Design of IMM Estimation with Differential Game Based Guidance System in Maneuvering Target Interception

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Abstract—A differential game based guidance law for passive ranging system (i.e. Image IR seeker) is presented in this paper. The interceptor system equipped with the IR seeker which can provide good stealth capability during its course of tracking the maneuvering invaders. With target motion uncertainty, the passive ranging system integrates the Extended Kalman Filter (EKF) with the Interacting Multiple Model (IMM) technique to minimize the errors in estimating the range and range rates incurred by the target's maneuvering motion. The guidance law of the missile interceptor is designed based on linear differential game theory while considering the motion of the target in 3-D space such that the distance between the missile and the target is minimized. Finally, extensive simulations incorporating aerodynamic model are demonstrated to verify the validity of the proposed system, and the simulation results reveal that the mission of intercepting a maneuvering target is accomplished successfully.

I. INTRODUCTION

The research on the passive ranging law applied to the estimator/seeker has gone through significant improvement over the last decade. Here we propose a passive ranging based missile which can provide good cover operation to assure successful missile interception. Since the sensing adopted by the interceptor is passive, the target may not be easy to execute effective evading strategies, thereby increasing the possibility of interception.

In the past decade, the Extended Kalman Filter (EKF) technique is applied to track the target based upon Modified Spherical Coordinates (MSC) while estimating the target range and range rates from the information of the IR seeker [1]. Because the image of IR seeker only provides 2-D information, the maneuvering movements of the target may cause certain degree of performances degradation in the estimation. Thus, our approach is to apply the Interacting Multiple Models (IMM) [2-3] method, which utilizes different types of target motion model, so that the performance of the maneuvering target estimation can be improved. In [4-6], the mentioned multiple target motion models consist of Coordinate Turn (CT) model, Constant Velocity (CV) model, and Constant Acceleration (CA) model. In most applications, IMM approaches are either limited to 2-D cases or to 3-D cases in which accelerations are usually constrained to certain planes. In this paper, the missile target to be tracked before interception belongs to a class of wingless ballistic missile (BM) equipped with Thrust Vector Control (TVC) and Divert Control System (DCS). To deal with this kind of target missile, our approach is to apply the BM model [7] whose advantage is that we can limit the number of IMM sub-filters to a few while maintaining performance of tracking.

In addition to the target tracking process, the intercepting missile's guidance law is another critical issue to accomplish the target interception. Forte and Shinar [8] designed the guidance law in a noise-corrupted environment based on game theory with the concept of mixed strategies. For the homing stage of the interception profile, Hull [9] proposed a linear-quadratic guidance law which can enhance the information content of the interception trajectory. Shinar [10] proposed a guidance law design which is based on the differential game and the concept of mixed strategies for the terminal phase of an intercepting missile against a maneuvering target. In [11-12], the design of the guidance law is based on a pursuit-evasion game and on the consideration of the estimator performance, and Vladimir [11] compared different kinds of game based guidance laws. An integrated estimation/guidance design was also proposed in [13], where the method can insure to destroy the maneuvering targets. Moreover, Shima [14] derived a sliding mode controller for a missile autopilot integrated with game based guidance, and Li [15] designed a low-altitude interceptor with logic-based guidance, which is steered by aerodynamic lift and divert thruster. Different from the above design in 2-D space under the assumption of perfect information, in this paper we employ the game based guidance with IMM estimation in 3-D space to deal with the target uncertainty and to find the better solution to the interception problem.

This paper is organized as follows: In section II, we formulate the interception problem and missile model. Then, the seeker/estimator design, the IMM technique and the BM model will be presented in section III and IV, respectively. Next, the game-based guidance law will be studied in section V. Simulation results are shown and discussed in section VI.

II. PROBLEM STATEMENT

The major objective, in this paper, is to employ a differential game based guidance with the passive ranging system to complete the missile-target interception in 3-D...
space, as shown in Fig. 1. The design of the guidance law should consider the estimation errors arising from the passive ranging law and maintain the performance of missile system.

Fig. 1. Missile-Target engagement in 3-D space

The airframe of missile [16] is shown in Fig. 2. Because it's wingless, the control surfaces should not exist. The main purpose of the wingless airframe is that the effect of the aerodynamics on this airframe will be minute and can be regarded as disturbances, which makes the control surfaces futile. The following show the forces and torques exerted on the missile from the mounted actuators.

Fig. 2. Missile model with DCS, main TVC and four extra TVCs

Translations:
\[
\mathbf{F}_{\text{prop}} = \mathbf{F}_{\text{mainTVC}} + \mathbf{F}_{\text{extTVC}} + \mathbf{F}_{\text{DCS}}
\]
\[
= \left[ N_{\alpha \phi} \cos \theta + 2 N_{\phi} (\cos \theta \phi + \cos \phi \theta) \right]
+ \left[ N_{\phi \phi} \sin \phi \cos \theta + F_{\text{DCS},xy} + 2 N_{\phi} \sin \phi \theta \right]
+ \left[ N_{\phi \theta} \sin \phi \sin \theta + F_{\text{DCS},yz} + 2 N_{\phi} \sin \phi \theta \right]
\]

(1)

Rotations:
\[
\mathbf{\tau}_{\text{prop}} = \mathbf{\tau}_{\text{mainTVC}} + \mathbf{\tau}_{\text{extTVC}} + \mathbf{\tau}_{\text{DCS}}
\]
\[
= \left[ N_{\alpha \phi} L_{\text{TVC}} \sin \phi \sin \theta + L_{\text{DCS}} F_{\text{DCS},y} + 2 N_{\phi} \sin \phi \theta \right]
- \left[ N_{\alpha \phi} L_{\text{TVC}} \sin \phi \sin \theta + L_{\text{DCS}} F_{\text{DCS},x} - 2 N_{\phi} \sin \phi \theta \right]
\]

(2)

III. SEEKER/ESTIMATOR DESIGN

This section presents the algorithm of the seeker/estimator with the passive ranging law. The functions of the IR seeker are to sense the heat regions located at the surface of the target body and to take their 2-D image frames, out of which two relative angles are extracted: elevation and azimuth angles, as shown in Fig. 3. Because of significant improvement on the currently integrated circuit technology, the capability of real-time calculation becomes much more powerful. Under this circumstance, the seeker of the missile system can extract the trajectories, velocity, and many important data of the target in real-time.

![Fig. 3. Geometry of Missile-Target Interception](image)

Here, we apply the well-known Extended Kalman Filter (EKF) to the estimator based on the representation in Modified Spherical Coordinates (MSC). Consider the engagement of an intercepting missile and an invading target in 3-D space as shown in Fig. 3, where \( \alpha \) and \( \beta \) are the elevation and azimuth angles, respectively. The standard MSC matrix includes six states: two angles, two angle rates, inverse range, and inverse time-to-go [1], which are together defined as a state vector in MSC \( \mathbf{\zeta} \) denoted below

\[
\mathbf{\zeta} = \begin{bmatrix} \alpha & \dot{\alpha} & \beta & \dot{\beta} \cos \alpha & \frac{1}{r} & \frac{r}{r'} \end{bmatrix}^T, \tag{3}
\]

On the other hand, the system state in Cartesian coordinates is defined as the vector of relative distances and velocities.

In MSC space, the first four states of \( \mathbf{\zeta} \) can be measured by the IR seeker, and the fifth and the sixth states are estimated through EKF. Typically, the standard EKF comprises two stages: time update and measurement update, where they both are defined as follows:

**Time Update stage**
\[
\mathbf{\dot{\zeta}}(n \mid n-1) = f[\mathbf{\zeta}(n-1 \mid n-1)]
\]
\[
\mathbf{A}_{\zeta}(n \mid n-1) = \frac{\partial f[\mathbf{\zeta}(n-1 \mid n-1)]}{\partial \mathbf{\zeta}(n-1 \mid n-1)} \tag{4}
\]
\[
\mathbf{P}(n \mid n-1) = \mathbf{A}_{\zeta}(n \mid n-1)\mathbf{P}(n-1 \mid n-1)\mathbf{A}_{\zeta}^T(n, n-1) + \mathbf{J}_{\zeta}(n)\mathbf{Q}^T \mathbf{J}_{\zeta}(n)
\]

**Measurement Update stage**
\[
\mathbf{\hat{\zeta}}(n \mid n) = \mathbf{\zeta}(n \mid n-1) + \mathbf{G}[\mathbf{y}(n) - \mathbf{H}\mathbf{\zeta}(n \mid n-1)]
\]
\[
\mathbf{G}(n) = \mathbf{P}(n \mid n-1)\mathbf{H}^T[\mathbf{H}\mathbf{P}(n \mid n-1)\mathbf{H}^T + \mathbf{R}]^{-1} \tag{5}
\]
\[
\mathbf{P}(n \mid n) = (\mathbf{I} - \mathbf{G}\mathbf{H})\mathbf{P}(n \mid n-1)
\]

where \( f[\mathbf{\zeta}(n-1 \mid n-1)] \) is the nonlinear transition matrix,
\[
\mathbf{J}_{\zeta}(n) = \frac{\partial f[\mathbf{\zeta}(n-1 \mid n-1)]}{\partial \mathbf{x}(n \mid n-1)}, \quad \mathbf{P}(n \mid n-1) \text{ is the covariance matrix,}
\]
\( \mathbf{Q} \) is the process noise, \( \mathbf{R} \) is the measurement noise matrix, \( \mathbf{y}(n) \) is the measurement vector, \( \mathbf{I} \) is a \( 6 \times 6 \) identity matrix, and \( \mathbf{H} \) is the measurement matrix shown as follows:

\[
\mathbf{H} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \cos \alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

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IV. INTERACTING MULTIPLE MODEL FILTER DESIGN

After the process of EKF, we employ the IMM method to perform the subsequent estimation. Each sub-filter is associated with a particular target motion, meaning that each sub-filter is designed assuming that the target is to follow a specific type of trajectory. The resulting estimator can thus increase the tracking performance that leads to more reliable intercepting and can hold against the maneuvering target.

A. State and Covariance Combination

As mentioned above, the IMM method incorporates a number of sub-filters, each of which describing a type of target motion, and the way of mixing is shown as follows:

Mixing Process stage

\[ \mathbf{x}_i(n|n-1) = \sum_{j=1}^{N} \mu_{ij}(n-1) \mathbf{x}_j(n|n-1) \]

where \( \mu_{ij}(n-1) \) is the mixing probability from sub-filter \( i \) to \( j \), \( N \) is the total number of sub-filters, and \( \Delta = \mathbf{x}_i(n-1|n-1) - \mathbf{x}_j(n-1|n-1) \).

Mixing Probability Update stage

The mixing probability is obtained from

\[ \mu_{ij}(n|n-1) = \frac{P_{ij}(n|n-1)}{\sum_{j=1}^{N} P_{ij}(n|n-1)} \]

where \( P_{ij}(n|n-1) = \sum_{j=1}^{N} \mu_{ij}(n-1)P_{ij}(n-1|n-1) \), and \( C_{ij} = \sum_{j=1}^{N} P_{ij}(n-1) \).

Mode Probability Update stage

The mode probability \( \mu_i(n) \) is defined as follows:

\[ \mu_i(n) = \frac{C_{i}^{\text{mix}}(n-1) \Lambda_i(n)}{C_{\text{mode}}^{n}} \]

with \( C_{\text{mode}}^{n} = \sum_{j=1}^{N} C_{j}^{\text{mix}}(n-1) \Lambda_j(n) \), where \( \Lambda_i(n) \) is the likelihood function that employs \( y(n) - \mathbf{H} \hat{x}_i(n|n-1) \) and \( \mathbf{H} \mathbf{P}_i(n|n-1) \mathbf{H}^T + \mathbf{R} \) as residuals.

Estimate and Covariance Combination stage

The whole estimated state and covariance (output from IMM) are defined as follows:

\[ \hat{x}(n|n) = \sum_{i=1}^{N} \mu_i(n) \hat{x}_i(n|n) \]

and

\[ \mathbf{P}(n|n) = \sum_{i=1}^{N} \mu_i(n) \mathbf{P}_i(n|n) \]

B. Sub-Filter Design

In this paper, we apply two types of sub-filter models: Constant Acceleration (CA) model and the Ballistic Missile (BM) model. CA model is used to track the target’s constant acceleration movement which is assumed to be affected by gravity only. Whereas the BM model is designed for handling the target maneuvering motion. Hence, we not only assume that the target’s motion are uncertain trajectories but also update their estimates through a direct estimation procedure during the interception. Note that the state vector represented in Cartesian space will be employed in the time update of target motions. In this case, the CA Model is defined by

\[ \mathbf{x}(n|n-1) = \mathbf{A} \mathbf{x}(n-1|n-1) - \mathbf{B} \mathbf{o}(n-1) + \mathbf{C} \mathbf{g} \]

where

\[ \mathbf{A} = \begin{bmatrix} \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{bmatrix} \]

\[ \mathbf{B} = \begin{bmatrix} \mathbf{B}_x & \mathbf{B}_y & \mathbf{B}_z \end{bmatrix} \]

\[ \mathbf{C} = \begin{bmatrix} \mathbf{C}_x & \mathbf{C}_y & \mathbf{C}_z \end{bmatrix} \]

\[ \mathbf{g} = \begin{bmatrix} \mathbf{g}_x & \mathbf{g}_y & \mathbf{g}_z \end{bmatrix} \]

\[ T \] is the sampling time, and \( \mathbf{o}(n-1) \) represent the intercepting missile acceleration.

While the target missile returns from exoatmosphere to the earth, it relies on the gravity to reach its assigned destination. Facing the interception, the target will utilize Divert Control System (DCS) or apply lateral forces to generate lateral movements to escape from the interceptor. Fig. 4 shows the assumption of the configuration of the target missile.
probability and model probability constantly, the IMM is able to identify the correct target model ultimately. Finally, all the estimation information of IR seeker will be transferred to the missile guidance system to produce the guidance commands.

V. GUIDANCE LAW DESIGN

The relative distance, the relative velocity, and the relative acceleration of the target with respect to the intercepting missile can be respectively represented as follows:

\[ \mathbf{r} = \mathbf{r}_t - \mathbf{r}_m, \quad \mathbf{v} = \mathbf{v}_t - \mathbf{v}_m, \quad \mathbf{a} = -\mathbf{a}_t + \mathbf{a}_m \]

(15)
given that the subscripts \( t \) and \( m \) represent the target and the missile, respectively. Note that accelerations of the missile and the target are assumed to arise from their own thrusts only, i.e., excluding the gravity effect. Furthermore, we assume that the earth is flat, and gravity exerted on both the missile and the target are the same and remain constant throughout the interception course. It is worthwhile to note that, in the 3-D space, \( \mathbf{v} \) and \( \mathbf{r} \) generate a plane which does not enclose the missile-target engagement, the relative distance is very large. Referring to Fig. 1 the two elements are the thrusts which are generated by the missile guidance system to produce the guidance commands. Estimation information of IR seeker will be transferred to the autopilot system.

By the same token, we define \( \mathbf{a}_t = \mathbf{a} - (\mathbf{a} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \). After taking the time derivative of equation (18) and taking into account the fact that inner product \( \mathbf{v} \cdot \hat{\mathbf{r}} = 0 \), we obtain the following

\[ \dot{\mathbf{v}}_\perp = \mathbf{a}_t - \mathbf{a}_m - (\| \mathbf{v} \| / \| \mathbf{r} \|) \hat{\mathbf{r}} - (\mathbf{v} \cdot \hat{\mathbf{r}} / \| \mathbf{r} \|) \mathbf{v}_\perp \]

(19)

In order to let the missile intercept the target, our design concept is to drive \( \mathbf{v}_\perp \) to zero as \( \| \mathbf{r} \| \) decreases. Clearly, if \( \| \mathbf{v} \| \rightarrow 0 \) along with \( \| \mathbf{r} \| \rightarrow 0 \), and \( \mathbf{v}_\perp \cdot \hat{\mathbf{r}} \) is always negative, the missile will intercept the target successfully. Then, in order to get the true acceleration command \( \mathbf{a}_f \), which is shown in Fig. 5, we first assume the total thrust force from TVC is \( N = N_L + N_M \), yielding the acceleration \( \mathbf{a}_m \) of the missile : \( (N_L, N_M) \) are the thrusts which are generated by the extra and the main TVCs of the missile, respectively.

\[ \| \mathbf{a}_m \| = N/m = \sqrt{\| \mathbf{a}_L \|^2 + \| \mathbf{a}_M \|^2} \]

(20)

where \( m \) is the missile's mass. Now, we can derive the desired acceleration command by the formula: \( \mathbf{a}_f = \mathbf{a}_d + \mathbf{a}_m \), which will be delivered to autopilot system.

In order to validate the aforementioned design concept for our guidance system, several important issues need to be investigated. First, from equation (19), at the beginning of the interception, the relative distance is very large. Referring to the realistic data of fly tests, the lethal region of ballistic missiles is always designed in 5 to 15 meters. Thus, taking the fact in the above discussions into account, we can acquire the following linear approximate dynamics of equation (19), namely,

\[ \dot{\mathbf{v}}_\perp = \mathbf{a}_t - \mathbf{a}_m - (\mathbf{v} \cdot \hat{\mathbf{r}} / \| \mathbf{r} \|) \mathbf{v}_\perp \]

(21)

A. Game Formulation

Now, let the control constraints be set as equation (22) [17], where \( \mathbf{A}_{t\perp} \) and \( \mathbf{A}_{m\perp} \) are the possible ranges of acceleration components \( \mathbf{a}_t \) and \( \mathbf{a}_m \), namely,

\[ \mathbf{a}_{m\perp} \in \mathbf{A}_{m\perp}, \quad \mathbf{a}_{t\perp} \in \mathbf{A}_{t\perp} \]

(22)

Before we actually solve the formulated guidance problem, we first select the desirable cost function as (23),

\[ J = \| \mathbf{C} v_\perp (t) \| \]

(23)

where \( \mathbf{v}_\perp \in R^{3\times 1}, \mathbf{a}_{m\perp} \in R^{3\times 1}, \mathbf{a}_{t\perp} \in R^{3\times 1}, \mathbf{C} \in R^{3\times 3} \) and \( (\mathbf{v} \cdot \hat{\mathbf{r}} / \| \mathbf{r} \|) \mathbf{v}_\perp = \mathbf{A}(t) \) is a continuous function. We have to find the optimal strategies of the missile relative to the target in order to achieve the desired goal \( \| \mathbf{v}_\perp \| \rightarrow 0 \) at the final interception time \( t_f \). Assume that the feedback strategy pair \( \{\mathbf{M}(\cdot), T(\cdot)\} \) satisfies the following definition:

1. \( \{\mathbf{M}(\cdot), T(\cdot)\} \) generates at least one solution of \( \mathbf{v}_\perp \)
2. \( \mathbf{a}_{m\perp} = \mathbf{M}(\mathbf{v}_\perp(t), t), \quad \mathbf{a}_{m\perp} \in \mathbf{A}_{m\perp} \)
3. \( \mathbf{a}_{t\perp} = \mathbf{T}(\mathbf{v}_\perp(t), t), \quad \mathbf{a}_{t\perp} \in \mathbf{A}_{t\perp} \)

Then, the optimal strategy pair of the admissible pairs should hold the saddle point inequality:
J[v_\perp(t),t,\mathbf{M}(t),\mathbf{T}(t)] \leq J[v_\perp(t),t,\mathbf{M}(\cdot),\mathbf{T}(\cdot)]
\hat{=} J'[v_\perp(t),t] \leq J[v_\perp(t),t,\mathbf{M}(\cdot),\mathbf{T}(\cdot)] \tag{25}

In order to simplify the differential game, the state transition matrix is derived as follows:
$$
\mathbf{Φ}(t,f) = -\mathbf{Φ}(t,f)t(-v \cdot \hat{\mathbf{r}}/\|\mathbf{r}\|)\mathbf{v}_\perp
$$
\tag{26}

Here, we define the following notations:
$$
\mathbf{z} = \mathbf{CΦ}(t,f)(\mathbf{v}_\perp), \mathbf{X}(t,f) = -\mathbf{CΦ}(t,f), \mathbf{Y}(t,f) = \mathbf{CΦ}(t,f)
$$
Thus, the differential game can be rewritten as follows:
$$
\mathbf{z} = \mathbf{X}(t,f)\mathbf{a}_{m\perp} + \mathbf{Y}(t,f)\mathbf{a}_{1\perp}, J = \|\mathbf{z}\|_{1\perp}
$$
\tag{27}

### B. Optimal Strategies

In this subsection, we define a Lyapunov function to establish an optimal strategy pair. At first, we set a function for equation (27), namely,
$$
W(x(t),z) = \|\mathbf{z}\|_{1\perp}, \quad W(x(\cdot),z) = \mathbb{D}(z,t), \quad W(x(\cdot),x) = \mathbb{D}(z,t)
$$
Thus, the differential game can be rewritten as follows:
$$
\mathbf{z} = \mathbf{X}(t,f)\mathbf{a}_{m\perp} + \mathbf{Y}(t,f)\mathbf{a}_{1\perp}
$$
\tag{28}

where \(W(t)\) is a short hand for \( W(z,t) = \|\mathbf{z}\|_{1\perp} \) and \(W(x(\cdot),z) = \mathbb{D}(z,t)\). Then, we recheck the function \( W(t) \), and make sure that the ultimate goal of \( \mathbf{a}_{m\perp} \) is to maximize \(dW(t)/dt\) whereas the ultimate goal of \( \mathbf{a}_{1\perp} \) is to minimize it. Thus, we can choose an optimal strategy pair by:
$$
\begin{align*}
\mathbf{a}_{m\perp} = \min_{a_{m\perp}} & \|a_{m\perp}\| \leq \rho_m(t) \quad & A_{m\perp} = \{a_{m\perp} : \|a_{m\perp}\| \leq \rho_m(t)\} \\
\mathbf{a}_{1\perp} = \max_{a_{1\perp}} & \|a_{1\perp}\| \leq \rho_1(t) \quad & A_{1\perp} = \{a_{1\perp} : \|a_{1\perp}\| \leq \rho_1(t)\}
\end{align*}
\tag{29, 30}
$$

Now, in our guidance system, \(z\) is a scalar, \(C \in \mathbb{R}^{m \times 3}\) and the ranges of the acceleration command of the missile and target can be expressed respectively as follows:
$$
A_{m\perp} = \{a_{m\perp} : \|a_{m\perp}\| \leq \rho_m(t)\}, \quad A_{1\perp} = \{a_{1\perp} : \|a_{1\perp}\| \leq \rho_1(t)\}
$$

By referring to the proof in [17], then the optimal strategy pair can be found as:
$$
\begin{align*}
a_{m\perp} = -\rho_m(t)(-\mathbf{CΦ}(t,f)\mathbf{I})/\|\mathbf{CΦ}(t,f)\| & \quad \text{sgn}(\mathbf{CΦ}(t,f)(-\mathbf{v}_\perp)) \\
a_{1\perp} = -\rho_1(t)(\mathbf{CΦ}(t,f)\mathbf{I})/\|\mathbf{CΦ}(t,f)\| & \quad \text{sgn}(\mathbf{CΦ}(t,f)(-\mathbf{v}_\perp))
\end{align*}
\tag{31, 32}
$$
where \(\mathbf{I}\) is the 3x3 identity matrix. Let the actual acceleration required by the autopilot of the missile system be denoted as \(\mathbf{a}_{\perp}\). The final result of the relationship between \(\mathbf{a}_{\perp}\) and \(\mathbf{a}_{\perp}\) is \(\mathbf{a}_{\perp} = (1+E)\mathbf{a}_{\perp}\), where \(E\) is a scalar, defined as follow:
$$
E = (1/\rho^2)E_g
$$
$$
E_g = (1/\|\mathbf{a}_{\perp}\|)(2(S_v - I_0)\mathbf{A}(q)(\mathbf{a}_{\perp} \times \mathbf{a}_{\perp})\mathbf{A}(q)^\top (S_v - I_0)\mathbf{a}_{\perp}) + \rho_1 a_{\perp} a_{\perp}
$$
where \(S_v = \mathbf{P} + I_0\) with \(\mathbf{P} = I_{3 \times 3}\), and the nonzero control is assumed, i.e., \(\|a_{\perp}\| = 0\).

Next, we can design the Lyapunov function-like candidate as:
$$
V_g(v_\perp) = (1/2)v_\perp^\top v_\perp.
$$
Thus, taking its time derivative leads to
$$
\dot{V}_g(v_\perp) = v_\perp^\top (a_{\perp} - (1+E)a_{\perp} - (v \cdot \hat{\mathbf{r}}/\|\mathbf{r}\|) v_\perp)
$$
\tag{33}

By referring to equations (31) and (32), we can obtain:
$$
\dot{V}_g(v_\perp) = v_\perp^\top (- (v \cdot \hat{\mathbf{r}}/\|\mathbf{r}\|) v_\perp + v_\perp^\top (\rho_1(t) + (1+E)\rho_1(t))
\times \mathbf{C}(\mathbf{CΦ}(t,f)\mathbf{I})^\top \mathbf{CΦ}(t,f)(-v_\perp))\mathbf{C}(\mathbf{CΦ}(t,f)\mathbf{I})\|\mathbf{CΦ}(t,f)(-v_\perp\|}
\text{and then } v_\perp(t) = \Phi(t,f) v_\perp(t_0), \text{ where } t_0 \text{ is the initial time of the system. For simplicity of equation notation, we define } \Phi(t,f) = \Phi. \text{ Thus,}
\tag{34}
$$
$$
\dot{V}_g(v_\perp) = -v_\perp^\top (v \cdot \hat{\mathbf{r}}/\|\mathbf{r}\|) + (\rho_1(t) + (1+E)\rho_1(t))
\times (\mathbf{C}(\mathbf{CΦ})^\top \mathbf{CΦ}/\mathbf{C}(\mathbf{CΦ}))\|\mathbf{CΦ}(v_\perp)\|
\$$

Because \(v \cdot \hat{\mathbf{r}}/\|\mathbf{r}\| = (1/\|\mathbf{r}\|)d\|\mathbf{r}\|/dt = (d/dt)\ln\|\mathbf{r}\|\) is bounded for \(\|\mathbf{r}\| \neq 0\), and the last term of equation (34) is a positive scalar upper-bounded by \(\rho\), we readily obtain the following inequality:
$$
\dot{V}_g(v_\perp) \leq -v_\perp^\top (\|\mathbf{r}\|/\|\mathbf{r}\| < 0
\tag{35}
$$

According to the Lyapunov stability theory, we conclude that the equilibrium point \(v_\perp = 0_{3x1}\) is globally asymptotically stable regardless of the presence of attitude error.

### VI. Simulation Results

So far, we have shown that the stability of the proposed missile guidance systems is guaranteed in Section V. This section will present the simulation results, and will try to make comparison between the current results and those from the previous works (only with sliding mode guidance) [16]. In order to make analysis significant, all the initial conditions of the missile and the target are the same as in the previous works. The behavior of maneuvering target is a free fall and with the escape strategy as shown in equation (32).

The simulation results are shown in Fig. 6 to Fig. 9. The performance of the tracking algorithm proposed in this paper is shown in Fig. 6. The tacking errors of relative distance and relative velocity are both less than 1%, and the tacking error of the relative acceleration converge is less than 5%. They mean that the maneuvering target can be tracked precisely by the interceptor deployed with the proposed system. The interceptor with differential game based guidance system (blue line) as proposed in this paper however successfully accomplishes the target interception mission at 104.19 sec. in Fig. 7. Compared with the previous works (red line) whose interceptors are equipped with sliding mode guidance, we found that they failed to intercept the maneuvering target even when their corresponding motions are adjusted more frequently. In Figs. 8 and 9, the \(v_\perp\), guidance commands, the missile's energy consumption are presented. By analyzing these simulation results, the interceptor with our proposed system not only completes the target interception, but also can
keep reasonably small tracking error and good performance on the critical parameters of missile interceptor.

VII. CONCLUSIONS

In this paper, to tackle maneuvering target, we proposed a differential game based guidance law with passive ranging system in 3-D space. By selection of suitable motion models of the attacking (target) and treat them as sub-filters in IMM for an extended Kalman Filter, the trajectory of the target subject to maneuvering can still be tracked very accurately. The performances of the hereby proposed differential game based guidance law are compared with sliding mode guidance, which demonstrates that our approach not only satisfies the requirement of the target interception but also exhibits better performance. Moreover, the stability is proved via Lyapunov stability analysis. Finally, the effectiveness of our approach is verified by numerical simulations.

Fig. 6. The estimation error of maneuvering target tracking.

Fig. 7. The 3-D trajectory of maneuvering target interception.

Fig. 8. The performance of interceptor with the previous work.

Fig. 9. The performance of interceptor with our proposed system.

REFERENCES