Control of Modular Aerial Robots: Combining Under- and Fully-Actuated Behaviors

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Abstract—This work presents the design of flight control algorithms for set-point stabilization of a class of modular aerial vehicles obtained by rigidly interconnecting a number of single ducted-fan aircraft. Interestingly enough, for such a modular configuration, certain types of interconnection structure between the different modules may lead to redundancy both in term of the overall number of actuators available onboard and in the number of degrees of freedom that can be actually governed simultaneously. The design of the control policy for such a complex dynamical behavior is handled by defining specific control allocation algorithms for each possible case (for which a taxonomy is given in the paper) and by deriving an overall control structure capable to switch among these policies according to the properties of the selected configuration. This approach results in an architecture that combines classical control schemes for under-actuated air vehicles – such as those employed in most Vertical Take-Off and Landing (VTOL) aircraft – with control strategies for fully-actuated vehicles.

I. INTRODUCTION

Ducted-fan air vehicles [3], [4], [8]–[10] constitute a special class of tail-sitter aircraft in which the propeller is protected by an annular airfoil denoted as duct. The aero-mechanical design foresees the presence of either a single propeller or a pair of counter-rotating propellers, driven by a single motor. As opposed to a helicopter, attitude controllability is not obtained by employing cyclic pitches on the main rotor, rather by vectorizing the thrust of the propeller through a set of aerodynamic control surfaces. The resulting low mechanical complexity has allowed the design of miniature vehicles able to accomplish numerous tasks both in civil and military applications, including surveillance, aerial photography, and many others [2].

Recently, a number of aerial robotic applications have considered scenarios requiring physical interaction between the vehicle and the surrounding environment. These operations, which include cooperative grasping and transportation [6], cleaning [1], docking [5], represent a new research direction (with significant challenges) for both the design and control of aerial vehicles. The specific features of ducted-fan vehicles, such as the presence of the shroud protecting the propeller and the compact layout, suggest that this kind of airframe could play an important role in this new endeavour.

However, ducted-fan and more general tail-sitter vehicle configurations are characterized by relevant payload and dynamic constraints that may reduce their effectiveness in some circumstances. First of all, as shown, for instance, in [8], the position of the payload, and thus the position of the center of mass, plays an important role in determining the stabilizability property of a given configuration, since the torque required to govern the attitude dynamics is produced by applying aerodynamic forces at a given distance from the center of mass itself. Another relevant limitation derives from the typical under-actuated nature of the dynamics of ducted fans. In certain situations, such as during navigation in cluttered environments or while making contact to a surface, the number of degrees of freedom that can be controlled independently may not be sufficient to achieve the required level of performance in accomplishing the given task. As a result, specific control problems, such as maintaining a hovering configuration in the presence of disturbances or handling a large payload, may not even be feasible at all [7].

As a means to overcome these limitations, the idea pursued in this paper is to focus on a modular vehicle structure introduced in [7], obtained by rigidly interconnecting two or more ducted-fan airframe modules. As shown in [7], this class of air vehicles is able to achieve both under-actuated and fully-actuated dynamical behaviors according to the specific geometric configuration employed to interconnect the different modules. As a natural continuation of our previous modeling effort, the goal of the present work is to develop a methodology for flight control that exploits efficiently the possibilities offered by the modularity of the airframe. Specifically, a characterization of the available degrees of actuation resulting from a set of meaningful modular configurations is used to devise an overall control strategy that handles the specific dynamical features of each individual case. This is accomplished while efficiently handling actuator redundancy – which is present in any configuration of interest – through a blending of control allocation algorithms and feedback control strategies based on a classical inner loop/outer loop decomposition. The proposed approach is validated by means of extensive simulations on the specific modular ducted-fan vehicle model presented in [7].

II. THE MODULAR AERIAL VEHICLE

We summarize in this section the main findings of [7] with the aim of concisely presenting the dynamical model of the class of modular aerial vehicles of interest, which
is obtained by rigidly interconnecting a number of ducted-fan airframes. Specifically, in Section II-A we first recall the main features of the single-module ducted-fan aircraft, showing in particular how the control forces and torques are generated. With this analysis at hand, in Section II-B it is shown how the force/torque generation mechanism of the modular vehicle depend on the geometric parameters that describe how each single module is rigidly interconnected in the structure. Finally, in Section II-C we present the Newton-Euler formulation of the dynamical model of the modular vehicle that will be used for control design.

A. Individual Module: the Ducted-Fan Aerial Vehicle

Following [8], each ducted-fan vehicle is comprised of two main subsystems. The first one consists of a fixed-pitch propeller driven by an electric motor. This subsystem generates the main thrust, $T$, required to counteract the gravity force by inducing an airflow inside the duct. The second subsystem consists of a set of control vanes realized by eight independent actuated aerodynamic surfaces, which are positioned below the propeller. The vanes are immersed in the airflow induced by the propeller and their angular positions are controlled to generate a certain number of lift forces.

Following Figure 1(a), with respect to a coordinate frame $F_b = \{O_b, i_b, j_b, k_b\}$ placed at the center of mass of the aircraft, the force and torque components generated by the ducted-fan are the propeller’s thrust $T$, the resultant force contribution $F_x$ and $F_y$, directed along the body $x$ and $y$ axis and a resultant torque component $\tau_z$ along the body $z$ axis. The last three forces and torques depend on the thrust $T$ and the angle of attack of the control vanes. For the sake of simplicity, in this paper we will consider these aerodynamic forces as control inputs, and neglect the dynamics of the servomechanisms that actuate the vanes.

As shown in [7], the force-torque vector produced by each ducted-fan module is given by $u := [T, F_x(T), F_y(T), \tau_z(T)]^T$. With an eye on Figure 1(a), for the single-module ducted-fan configuration we assume that the center of pressure of the aerodynamic forces $F_x$ and $F_y$ is located at a constant distance $d$, along the body $z$ axis, from the center of gravity of the vehicle, while the thrust force $T$ is applied to the center of mass of the system. This fact is exploited in the following section where the resultant control wrench vector for a generic modular system, including also the single module case, is derived.

B. Modular Configuration

Consider now a modular system composed by a number $N$ of rigidly interconnected equal ducted fan vehicles, as shown for example in Fig. 1(b). Let $F_{b_m} = \{O_{b_m}, i_{b_m}, j_{b_m}, k_{b_m}\}$ be a reference frame attached to the center of mass of the overall system and let the unit vectors $\vec{r}_b$ for each single module be selected to point in the same direction; specifically, we assign the positive direction to be that of the gravity force; for the configuration in which the $z$-axis are not aligned, the $x$-axis is chosen to point to the center of mass of the modular system, as shown in the examples given in [7].

With this choice of reference frame, let the vector $\ell^m_{bi} := [r_{i1}, 0, h_{i}]^T$ expressed in the reference frame $F_{b_i}$, $i \in \{1, 2, ..., N\}$, be the position of the center of mass of each module with respect to the center of mass of the overall configuration. In $\ell^m_{bi}$, the parameters $r_{i1} \in \mathbb{R}_{>0}$ and $h_{i} \in \mathbb{R}$ denote respectively the horizontal and vertical offset of each ducted-fan in the formation. Finally, the relative orientation between the reference frames $F_{bi}$ and the reference frame $F_{b_m}$ is described by the angles $\psi_i$, $i \in \{1, 2, ..., N\}$. The following relation holds for each pair of single modules:

$$\|R_{\psi_i} \ell^m_{bi} - R_{\psi_j} \ell^m_{bj}\| > 0, \quad i \neq j, \quad i, j \in \{1, 2, ..., N\}$$

where $R_{\psi_i}$ is the rotation matrix associated with $\psi_i$, that is

$$R_{\psi_i} := \begin{bmatrix} \cos \psi_i & -\sin \psi_i & 0 \\ \sin \psi_i & \cos \psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The relation (1) implies that the center of mass of two single module are physically separated.

Definition 1 [7] A modular ducted-fan aerial robot $\mathcal{M}$ is given by the 4-tuple $(N, \Psi, \mathcal{R}, \mathcal{H})$ where

- $N$ is the number of equal modules generating a force/torque vector $u \in \mathbb{R}^4$ given by (??);
- $\Psi := \{\psi_i \in \mathbb{R} | i = 1, 2, ..., N\}$ is the set of the orientations $\psi_i$ of each frames $F_{bi}$ with respect to the frame $F_{b_m}$;
- $\mathcal{R} := \{r_i \in \mathbb{R} | i = 1, 2, ..., N\}$ is the set of horizontal distances $r_i$ between each module and the center of gravity of the formation;
- $\mathcal{H} := \{h_i \in \mathbb{R} | i = 1, 2, ..., N\}$ is the set of vertical distances $h_i$ between each module and the center of gravity of the formation.

Let $\mathbf{u} := [u_1^T, u_2^T, ..., u_N^T]^T$ be the vector of all the force/torque components $u_i$ produced by all the $N$ modules
in the formation. The resultant control force and torque vectors \( f_c = [f_x, f_y, f_z]^T \) and \( \tau_c = [\tau_x, \tau_y, \tau_z]^T \) applied by all the modules to the center of mass of the formation are given by \( f_c = B_f(\Psi)u, \tau_c = B_r(\Psi, R, \mathcal{H})u \) where

\[
B_f(\Psi) := \begin{bmatrix}
R_{\psi}G_f, ..., R_{\psi}G_f
\end{bmatrix},
\]

\[
B_r(\Psi, R, \mathcal{H}) := \begin{bmatrix}
R_{\psi}G_r(r_1, h_1), ..., R_{\psi}G_r(r_N, h_N)
\end{bmatrix}
\]

having defined

\[
G_f := \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & -d + h_i & 0 \\
-r_i & d - h_i & 0 & 0 \\
0 & 0 & -r_i & 1
\end{bmatrix},
\]

\[
G_r(r_i, h_i) := \begin{bmatrix}
0 & 0 & -d + h_i & 0 \\
r_i & d - h_i & 0 & 0 \\
0 & 0 & -r_i & 1
\end{bmatrix}.
\]

C. Vehicle Dynamics

In this subsection we derive the mathematical model of the modular system in the configuration space \( SE(3) = \mathbb{R}^3 \times SO(3) \) by using the Newton-Euler equations of motion for a rigid body. We consider an inertial reference frame \( F_i = \{O_i, i, \mathbf{i}, \mathbf{j}, \mathbf{k} \} \) and the body reference frame \( F_{b,m} \) attached to the center of mass of the modular system and aligned with the principal axis of inertia of the rigid body. The origins of the two reference frames are connected by the position vector of the center of mass, denoted by \( p = [x, y, z]^T \), whereas the relative orientation of the two frames is described by the rotation matrix \( R \). An Euler-angles representation is selected for the rotation matrix \( R \), yielding

\[
R = \begin{bmatrix}
C_{\psi}C_{\theta} & -S_{\psi}C_{\phi} + C_{\psi}S_{\phi}S_{\theta} & S_{\psi}S_{\phi} + C_{\phi}S_{\theta} \\
S_{\psi}C_{\theta} & C_{\psi}C_{\phi} + S_{\psi}S_{\phi}S_{\theta} & -C_{\psi}S_{\phi} + S_{\phi}S_{\theta} \\
-S_{\theta} & C_{\phi}S_{\theta} & C_{\phi}C_{\theta}
\end{bmatrix}
\]

where \( \phi, \theta, \psi \) are respectively the roll, pitch and yaw angles and, for the sake of brevity, \( C_{\alpha} := \cos \alpha, S_{\alpha} := \sin \alpha, T_{\alpha} := \tan \alpha \). By defining the vectors \( \Theta := [\phi, \theta, \psi]^T, \Theta := [\phi, \theta]^T \) and the matrix

\[
Q(\Theta) = \begin{bmatrix}
1 & S_{\phi}T_{\theta} & C_{\phi}T_{\theta} \\
0 & C_{\phi} & -S_{\phi} \\
0 & S_{\phi}/C_{\phi} & C_{\phi}/C_{\theta}
\end{bmatrix}
\]

the following kinematics relation can be derived

\[
\dot{\Theta} = Q(\Theta)\omega
\]

where \( \omega \) is the angular velocity expressed in the body frame \( F_{b,m} \). The dynamical model of the overall vehicle is described by the equations

\[
M_m\ddot{p} = Rf_c + M_me_3
\]

\[
J_m\dot{\omega} = -\omega \times J_m\omega + \tau_c + k_nM_me_3
\]

where \( M_m \) is the total mass of the system, \( J_m \) denotes the inertia of the modular vehicle, \( g \) is the gravity acceleration, \( e_3 \) the unit vector \( e_3 : = [0, 0, 1]^T \), whereas \( k_nM_mg \) approximates the resistance aerodynamic torque with \( k_n \) a constant coefficient collecting aerodynamic parameters. The operator \( \omega \times \) is given by \( \omega \times := \text{Skew}(\omega) \), where as customary \( \text{Skew}(\text{col}(x_1, x_2, x_3)) \) denotes the skew-symmetric matrix with the first, second and third row respectively given by \([0, -x_3, x_2], [x_3, 0, -x_1] \) and \([-x_2, x_1, 0] \).

Denoting with \( M \) and \( J \) respectively the mass and the inertia matrix of a single ducted-fan module, where \( J = \text{diag}(J_{xy}, J_{xy}, J_z) \), neglecting the mass of the payload and of the links connecting the modules, one obtains

\[
J_m = NJ + M \sum_{i=1}^{N} \begin{bmatrix}
r_i^2 \sin^2 \psi_i & 0 & 0 \\
0 & r_i^2 \cos^2 \psi_i + h_i^2 & 0 \\
0 & 0 & r_i^2
\end{bmatrix}
\]

and \( M_m = NM \).

III. CONTROL PROBLEM

The goal of this section is to synthesize, on the basis of the dynamical model of the modular vehicle, a control strategy capable of handling the actuator redundancy of the different configurations. In particular, in Section III-A an analysis of the actuator redundancy as a function of the geometric parameters of the modular configuration is carried out and employed for the synthesis of the control allocation policies. With the control allocation at hand, in Section III-B a control strategy is proposed that is applicable to the entire class of modular aerial robots considered in this work.

A. Control Allocation and Redundancy Analysis

To analyze the actuator redundancy of the modular system (hereby denoted by \( M \)), let us first partition the control wrench \([f_c^T, \tau_c^T]^T \) as follows

\[
f_x = B_1u, \quad f_y = B_2u, \quad \tau_x = B_3u,
\]

where \( \tau_c = [\tau_x, \tau_y]^T \). It can be shown that the matrices \( B_1, B_2 \) and \( B_3 \) have the following property [7]

\[
\rho(B_1) = 1, \quad \rho(B_2) = 1, \quad \rho(B_3) = 4,
\]

where \( \rho(A) \) stands for the rank of a matrix \( A \). Furthermore, the overall matrix \( B = [B_1^T, B_2^T]^T = [B_1^T, B_2^T, B_3^T]^T \) satisfies

\[
\rho(B) = \begin{cases}
4, & \text{if } \text{Im}(B_1^T, B_2^T) \subset \text{Im}(B_3^T) \\
5, & \text{if } \{\text{Im}(B_1^T) \text{ or Im}(B_2^T)\} \subset \text{Im}(B_3^T) \\
6, & \text{if } M \text{ is fully-actuated}
\end{cases}
\]

Let the control input \( u \) be chosen as

\[
u = Hv := H_1v_1 + H_2v_2,
\]

where \( v := \text{col}(v_1, v_2) \in \mathbb{R}^{4N} \) is a vector of virtual inputs such that \( v_1 \in \mathbb{R}^4 \) and \( v_2 \in \mathbb{R}^{4N-4} \). The matrix \( H \in \mathbb{R}^{4N \times 4N} \), partitioned as

\[
H := [H_1, H_2]
\]
is an invertible matrix to be designed to allocate the virtual inputs to the real ones, with \( H_1 \in \mathbb{R}^{4N \times 4} \) and \( H_2 \in \mathbb{R}^{4N \times (4N-4)}. \) The matrix \( H \) in particular is chosen such that also the following condition holds true

\[
\text{Im}(H_2) = \text{Ker}(B_1^T).
\]

(6)

With the above construction at hand, the following result holds:

**Proposition 1** Let \( \mathcal{M} \) be a modular system with \( N > 1 \) and \( H \) in (5) be chosen such that (6) holds true. Then, from (4) the wrench vector (3) can be rewritten as

\[
\begin{align*}
    f_x &= B_1 H_1 v_1 + B_1 H_2 v_2, \\
    f_y &= B_2 H_1 v_1 + B_2 H_2 v_2, \\
    \tau_e &= B_3 H_1 v_1
\end{align*}
\]

(7-9)

where

- \( B_3 H_1 \in \mathbb{R}^{4 \times 4} \) is an invertible matrix;
- \( B_1 H_2 \in \mathbb{R}^{1 \times (4N-4)} \) and \( B_2 H_2 \in \mathbb{R}^{1 \times (4N-4)} \) are such that \( \rho(B_1 H_2) \neq 0 \) and \( \rho(B_2 H_2) \neq 0 \) if and only if \( \mathcal{M} \) is fully-actuated;
- if \( \rho(B) = 5 \) then either \( \rho(B_1 H_2) = 1 \) or \( \rho(B_2 H_2) = 1 \).

It is worth noting that the last condition in Proposition 1 determines which degree of freedom (\( x \) or \( y \)) of the modular configuration is not directly actuated.

When the system is under-actuated, the vector of virtual inputs \( v_2 \) has no effect on the forces and torques applied to the robot. Indeed \( v_2 \) can be designed in order to satisfy some property of the joint space of the aerial vehicle, such as optimizing some cost function related to the energy employed by the actuators, without having effect on the actual task space dynamics of the system, namely on the rigid-body dynamics. On the other hand, when the system \( \mathcal{M} \) is fully-actuated, one can exploit the redundancy left in the input \( v_2 \) following the construction proposed above. In particular, let \( w := [w_1^T, w_2^T]^T \), with \( w_1 \in \mathbb{R}^2 \) and \( w_2 \in \mathbb{R}^{4N-6} \), and \( L = [L_1, L_2] \in \mathbb{R}^{(4N-4) \times (4N-4)} \), with \( L_1 \in \mathbb{R}^{(4N-4) \times 2} \) and \( L_2 \in \mathbb{R}^{(4N-4) \times (4N-6)} \), a new full rank matrix which can be chosen to allocate the virtual input \( v_2 \). In particular \( L \) is chosen such that \( \text{Im}(L_2) = \text{Ker}(H_2^T B_1^T, B_2^T) \). Then by applying \( L \) to the vector \( v_2 \) we obtain \( v_2 = Lv = L_1 w_1 + L_2 w_2 \), and, following similar arguments as in Proposition 1,

\[
\begin{align*}
    f_x &= B_1 H_1 v_1 + B_1 H_2 L_1 w_1, \\
    f_y &= B_2 H_1 v_1 + B_2 H_2 L_1 w_1, \\
    \tau_e &= B_3 H_1 v_1
\end{align*}
\]

(10-12)

For the fully-actuated control allocation the redundancy in the joint space can then be exploited through the choice of the input \( w_2 \) which does not affect the rigid-body dynamics.

The other relevant case to be considered is the one in which \( \rho(B) = 5 \), namely an additional degree of freedom with respect to the standard under-actuation can be controlled through the allocation. We just consider then the case in which \( \text{Im}(B_1^T) \subset \text{Im}(B_2^T) \), since the other case in which \( \text{Im}(B_1^T) \subset \text{Im}(B_2^T) \) follows similarly. Let \( r := [r_1, r_2]^T \) with \( r_1 \in \mathbb{R}, \) with \( r_2 \in \mathbb{R}^{4N-5}, \) \( M = [M_1, M_2] \in \mathbb{R}^{(4N-4) \times (4N-4)}, \) with \( M_1 \in \mathbb{R}^{4N \times 4} \) and \( M_2 \in \mathbb{R}^{(4N-4) \times (4N-5)} \), a full rank matrix in which

\[
\text{Im}(M_2) = \text{Ker}(H_2^T B_1^T).
\]

Then by applying the transformation \( M \) to the virtual inputs \( v_2 \) we obtain \( v_2 = Mr = M_1 r_1 + M_2 r_2 \), resulting into the wrench vector \( f_x = B_1 H_1 v_1 + B_1 H_2 M_1 r_1, f_y = B_2 H_1 v_1, \) \( \tau_e = B_3 H_1 v_1 \). For the case in which \( \rho(B) = 5 \), the redundancy in the joint space can be then exploited by arbitrarily assigning the vector \( r_2 \) which has no effect on the vehicle dynamics. On the contrary, the input \( r_1 \) can be selected to influence the lateral dynamics of the modular vehicle (or the longitudinal in the symmetric case in which \( \text{Im}(B_1^T) \subset \text{Im}(B_2^T) \)).

The availability of additional inputs having effect on the task space of the vehicle, namely on the rigid body dynamics, is then employed in the next subsection to derive the feedback control strategy.

### B. Control Strategy

In this section, we present a control strategy that comprises both the under- and the fully-actuated characteristics of the modular system. To begin, we describe each single control modality, which is based each control allocation policy developed in the previous section. Then, the overall controller is described. For simplicity, we restrict our attention to a set-point stabilization problem, where the constant set-points for the position and attitude are defined by \( p^* = [x^*, y^*, z^*]^T \) and \( \theta^* = [\phi^*, \theta^*, \psi^*]^T \).

1) **Fully actuated control:** In the fully-actuated configuration, application of equations (10)-(11)-(12) to the model (2) yields

\[
M_m \dot{p} = R \left[ B_{12} H_1 v_1 + B_{12} H_2 L_1 w_1 \right] + M_m g e_3 \quad (13)
\]

\[
J_m \dot{\omega} = -\omega \times J_m \omega + v_1' + k_n M_m g e_3 \quad (14)
\]

where \( B_{12} := [B_1^T, B_2^T]^T \), \( v_1' \in \mathbb{R} \) and \( v_1'' \in \mathbb{R}^3 \) are such that \( B_1 H_1 v_1 = [v_1', v_1'']^T \). In order to stabilize the attitude dynamics, we choose the following control law

\[
\dot{v}_1' = -K_p^\theta \Theta_{\psi} - \Theta_{\psi}^* - K_D^\theta \omega - k_n M_m g e_3
\]

where \( K_p^\theta, K_D^\theta \) are diagonal and positive definite matrices. For the position dynamics, we select

\[
\begin{bmatrix}
    w_1 \\
    v_1'
\end{bmatrix}
= R^{-1} \left[-K_p^\theta (p - p^*) - K_D^p \dot{p} - M_m g e_3 \right]
\]

where \( K_p^\theta, K_D^p \) are diagonal and positive definite matrices.
2) Under-actuated control: To control the modular configuration in the under-actuated mode, we first select \( w_1 = 0 \) in (13). Consequently, the dynamics of the position vector \( p(t) \) are written as

\[
M_m \dot{p} = R \begin{bmatrix} B_{12} H_1 v_1 \\ v'_1 \end{bmatrix} + M_m g e_3.
\]

To overcome the absence of direct actuation of the lateral/longitudinal dynamics, a cascade control structure similar to the one described in [8] is employed. This control strategy is decomposed into the inner-loop controller

\[
v''_1 = -K_P \Theta \left[ \tan \Theta - A(\Theta) \Theta_{out} \right] + \Theta_D \omega - k_n M_m g e_3
\]

where \( A(\Theta) \) is defined as

\[
A(\Theta) := \begin{bmatrix} -C_\psi & S_\psi C_\theta / C_\phi \\ S_\psi / C_\theta & C_\phi / C_\theta \end{bmatrix}.
\]

The outer-loop controller generates the references for the inner loop, according to the selection

\[
\Theta_{out} = \begin{bmatrix} \Theta_{out}^x \\ \Theta_{out}^y \end{bmatrix} = K_p^x \begin{bmatrix} y - y^* \\ x - x^* \end{bmatrix} + K_p^y \begin{bmatrix} \dot{y} \\ \dot{x} \end{bmatrix}
\]

where \( K_p^x, K_p^y \) are diagonal and positive definite matrices. Finally, for the vertical dynamics, we select the following control law

\[
v'_1 = 1 \frac{C_\phi}{C_\theta C_\phi} [-k_P^z (z - z^*) - k_P^z \dot{z} - M_m g]
\]

where \( k_P^z, k_P^z \) are positive control gains.

3) Under-actuated control with \( \rho(B) = 5 \): Let us consider the intermediate case in which the system is not fully actuated but \( \rho(B) = 5 \). Without loss of generality, assume that \( \text{Im}(B_2) \subset \text{Im}(B_3) \). Consequently, the position dynamics become

\[
M_m \ddot{p} = R \begin{bmatrix} B_{12} H_1 v_1 + B_{12} H_2 M_1 r_1 \\ B_2 H_1 v_1 \end{bmatrix} + M_m g e_3.
\]

We retain the control law (15) for the vertical dynamics, whereas the extra degree of freedom allows a direct stabilization of the longitudinal dynamics. The control law \( r_1 = (B_{12} H_1 M_1)^{-1} (-B_{12} H_1 v_1 + r'_1) \) where \( r'_1 = \frac{1}{C_\theta C_\phi} [-k_P^z (x - x^*) - k_P^z \dot{x}] \) and \( k_P^z, k_P^z \) are positive gains. The attitude control law is

\[
v'''_1 = -K_P \Theta \begin{bmatrix} \Theta_{out}^y \\ \Theta_{out}^y \end{bmatrix} + \Theta_D \omega - k_n M_m g e_3
\]

where \( \Theta_{out}^y \) is the first element of the vector defined in (III-B.2). This choice allows us to control independently the pitch and yaw angles while the angle \( \phi \) is controlled by a cascade structure, that in turn allows to control the lateral dynamics \( y \).

4) Switching control mode: A possible structure of the overall controller that accomplishes a blending of the previous three strategy is shown below. In this structure, the controller of the vertical dynamics remains fixed, independently of the choice of the control mode, while the controllers for the \( x-y \) dynamics and the attitude are commanded by five inputs, respectively, \( m_x, m_y, m_z, m'_x, m'_y \) and \( m_{\Theta} \) defined as follows:

\[
m_{\Theta} = \begin{cases} A(\Theta) \Theta_{out} - \tan \Theta + \Theta - \Theta^*, & \text{if F.A.} \\
0, & \text{if U.A.} \\
0, & \text{if U.A.} \end{cases}
\]

where

\[
m_{U.A.(5)} = \begin{bmatrix} -\tan \dot{\Theta} + \frac{S_\theta C_\theta}{C_\phi} \Theta_{out}^y + \frac{C_\phi}{C_\phi} \Theta_{out}^x \theta \theta - \theta^* \end{bmatrix}
\]

and

\[
(m_x, m_y, m_z, m'_x, m'_y) = \begin{cases} (1, 1, 0, 0), & \text{if F.A.} \\
(0, 0, 1, 1), & \text{if U.A.} \\
(1, 0, 0, 1), & \text{if U.A.(5)} \end{cases}
\]

The control wrench becomes

\[
\begin{bmatrix} f^*_x \\ f^*_y \\ f^*_z \end{bmatrix} = \begin{bmatrix} \frac{-k_P^z (x - x^*) - k_P^z \dot{x}}{C_\phi} m_x + f^*_x (f^*_z, \tau^*_c) m'_x \\ \frac{-k_P^z (y - y^*) - k_P^z \dot{y}}{C_\phi} m_y + f^*_y (f^*_z, \tau^*_c) m'_y \\ \frac{-k_P^z (z - z^*) - k_P^z \dot{z}}{C_\phi} M_m g \\ \tau^*_c = -K_P \Theta \begin{bmatrix} \tan \Theta - A(\Theta) \Theta_{out} + m_{\Theta} \theta - \theta^* \end{bmatrix} + \begin{bmatrix} \psi - \psi^* \end{bmatrix} - K_D \Theta \omega - k_n M_m g e_3 
\end{bmatrix}
\]

where

\[
\begin{bmatrix} f^*_x (f^*_z, \tau^*_c) \\ f^*_y (f^*_z, \tau^*_c) \end{bmatrix} = (B_{12} H_1)(B_{12} H_1)^{-1} \tau^*_c
\]

are residual coupling forces due to the cascade control structure that are neglected. Finally, the remaining redundant inputs \( v_2 \) could be selected appropriately by a supervisor module.

IV. Simulation Results

In this section, we describe simulation results obtained by using the vertical modular system described in Figure 1(b). Choosing for this model, without loss of generality, \( r_1 = r_2 = 0, h_1 = -h_2 = h, \psi_1 = \psi_2 = 0 \), one obtains
for the matrices $B_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -d + h & 0 & 0 & 0 & -d - h & 0 \\ 0 & d - h & 0 & 0 & 0 & -d + h & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, and

$B_3 = \begin{bmatrix} -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -d + h & 0 & 0 & 0 & -d - h & 0 \\ 0 & d - h & 0 & 0 & 0 & -d + h & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

The physical parameters of the model and the saturation of the forces acting on a single Ducted Fan module are listed in Table I. In particular, $T_x, T_y$ represent the saturation of the propeller’s thrust, $F_x, E_x, F_y, E_y$ represent the saturation of the forces generated by the flaps, respectively on the $x$ and $y$ axis, whereas $T_z, \tau_z$ are the saturation levels for the torque around the $z$ axis.

Figure 2 compares the time history of position and attitude when the switching controller operates in fully actuated and under actuated mode. The maneuver consists in bringing the vehicle from the initial position $p(0) = [0, 0, 0]^T$ to the set point $p^* = [1, 1, -1]^T$ while maintaining the attitude fixed at $\Theta_v = [0, 0, 0]^T$ starting at $\Theta_v(0) = [0.005, 0.005, 0.005]^T$.

The controller parameters are $K_{p\theta} = 40$, $K_{p\psi} = 20$, $K_{pD} = 20$, $K_{pD} = 10$, $K_{pD} = \text{diag}(0.5, 0.5)$ and $K_{pD} = \text{diag}(2, 2)$. It is possible to notice the independent actions of the two controllers for the $z$ and $\psi$ dynamics. On the other hand, because of the cascade control structure in the under-actuated mode, the correlation between the the angles $\phi, \theta$ and the two position $x, y$ is clearly visible. In particular, it is evident that the roll angle is used to pilot the longitudinal dynamics and the pitch one is used to control the lateral displacement.

V. CONCLUSION

This work has presented the design of a control strategy for a class of modular aerial vehicles obtained by rigidly interconnecting a number of ducted-fan modules. The proposed controller is able to take into account for the redundancy of actuators; in particular, it implements a mode switch between control policies designed for standard under-actuated configurations, an intermediate under-actuated configuration, and a fully-actuated case. In each modality, the residual degrees of freedom are handled by means of control allocation methods.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$J_{xy}$</th>
<th>$J_{yz}$</th>
<th>$J_{xz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 Kg</td>
<td>$1.8 \cdot 10^{-2}$ Kg·m$^2$</td>
<td>$9 \cdot 10^{-2}$ Kg·m$^2$</td>
<td>$9 \cdot 10^{-2}$ Kg·m$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k_n$</th>
<th>$d$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9.07 \cdot 10^{-4}$</td>
<td>$0.2$ m</td>
<td>$0.7$ m</td>
</tr>
</tbody>
</table>

$[T, T] = [22, 2]$ N $[F_x, E_x] = [4, -4]$ N $[F_y, E_y] = [4, -4]$ N $[\tau_x, \tau_z] = [0.5, -0.5]$ N·m

Table I

PARAMETERS OF THE MODULAR SYSTEM

Simulations show the effectiveness of the proposed strategy. Current work focuses on the implementation of more general tracking controllers and definition of robust control allocation schemes as well as experimental verification of the proposed methodology.

REFERENCES


