How Useful are Mean-Variance Considerations in Stock Trading via Feedback Control?

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Abstract—In classical finance, when a stochastic investment outcome is characterized in terms of its mean and variance, it is implicitly understood that the underlying probability distribution is not heavily skewed. For example, in the “perfect” case when outcomes are normally distributed, mean-variance considerations tell the entire story. The main point of this paper is that mean-variance based measures of performance may be entirely inappropriate when a feedback control law is used instead of buy-and-hold to modulate one’s stock position as a function of time. For example, when using a feedback gain $K$ to increment or decrement one’s stock position, we see that the resulting skewness measure $S(K)$ for the trading gains or losses can easily become dangerously large. Hence, we argue in this paper that the selection of this gain $K$ based on a classical mean-variance based utility function can lead to a distorted picture of the prospects for success. To this end, our analysis begins in a so-called idealized market with prices generated by Geometric Brownian Motion (GBM). In addition to the “red flag” associated with skewness, a controller efficiency analysis is also brought to bear. While all feedback gains $K$ lead to efficient (non-dominated, Pareto optimal) controllers, in the mean-variance sense, we show that the same does not hold true when we use a return-risk pair which incorporates more information about the probability distribution for gains and losses. To study the efficiency issue in an application context, the paper also includes a simulation for Pepsico Inc. using the last five years of historical data.

I. INTRODUCTION

Over the last few years, concepts from modern feedback control theory have been introduced into the stock trading arena; e.g., see [1]-[21]. In contrast to much of modern finance, a number of these papers address trading in a model-free manner; e.g., see [1]-[3]. That is, the controllers are not derived from a predictive model of future stock prices such as a stochastic differential equation. Instead, the adaptive properties of feedback are exploited. For example, a long stock position is simply modified using a feedback rule which is not based on underlying economic or financial principles. Instead, the instantaneous investment $I(t)$ is modulated based on some aspect of performance such as the cumulative trading gain or loss $g(t)$. In this paper, we begin our analysis with a classical linear time-invariant feedback trading law.

When the trader goes long, the investment is given by

$$I(t) = I_0 + Kg(t)$$

where $I_0 = I(0) > 0$ is the initial investment in a stock at time $t = 0$ and $K \geq 0$ is the feedback gain. Furthermore, we restrict our attention to the class of idealized markets with prices described by Geometric Brownian Motion (GBM) and study some basic issues associated with the profit-loss performance parameterized in the feedback gain $K$. This model is used extensively in finance; e.g., see [22]-[24].

Motivation for our work is derived by first considering the classical buy-and-lose strategy in lieu of linear feedback above. In this case, the trading gain or loss at the terminal time $g(T)$ is governed entirely by the log-normally distributed stock price with skewness, see [29], given by

$$S = (e^{\sigma^2 T} + 2)\sqrt{e^{\sigma^2 T} - 1}.$$ 

In practice, in financial markets, it is often the case that this leads to $S$-values which are sufficiently small to justify reliance on mean-variance performance evaluation. Even when a stock trades with annualized volatility as high as fifty percent, that is, $\sigma = 0.5$, the resulting skewness after one year, let $T = 1$, is $S \approx 1.75$, which is rather modest.

Many of the well-known tools in investment theory are based on the statistics of the return such as the mean and variance; e.g., see [25]-[27]. Once such statistics are available, various strategy options can be assessed using a number of different measures. More generally, it is well known that mean-variance considerations can be inadequate for various classes of problems in finance; e.g., see [31] and more recent work such as [32] accounting for higher order moments. Further to this literature, the main point of this paper is that mean-variance based measures of performance may be entirely inappropriate when a feedback control law is used instead of buy-and-hold to modulate one’s stock position as a function of time. For example, when using a feedback gain $K$ to increment or decrement one’s stock position, we see that the resulting skewness measure $S(K)$ for the trading gains or losses can easily become dangerously large. In fact, we see in Section 3 that the skewness formula above, when modified by linear feedback, becomes

$$S(K) = (e^{K^2 \sigma^2 T} + 2)\sqrt{e^{K^2 \sigma^2 T} - 1}.$$ 

This formula clearly shows that when $K$ increases, the skew can get very large. For example, with modest annualized
volatility of 5% and feedback gain $K = 10$, with $\sigma = \sqrt{0.05}$ and $T = 0.4$ above, we obtain $S(10) \approx 23.73$. In other words, this is the first issue which we pursue.

The second issue which we consider, related to the first, is the issue of “efficiency” which is central to modern finance; e.g., see [30]. First, we argue that no matter what linear feedback gain $K$ is used, the resulting risk-return pair $\text{Var}(g(T)), E(g(T))$ is always efficient. In terms of the language used in the control literature, this can be understood to mean that this pair is non-dominated or Pareto optimal. Our point of view is that all feedback gains being deemed efficient is a distortion resulting from the inappropriate use of mean and variance with a skewed distribution.

Motivated by the efficiency and skewness considerations above, we consider the possibility of alternative risk-return coordinates which implicitly include information about the skewness. To this end, we propose an alternative risk-return pair and show via an example that inefficiency occurs for a certain range of the feedback $K$. That is, a feedback $K$ which is efficient from the classical mean-variance point of view may no longer be efficient with the new risk-return measure.

II. Idealized Market and Results Used Here

We begin with a Geometric Brownian Motion (GBM) market with prices governed by

$$\frac{dp}{p} = \mu dt + \sigma dZ$$

where $\frac{dp}{p}$ is the percentage change in the stock price over the time increment $dt$ and $Z(t)$ is a standard Brownian Motion. The parameter $\mu$, is the so-called drift and the parameter $\sigma$, the annualized standard deviation, is the so-called volatility. Now, with the linear feedback trading strategy $I = I_0 + K g$ given above, the resulting stochastic equation for $g$ is given by

$$dg = \frac{dp}{p} (I_0 + Kg).$$

In addition, when trading gains and losses are considered in the sequel, the notion of an idealized market is implicit.

A. Idealized Market Assumptions

We note that the GBM price generation model above is an idealization in that the solutions $p(t)$ and $g(t)$ must be continuous. We point out that this is a serious assumption because real markets often experience jumps based on such factors such as breaking news, overnight blackouts when trading cannot occur, earnings announcements, etc. In our modeling above, it is assumed that the amount invested $I(t)$ can be continuously updated with zero transaction costs in a perfectly liquid market with equal bid and ask prices.

We also assume the trader is a price taker. This assumption would not necessarily hold for a large hedge fund whose prices typically change during a large transaction involving a very large number of shares. Finally, in this paper and in [1], it is assumed that the trader has adequate resources so that no transactions are stopped due to a failure to meet collateral requirements. This assumption is satisfied if the account maintains a suitably large cash balance so that margin considerations are not in play.

III. The Skewing Effect of Feedback

As mentioned in the introduction, if a linear feedback control with gain $K \geq 0$ is used, our claim is that the resulting skew $S(K)$ of the probability distribution for $g(T)$ can be so large as to render mean-variance information of questionable worth. For the sake of self-containment, we provide the reader with the definition of skewness which is used. For the random variable $g = g(T)$, as shorthand, we denote its mean by $M_g$ and its standard deviation by $\sigma_g$. Then, associated with the feedback gain $K$ is the skewness $S(K) = \frac{(g - M_g)^3}{\sigma_g^3}$.

Now, to demonstrate the skewing effect of feedback, we establish the following result.

Theorem 3.1: Consider the idealized GBM Market with drift $\mu$, volatility $\sigma$ and linear feedback controller with gain $K$. Then, at time $T > 0$, the resulting trading gain or loss $g(T)$ has probability density function with skewness which is independent of $\mu$ and given by

$$S(K) = (e^{K^2 \sigma^2} + 2) \sqrt{e^{K^2 \sigma^2} - 1}.$$  

Sketch of Proof: From the GBM model, we begin by noting that for any sample path $p(\cdot)$ for the price, it is easy to show that the resulting trading gain at time $T$ is given by

$$g(T) = \frac{I_0}{K} \left[ a \left( \frac{p(T)}{p(0)} \right)^K - 1 \right]$$

where $a = e^{\frac{1}{2} \sigma^2 (K - K^2) T}$. It follows that the desired quantity $S(K)$ is precisely equal to the skewness of the log-normally random variable

$$X = \frac{p(T)}{p(0)}^K$$

whose scale parameter is given by $\sigma_X = K \sigma \sqrt{T}$. Now using the skew formula for the log-normal random variable in Section 1, we obtain

$$S(K) = (e^{K^2 \sigma^2} + 2) \sqrt{e^{K^2 \sigma^2} - 1}.$$  

This completes the proof of the theorem. □

A. Illustrative Example

To illustrate how feedback affects the skew on the probability distribution for $g(T)$, we consider Geometric Brownian Motion for the stock price with $\sigma^2 = 0.05$ which represents about a 22.36% annualized volatility. Taking $T = 0.4$ to represent a little over 100 trading days, in Figure 1 below,
we provide a plot for the feedback-induced skew which results when the probability distribution for the trading gain \( g(T) \) is considered. The key point to note is that for practical typical feedback gain values, say \( 0 \leq K \leq 10 \), the skew becomes unreasonably large; e.g., as indicated in the introduction, \( S(10) \approx 23.73 \) which is an order of magnitude larger than the skew \( S(1) \approx 0.4813 \) associated with buy and hold at time \( T = 0.4 \).

**B. Example Continued: Mean-Variance Optimization**

To further drive home the point that reliance on mean-variance with feedback may be inappropriate, we continue with the running example above and include the drift parameter \( \mu = 0.25 \) representing an annualized return of 25%. To carry out our analysis, we make use of two formulae, which are easily derived from the results in [1]. Namely, for the stochastic GBM feedback equations under consideration, we have closed-form mean and variance expression given by

\[
\begin{align*}
\mathbb{E}[g(T)] &= \frac{I_0}{K} (e^{\mu KT} - 1) ; \\
\text{Var}[g(T)] &= \frac{I_0^2}{K^2} e^{2\mu KT} (e^{2\sigma^2 K^2 T} - 1) .
\end{align*}
\]

Using these formulae, we now consider a classical mean-variance optimization, for example, see [26], to find a so-called “optimal” feedback gain \( K \). More specifically, to work on a per-dollar basis we first set \( I_0 = 1 \) so that \( g(T) \) corresponds to the rate of return. Now, with terminal time \( T = 0.4 \) and risk aversion coefficient \( A \geq 0 \), we construct a classical quadratic objective function as in the literature for our running example. Namely, we take

\[
J(K) = \mathbb{E}[g(T)] - 0.5A \text{Var}[g(T)]
\]

\[
= \frac{I_0}{K} (e^{\mu KT} - 1) - \frac{0.5A}{K^2} e^{\mu K T} (e^{\sigma^2 K^2 T} - 1)
\]

to be maximized. For this case involving drift \( \mu = 0.25 \), a trader seeing this upward trending bull market might declare “risk on” and reasonably set \( A = 0.1 \) to capture as much of the stock gain as possible without completely ignoring downside risk. For this case, from the resulting plot of \( J(K) \) below, we obtain optimal feedback \( K = K^* \), optimal cost \( J = J^* \) and resulting skewness \( S^* = S(K^*) \) given by \( K^* \approx 9.81; \ J^* \approx 0.15; \ S^* \approx 21.42 \). To demonstrate how misleading this mean-variance optimization can be, we construct the **true cumulative distribution function** associated with the linear feedback controller. That is, via a lengthy but straightforward calculation,

\[
P(g(T) \leq \gamma) = \phi \left( \frac{\log(I_0 + K\gamma) - \log(aI_0) - K\mu_Y}{K\sigma_Y} \right)
\]

where \( \phi(\cdot) \) denotes the cumulative distribution of the standard normal random variable \( \mathcal{N}(0, 1) \) and

\[
\mu_Y = (\mu - \frac{1}{2}\sigma^2)T; \ \sigma_Y^2 = \sigma^2 T.
\]

Now, computing the mean and variance at the optimum feedback gain \( K^* \approx 9.81 \), we obtain

\[
M^*_g \approx \mathbb{E}[g(T)] \approx 0.1699; \ \sigma^*_g \approx \text{Var}[g(T)] \approx 0.4327.
\]

Using the normal distribution \( \mathcal{N}(M^*_g, \sigma^*_g) \), we can calculate the implied probability \( P(g(T) \geq \gamma) \), which we call it \( P_N(\gamma) \). We compare this quantity with the true probability which we call \( P(\gamma) \) and consider rates of return \( \gamma \geq 0 \). These two, cumulative distributions are provided in Figure 3 below. In the figure, the large differences between the two distribution functions is immediately evident. For example,
when a target return of 20% is sought, the normal distribution understates the probability of success by nearly 50%. On the other hand, when a high target return such as 20% is sought, the normal distribution understates the probability of success by about 25%. To conclude, mean-variance optimization provides a distorted view of a trader’s prospect for success.

IV. SIMULTANEOUS LONG-SHORT (SLS) FRAMEWORK

In this section, we provide an extension of the skew formula $S(K)$ above which applies to another type of linear feedback controller called Simultaneous Long-Short introduced in [3] and pursued further in [1] and [2]. The key idea in these papers is that the trader has investment $I_L(t)$ long and $I_S(t)$ short with the overall investment being

$$ I(t) = I_L(t) + I_S(t). $$

Letting $g_L(t)$ and $g_S(t)$ denote the the cumulative trading gains resulting from the long and short trades respectively and take $g(t) = g_L(t) + g_S(t)$ to be the overall trading gain, the two associated feedback control laws are

$$ I_L(t) = I_0 + K g_L(t); \quad I_S(t) = -I_0 - K g_S(t) $$

with $I_0 > 0$ and $K > 0$ being the same for both trades and negative investment $I_S(t)$ representing a short position. Using the mean and variance formulæ

$$ \mathbb{E}[g(t)] = \frac{I_0}{K} e^{\mu K t} + e^{-\mu K t} - 2; $$

$$ \text{Var}[g(t)] = \frac{I_0^2}{K^2} (e^{2 \sigma^2 K^2 t} - 1) (e^{2 \mu K t} - 1 + e^{-\mu K t} + e^{-2 \sigma^2 K^2 t}) $$

for this type of trading, a formula for the resulting skew $S(K)$ in the overall trading gain $g(t)$ is now provided. Similar to the earlier analysis in Section 3, $S(K)$ can become unreasonably large.

**Theorem 4.1:** Consider the idealized GBM Market with drift $\mu$, volatility $\sigma$ and SLS linear feedback controller with gain $K$. Then, at time $T > 0$, the resulting trading gain or loss $g(T)$ has probability density function with skewness given by

$$ S(K) = C(K, \mu, \sigma, T) [A(K, \mu, \sigma, T) - B(K, \mu, \sigma, T)] $$

where

$$ A(K, \mu, \sigma, T) = e^{K \mu T} + e^{-K \mu T} - 2; $$

$$ B(K, \mu, \sigma, T) = 3(e^{K \sigma^2 T} + e^{-K \sigma^2 T})^2; $$

$$ C(K, \mu, \sigma, T) = \frac{\sqrt[3]{e^{K^2 \sigma^2 T} - 1}(e^{K \mu T} + e^{-K \mu T})}{(e^{2K \sigma^2 T} + e^{-2K \mu T} + e^{-2 \sigma^2 K^2 T})^2}. $$

**Sketch of Proof:** To simplify notation in the calculations to follow, we work with the scaled random variable $X = p(t)/p(0)$ and define $M_p = \mathbb{E}(X^K)$; $M_n = \mathbb{E}(X^{-K})$ and $c = e^{-\frac{1}{2} \sigma^2 (K+K^2) T}$. Now using the fact that the stochastic differential equation for $g$ is integrable, for sample path $p(\cdot)$ we obtain

$$ g(T) = \frac{I_0}{K} (a X^K + c X^{-K} - 2). $$

Next, we use the fact that the GBM process leads to $X$ being log-normal. More precisely, we have $\log X \sim N(\mu_X, \sigma^2_X)$ with $K$-th moment

$$ \mathbb{E}[X^K] = e^{K \mu_X + \frac{1}{2} K^2 \sigma^2_X}. $$

Now, a lengthy but straightforward computation leads to

$$ M_g = \frac{I_0}{K} \left( a \mathbb{E}(X^K) + c \mathbb{E}(X^{-K}) - 2 \right) = \frac{I_0}{K} \left( e^{K \mu_T} + e^{-K \mu_T} - 2 \right) $$

and denominator in the skewness given by

$$ \sigma_g^3 = \frac{I_0^3}{K^3} \left[ (e^{K^2 \sigma^2 T} - 1)(e^{2K \mu T} + e^{-2K \mu T} + e^{-\sigma^2 K^2 T}) \right]^2. $$

Next, to obtain the numerator in the skewness formula, we expand $(g - M_g)^3$ and use the fact that $M_p M_n = \frac{1}{a c}$. A lengthy calculation yields

$$ \mathbb{E}[(g - M_g)^3] = \frac{I_0^3}{K^2} \left\{ a^3 \mathbb{E}(X^{3K}) - 3 M_p \mathbb{E}(X^{2K}) + 2 M_p^2 \right\} $$

$$ + 3 a \left[ (-1 + 2 M_p - \frac{1}{M_p}) \mathbb{E}(X^{2K}) \right] $$

$$ + 3 c \left[ (-1 + 2 M_n - \frac{1}{M_n}) \mathbb{E}(X^{-2K}) \right] $$

$$ + a^2 \left[ (-3 + 2 M_p + 2 M_n - \frac{1}{M_p} M_n) \mathbb{E}(X^{-3K}) \right]. $$

Now substituting for $\mathbb{E}(X^{2K}), \mathbb{E}(X^{-2K}), \mathbb{E}(X^{3K}), \mathbb{E}(X^{-3K})$ and in last step for $\mu_X$ and $\sigma_X$ leads to

$$ \mathbb{E}[(g - M_g)^3] = -3(2 e^{K^2 \sigma^2 T} + e^{-\frac{1}{2} K^2 \sigma^2 T})^2 (e^{K \mu_T} + e^{-K \mu_T}) $$

$$ + (e^{3 K^2 \sigma^2 T} - 3 e^{K^2 \sigma^2 T} + 2) (3 e^{K \mu_T} + e^{-K \mu_T}). $$

Then, dividing by the expression for $\sigma_g^3$, and further simplifying we arrive at

$$ S(K) = C(K, \mu, \sigma, T) [A(K, \mu, \sigma, T) - B(K, \mu, \sigma, T)]. $$

This completes the proof of the theorem. □

V. CONTROLLER EFFICIENCY CONSIDERATIONS

In accordance with the discussion in Section 1, we now look at efficiency issues. The starting point for our analysis is that both the expected value and the variance of the trading gain are increasing with respect to the feedback $K$. This is true for both of the linear feedback trading schemes discussed earlier in this paper. As a consequence of this monotonicity, when the expected value is plotted against the variance as a function of $K$, no pair $(\text{Var}(g(T,K)), \mathbb{E}(g(T,K)))$ dominates any other from a two-coordinate risk-return point of view. That is, all mean-variance pairs are efficient.

Our hypothesis is that this “all-$K$ efficient result” gives an erroneous impression about efficiency because skew is neglected. When a different return-risk pair is used which incorporates more information about the distribution of $g(T)$ beyond the second moment, our hypothesis is that feedback gains in certain ranges can be ruled out based on efficiency considerations. That is, even the trader with a utility function reflecting very low risk aversion will limit the selection of the feedback $K$ to those values in the efficiency regime.
Said another way, on an equal return basis, the classical point of view is that all traders, independent of their utility functions, prefer less risk. Similarly, on an equal risk basis, all traders, independent of their utility functions, prefer a higher return.

### A. Preamble on Efficiency Basics

We quickly review the notion of efficiency which is standard in the finance literature, for example, see [30]. Efficiency considerations also arise occasionally in the control literature in the context of Pareto optimality analysis.

Indeed, we consider a process with possible outcomes \( X \in \mathcal{X} \subseteq \mathbb{R}^2 \) with components \( X_1 \) and \( X_2 \) representing some measure of risk and return respectively. In finance and engineering, when dealing with an investment with gain or loss \( g \), the most classical measure of risk is the variance, \( \text{Var}(g) \) and the most classical measure of return is the mean \( \mathbb{E}(g) \).

Now, given a possible outcome \( X \in \mathcal{X} \), it is said to be inefficient if there exists some \( X' \in \mathcal{X} \) with either \( X'_1 \leq X_1 \) and \( X'_2 > X_2 \) or alternatively, \( X'_1 < X_1 \) and \( X'_2 \geq X_2 \). In other words, \( X' \) either has a higher return than \( X \) with no additional risk or it has a lower risk than \( X \) with at least as much return. We observe that \( \mathcal{X} \) can be partitioned into a union of two disjoint sets: the inefficient set and its complement, the efficient set. In the case where \( X \) is inefficient as demonstrated by \( X' \), under some basic assumptions about utility functions, it can be argued that both investors will discard \( X \) in favor of \( X' \).

### B. Alternative Risk-Return Pair

As previously stated, the monotonicity of the mean and variance of \( g(T) \) implies that all feedback controllers are efficient. The question we address in this section is the following: Might there be alternative risk-return measures which lead to a different conclusion in the presence of skewness? The example which we give below enables us to answer this question with a qualified “yes.” That is, for the idealized GBM market and the alternative risk-return pair which we describe, we show via an example that the linear feedback controller \( I(t) = I_0 + Kg(t) \) leads to risk-return combinations which are inefficient for low values of \( K \).

That is, there exists some feedback gain \( K^* < \infty \) with the property that feedback gain \( K < K^* \) has associated risk-return pair which is inefficient.

#### Construction of Proposed Risk-Return Pair

We begin by fixing a target return \( \gamma \geq 0 \) and take off from the fact that the idealized GBM market leads to a finite lower bound on the support of the trading gain \( g(T) \). We let \( W(K) \) denote the worst-case trading loss produced by the model and let \( P_\gamma(K) = P(g(T) \geq \gamma) \); i.e., the probability of a successful trade. That is, instead of variance and mean of \( g(T) \), we envision a trader whose underlying utility function depends on these new variables which are functions of the feedback gain \( K \). Since \( \gamma \) is a parameter, one can vary this parameter and work with a family of curves which serve as a “menu” corresponding to differing returns.

#### C. Example Demonstrating Realization of Inefficiency

To show how efficiency might arise, we continue with the SLS trading scheme described in Section 4 using the parameters given in Section 3. We consider the target return \( 10\% \) described by \( \gamma = 0.1 \). Via a lengthy calculation we obtain the worst-case loss and the corresponding probability of success

\[
W(K) = \frac{2I_0}{K}[1 - e^{-\frac{1}{2}(K^2 t)}],
\]

\[
P_\gamma(K) = 1 - \Phi(\frac{y_+ - \mu Y}{\sigma_Y}) + \phi(\frac{y_+ - \mu Y}{\sigma_Y})
\]

where \( \phi(\cdot) \) denotes the cumulative distribution for the standard \( N(0, 1) \) random variable, \( \mu_Y, \sigma_Y \) and \( a \) are given in Section 3 and

\[
y_\pm = \log \left[ \frac{1}{2a} (2 + \frac{K\gamma}{I_0} \pm \sqrt{(2 + \frac{K\gamma}{I_0})^2 - 4ac}) \right]^\pm.
\]

Examining the \((W(K), P_\gamma(K))\) plot in Figure 4, we note that the point where the worst-case trading loss \( W(K) \) is maximized can readily be characterized. That is, by setting the derivative of \( W(K) \) to zero, it is straightforward to show that this point of maximality is characterized by \( K^* \approx 8.91 \) and \( W(K^*) \approx 0.13 \). Noting that the right side of

![Figure 4. Demonstration of Inefficiency in SLS Trading](image)

the figure corresponds to low gain \( K \), we see that there are many \((W(K), P_\gamma(K))\) pairs which are inefficient. That is, the risk-return pair associated with feedback gain \( K \) is inefficient because the same probability of success is guaranteed with some other gain \( K' \) with a lower level of risk.

### VI. Numerical Example

The historical price for Pepsico Inc. (ticker PEP) over the 5 year period February 12, 2007 to February, 10, 2012 was selected to generate an empirical plot using the new risk-return pair \((W(K), P_\gamma(K))\). We used a time interval \( T = 1 \) which was represented by 252 trading days and carried out a number of back-tests using the SLS trading strategy. To
generate about 1000 one-year long sample paths, we used each of the days in the first four years as a starting point. Using $I_0 = 1$ and $\gamma = 0.1$, we generated empirical estimates of $P_r(K)$ and $W(K)$ and plot this pair as we increased $K$ over the interval $[0, 10]$. As seen in Figure 5, a regime of inefficiency was detected for $K \geq 5.2$ which corresponds to the maximum of $W(K)$.

![Figure 5. Efficiency Plot for Pepsico 2007-2012](image)

**VII. CONCLUSION**

In this paper, the focal point was the skewing effects of feedback controller gains on the probability distribution for the trading gain or loss $q(T)$. It was demonstrated that there are pitfalls associated with reliance on mean-variance based measures of performance. That is, when the feedback leads to a level of skewness $S(K)$ which is large, performance metrics based on mean and variance provide a distorted picture of the prospects for success.

Based on the work presented herein, one obvious question presents itself for consideration in future research: What type of objective function $J(K)$ should be used in carrying out an optimization problem for selection of the feedback gain $K^*$? Based on our arguments presented in Section 5, one possibility would be to use $(W(K), P_r(K))$ as the risk-return pair entering into an appropriate objective function.

**REFERENCES**


