

Coefficients for the designed observer are set to be

\[
L_1 = 1, L_2 = 10
\]

\[
k_1 = 6, k_2 = 6, k_3 = 7, m_{2,1} = 0.9, m_{3,1} = 0.9, m_{4,1} = 0.9
\]

\[
k_4 = 1, k_5 = 9, k_6 = 5, m_{2,2} = 0.9, m_{3,2} = 0.9, m_{4,2} = 0.9
\]

Figures 2-4 show the state responses of the observer.

As shown in Figures 3-7, the estimates of \(x_2, x_3, x_5\) and \(x_6\) will tend to their real values in a relatively short time compared to the slow process of the boiler system.

However, as shown in Figure 2 and Figure 5, although the estimates \(\hat{x}_1\) and \(\hat{x}_2\) will converge to \(x_1\) and \(x_2\) promptly, the state \(x_1\) and \(x_2\) will not converge to the desired pressure of 150 and 30. This is due to the fact the controller (8) and (7) have used the initial parameter estimates and not the actual values of \(c_1-c_4.\)

VI. Adaptive Control of the Nonlinear Boiler System

In order to address the divergence of \(x_1\) and \(x_2\), in this section, an adaptive controller is developed based on the parameter estimator designed in the preceding section. The flow-chart for how we implement the proposed adaptive control is depicted in Figure 8.

The adaptive control scheme can be described as follows: After each 20 seconds, we will use the estimates of \(c_1-c_4\) to replace the values in the controller and then implement the new adaptive control in the next period. Due to the finite-time convergence of the parameter observer, there is a finite time \(t^*\) such that the estimates

\[
\hat{c}_1 = c_1, \quad \hat{c}_2 = c_2, \quad \hat{c}_3 = c_3, \quad \hat{c}_4 = c_4, \quad t \geq t^*.
\]
Consequently, with the real values of $c_1$-$c_4$ implemented in the controller (8)-(7), the states $p_1$ and $p_2$ will be regulated to their desired pressures.

The computer simulation is conducted with the implementation of the adaptive control scheme.

As shown in Figure 9 and 10, the states $p_1$ and $p_2$ are regulated to the desired pressure after the real values of the parameters are identified.

**Remark 6.1:** In this paper, for the simplicity, we adopt the constant controller (8)-(7) which cannot stabilize the system (1) as shown in [26]. This is also verified in the
similation (Figure 9 and 10). To achieve better performance, we can use more advanced controllers such as finite-time or exponential stabilizers designed in [26]. One disadvantage of using time-varying controller is that the observer design will be more complicated than the one we are currently using. But the regulation performance can be significantly improved.

VII. CONCLUSIONS

A finite-time convergent observer with time-varying coefficients is constructed to identify the explicit values of unknown parameters of a nonlinear boiler system. Then an adaptive control scheme is developed to regulate the states of the boiler system. The major contributions of the paper are (i) the new approach is able to handle the case when all parameters are unknown; (ii) faster convergence rate can be achieved to identify the unknown parameters and regulate the boiler system.

ACKNOWLEDGEMENT

This work is supported in part by the U.S. NSF under Grant No. HRD-0932339, Velaro Graduate Scholarship, Natural Science Foundation of China (61074013), and the Scientific Research Foundation of Graduate School of Southeast University.

REFERENCES