Abstract—The tracking synchronization problem of multi-agent systems with unknown nonlinear and heterogeneous dynamics is studied. A feedback linearization approach is used with neural networks to compensate for the uncertainties caused by the unknown and nonlinear dynamics of agents. Using the Lyapunov theory, an adaptive distributed control scheme is designed, and verified through the secondary voltage control of the practical microgrids.

I. INTRODUCTION

Cooperative multi-agent systems have earned much attention due to their flexibility and computational efficiency. In these systems, the coordination and synchronization process necessitates that each agent exchange information with other agents according to some restricted communication protocol [1-4]. Cooperative control problems for consensus and synchronization of multi-agent systems are mainly categorized into the regulator synchronization problem and the tracking synchronization problem. In the regulator synchronization problem all agents synchronize to a common value that is not prescribed or controllable. In the tracking synchronization problem, all agents synchronize to a leader node that acts as a command generator. The leader node is only connected to a small portion of the agents [4-6].

The synchronization problem of multi-agent systems has been previously investigated for different types of systems. The synchronization problem for multi-agent systems with single and double integrator linear dynamics has been studied in [7-8]. This problem for multi-agent systems with high order dynamics (i.e. with greater than two integrators) is considered in [9-10]. In [11-13], the synchronization problem in the case of identical nonlinear dynamics is studied. Adaptive control for the consensus problem of multi-agent systems with unknown nonlinear dynamics is studied in [14-17].

In this paper, the tracking synchronization problem of multi-agent systems with unknown nonlinear and heterogeneous dynamics is of concern. Input-output feedback linearization is used to provide a direct relationship between the dynamics of agent outputs and control inputs. Applying input-output feedback linearization, the synchronization problem is transformed to a high order synchronization problem with unknown nonlinear dynamics. Linear-in-parameter neural networks (NN) are used to compensate for the uncertainties caused by the unknown and nonlinear dynamics of agents. The Lyapunov theory is used to design fully distributed controllers such that the controller at each agent only requires its own information and the information of its neighbors.

The proposed adaptive distributed control is applied to the design of the secondary voltage control of microgrids as a practical example to demonstrate the effectiveness of the approach in handling nonlinear heterogeneous multi-agent systems. Microgrids are small-scale power systems containing distributed generators (DG), and can be taken as nonlinear heterogeneous multi-agent systems mathematically. Microgrids may get islanded from the main power grid. Once islanded, the DG voltage amplitudes start to deviate. To maintain these voltage amplitudes in stable ranges, the so-called primary control is applied. However, primary control may not return the DG voltage amplitudes to the nominal voltage. This function is provided by the secondary control, which compensates for the voltage and frequency deviations caused by the primary control [18-19]. The dynamics of DGs are nonlinear and non-identical. Therefore, the secondary voltage control resembles the tracking synchronization of a multi-agent system with nonlinear and non-identical dynamics. The proposed adaptive and distributed control scheme is used to implement an adaptive secondary voltage control that compensates for the uncertainties of DG parameters.

II. PRELIMINARIES OF GRAPH THEORY

The communication network of a multi-agent cooperative system can be modeled by a directed graph (digraph). A digraph is usually expressed as $G=(V,E,A_G)$ with a nonempty finite set of $N$ nodes $V_G=\{v_1,v_2,\ldots,v_N\}$, a set of edges or arcs $E_G \subseteq V_G \times V_G$, and the associated adjacency matrix $A_G=[a_{ij}] \in \mathbb{R}^{N \times N}$. In a microgrid, DGs are considered as the nodes of the communication digraph. The edges of the corresponding digraph of the communication network denote the communication links.

In this paper, the digraph is assumed to be time invariant, i.e., $A_G$ is constant. An edge from node $j$ to node $i$ is denoted by $(v_j,v_i)$, which means that node $i$ receives the information from node $j$. $a_{ij}$ is the weight of edge $(v_j,v_i)$, and $a_{ij}=0$ if $(v_j,v_i) \in E_G$, otherwise $a_{ij} \neq 0$. Node $i$ is called a neighbor of node $j$, if $(v_j,v_i) \in E_G$. The set of neighbors of node $j$ is denoted as $N_j=\{i|(v_j,v_i) \in E_G\}$. For a digraph, if node $i$ is a neighbor of node $j$, then node $j$ can get information from node $i$, but not
necessarily vice versa. The in-degree matrix is defined as 
\[ D = \text{diag} \{d_i\} \in R^{N \times N} \] with 
\[ d_i = \sum_{j \in N} a_{ij} \] . The Laplacian matrix is 
defined as 
\[ L = D - A_G \] . A direct path from node \( i \) to node \( j \) is a 
sequence of edges, expressed as \{\((v_i, v_k), (v_k, v_j), \ldots, (v_m, v_j)\)\}. A 
digraph is said to have a spanning tree, if there is a root node 
with a direct path from that node to every other node in the 
graph [16].

III. PROBLEM FORMULATION

Consider \( n \) nonlinear and heterogeneous systems or 
agents that are distributed on a communication digraph \( G_r \) 
with the node dynamics 
\[
\begin{align*}
\dot{x}_i &= M_i(x_i) + N_i(x_i)u_i, \\
y_i &= h_i(x_i)
\end{align*}
\]  
(1)

where \( x_i(t) \in R^k \) is the state vector, \( u_i(t) \in R \) is the control 
input, and \( y_i(t) \in R \) is the output of \( i \)th node, and 
\( M_i(\cdot): R^k \rightarrow R^k \) and \( N_i(\cdot): R^k \rightarrow R^k \) are unknown and locally 
Lipschitz in \( R^k \). The agent state dynamics and state 
dimensions \( n_i \) do not need to be the same. The usual 
assumptions are made to ensure existence of unique 
solutions.

In the tracker synchronization problem, it is desired to 
design distributed control inputs \( u_i(t) \) to synchronize the 
output of all nodes to the output of a leader node \( y_0(t) \), i.e., 
one requires \( y_i(t) \rightarrow y_0(t), \forall i \). The leader node can be 
viewed as a command generator that generates the desired 
trajectory 
\[
\begin{align*}
\dot{x}_0 &= M_0(x_0) \\
y_0 &= h_0(x_0)
\end{align*}
\]  
(2)
The functions \( M_0 \) and \( h_0 \) are assumed to be of class \( C^\infty \).

A direct relationship between the dynamics of the 
outputs \( y_i(t) \) and the control inputs \( u_i(t) \) is generated by 
Differentiating the output of the \( i \)th agent yields 
\[
\dot{y}_i = L_{M_i}h_i + L_{N_i}h_iu_i.
\]  
(3)

If \( L_{N_i}h_i \) is equal to zero, the differentiation process is 
continued until \( u_i \) appears in the \( r \)th order derivative of \( y_i(t) \).

Definition 1: For the smooth function \( h(x) : R^k \rightarrow R \) and 
smooth vector field \( f(x) : R^k \rightarrow R^k \),

\[
L_j^k h = L_j(L_j^{k-1}h) = \frac{\partial(L_j^{k-1}h)}{\partial x} f,
\]  
where \( L_j h \) is the Lie derivative.

Assumption 1: There exists an \( r > 1 \) such that 
\[ a. L_{N_i}L_{M_i}^{-1}h_i = 0, \text{ for } l < r - 1, \forall i. \]
\[ b. L_{N_i}L_{M_i}^{-r-1}h_i \neq 0, \forall i. \]

According to Definition 1 and Assumption 1a, the \( r \)th 
derivative of \( y_i(t) \) can be written as 
\[
y_i^{(r)} = L_{M_i}r h_i + L_{N_i}L_{M_i}^{-r-1}h_iu_i.
\]  
(4)

Equation (4) and the first \( r \) derivatives of \( y_i \equiv y_{i1} \) can be 
written as 
\[
\begin{align*}
\dot{y}_{i1} &= y_{i2} \\
\dot{y}_{i2} &= y_{i3} \\
&\vdots \\
\dot{y}_{i,r-1} &= y_{i,r} \\
\dot{y}_{i,r} &= f_i(x_i) + g_i(x_i)u_i
\end{align*}
\]  
(5)

where \( f_i(x_i) = L_{M_i}r h_i \) and \( g_i(x_i) = L_{N_i}L_{M_i}^{-r-1}h_i \). The 
functions \( f_i(x_i) \) and \( g_i(x_i) \) are nonlinear and are assumed 
to be unknown in the control design process. In [16], and 
adaptive distributed control is proposed when \( g_i(x_i) = 1 \). 
This paper improves the control method in [16], and 
proposes a control method to solve the tracking 
synchronization problem for multi-agents systems in which 
both functions \( f_i(x_i) \) and \( g_i(x_i) \) are nonlinear and unknown.

Assumption 2: The functions \( g_i(x_i) \) satisfy \( g_2 < g_i(x_i) \), 
where \( g_2 \) is a positive constant. A positive function \( g_0(x_i) \) 
exists such that 
\[
\begin{align*}
\dot{y}_{i1} &= y_{i2} \\
\dot{y}_{i2} &= y_{i3} \\
&\vdots \\
\dot{y}_{i,r-1} &= y_{i,r} \\
\dot{y}_{i,r} &= f_i(x_i) + g_i(x_i)u_i
\end{align*}
\]  
(6)

Using the input-output feedback linearization, the 
dynamics of each agent is decomposed into the \( r \)th-order 
dynamical system in (5) and a set of internal dynamics 
denoted as 
\[
\dot{\mu}_i = W_i(y_{i1}, \ldots, y_{i,r}, \mu_1, \ldots, \mu_{r-1}), i = 1, \ldots, n-r. \]  
(7)

The commensurate reformulated dynamics of the leader 
node in (2) can be expressed as 
\[
\begin{align*}
\dot{y}_{i1} &= y_{i2} \\
\dot{y}_{i2} &= y_{i3} \\
&\vdots \\
\dot{y}_{i,r} &= f_0(x_0)
\end{align*}
\]  
(8)

where \( y_0 \equiv y_{01} \). All required derivatives of \( y_0 \) exists since 
\( M_0(\cdot), h_0(\cdot) \in C^\infty \).

Define \( Y=[y_{i1}, y_{i2}, \ldots, y_{i,r}]^T \) and \( Y_0=[y_{01}, y_{02}, \ldots, y_{0,r}]^T \).
The synchronization problem is solved if a distributed \( u_i \) in (5) is 
found such that \( Y_i \rightarrow Y_0, \forall i \).

IV. THE LYAPUNOV DESIGN OF ROBUST ADAPTIVE 
CONTROLLER

In this section, first, the sliding mode error is introduced. 
Then, the Lyapunov function technique is used to design 
distributed controllers for each node such that the controller 
at each node only requires its own information and the 
information of its neighbors on the communication digraph.

A. Sliding Mode Error

To solve the synchronization problem, the cooperative 
team objectives are expressed in terms of the local neighborhood tracking error
where \( b_i \) is the pinning gain by which node \( i \) is connected to the leader node. Define 
\[
e_i = \begin{bmatrix} e_{i,1} & e_{i,2} & \cdots & e_{i,r-1} \end{bmatrix}^T, 
\]
\[
\delta_{i,m} = y_{i,m} - y_{0,m}, \quad \delta_i = Y_i - Y_0, \quad e = [e_1 \quad e_2 \cdots \quad e_N]^T, \quad \text{and} 
\]
\[
\delta = \begin{bmatrix} \delta_1 & \delta_2 & \cdots & \delta_N \end{bmatrix}^T. 
\]
The global neighborhood tracking error \( e \) can be written as 
\[
e = (L + B)\delta. 
\]
where \( B = \text{diag} \{ b_i \} \).

**Lemma 1:** Let the digraph \( Gr \) have a spanning tree and \( b_i \neq 0 \) for at least one root node. Then 
\[
\|\delta\| \leq \|e\| / \sigma(L + B), 
\]
and \( e = 0 \) if and only if all nodes synchronize.

The sliding mode error \( r_i \) for each agent is defined as 
\[
r_i = \lambda_i e_{i,1} + \lambda_2 e_{i,2} + \cdots + \lambda_{r-1} e_{i,r-1} + e_{i,r}. 
\]
The parameters \( \lambda_i \) are chosen such that the polynomial \( \lambda_1 + \lambda_2 s + \cdots + \lambda_{r-1} s^{r-2} + s^{-1} \) is Hurwitz. Therefore, on the sliding surface \( r_i = 0 \), \( e_i \) exponentially goes to zero. The derivative of the sliding mode error can be written as 
\[
\dot{r_i} = \rho_i + \sum_{j \in N_i} a_{ij}(f_j + g_iu_i - f_j - g_ju_j) + b_i(f_i + g_iu_i - f_0), 
\]
where 
\[
\rho_i = \lambda_1 e_{i,2} + \lambda_2 e_{i,3} + \cdots + \lambda_{r-1} e_{i,r}. 
\]
Defining \( d_i = \sum_{j \in N_i} a_{ij} \), (12) can be reformulated as 
\[
\dot{r_i} = \rho_i + (d_i + b_i)(f_i + g_iu_i) - b_i f_0 - \sum_{j \in N_i} a_{ij}(f_j + g_ju_j). 
\]
Define 
\[
E_{i,1} = \begin{bmatrix} e_{i,1} & e_{i,2} & \cdots & e_{i,r-1} \end{bmatrix}, 
\]
\[
\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
-\lambda_1 & -\lambda_2 & -\lambda_3 & \cdots & -\lambda_{r-1} \end{bmatrix}, 
\]
then, 
\[
\dot{E}_{i,1} = E_{i,1}\Lambda^T + \rho_i. 
\]
Note that \( \Lambda \) is Hurwitz. Therefore, given any positive real number \( \beta \), a symmetric and positive definite matrix \( P_i \) exists such that 
\[
\Lambda^T P_i + P_i\Lambda = -\beta I, 
\]
where \( I \) is the identity matrix.

**B. The Adaptive and Distributed Controller Design**

The following energy function is considered [17].

\[
V_{ri} = \frac{1}{2} \lambda_i^2. 
\]
Differentiating \( V_{ri} \) and replacing \( \dot{r}_i \) from (12) yields 
\[
\dot{V}_{ri} = \frac{1}{2} (g_{i0} + \frac{g_{i0}}{g_i} \dot{g}_i)^2 + r_i g_i^{-1} \sum_{j \in N_i} a_{ij}(f_j + g_ju_j) + g_i g_{i0} \dot{g}_i - b_i f_0, 
\]
or equivalently 
\[
\dot{V}_{ri} = \frac{1}{2} (g_{i0} + \frac{g_{i0}}{g_i} \dot{g}_i)^2 + r_i \tilde{f}_i + r_i g_i^{-1} (d_i + b_i)u_i, 
\]
where 
\[
\tilde{f}_i = \rho_i + (d_i + b_i)f_i - b_i f_0 + g_i g_{i0} \dot{g}_i \frac{g_{i0}}{g_i}. 
\]
is a function of the own information of \( i \)th agent, and 
\[
\tilde{g}_i = -\sum_{j \in N_i} a_{ij}(f_j + g_ju_j) \frac{g_{i0}}{g_i}, 
\]
is a function of the own information of \( i \)th agent and the information of its neighbors on the communication digraph.

The unknown nonlinear function \( \tilde{f}_i \) is approximated on a prescribed compact set \( \Omega^f \) by linear-in-parameter NN [21] 
\[
\tilde{f}_i = \Phi_{\tilde{f}_i}(r_i, \rho_i, x_i, b_i, f_0) + \epsilon_{\tilde{f}_i}, 
\]
where the NN weight vector is \( W_{\tilde{f}_i} \in R^{l_{\tilde{f}}}, \epsilon_{\tilde{f}_i} \) is the NN estimation error, and \( \Phi_{\tilde{f}_i} \in R^{l_{\tilde{f}}} \) consists of a set of \( l_{\tilde{f}} \) basis functions. The estimations of weight vectors are denoted as \( \hat{W}_{\tilde{f}_i} \). The error of the NN weights is defined as 
\[
\tilde{W}_{\tilde{f}_i} = W_{\tilde{f}_i} - \hat{W}_{\tilde{f}_i}. 
\]
The unknown nonlinear function \( \tilde{g}_i \) is approximated by linear-in-parameter NNs 
\[
\tilde{g}_i = \Phi_{\tilde{g}_i}(x_i, r_{-i}, x_{-i}, \hat{W}_{\tilde{f}_{-i}}^T, \hat{W}_{\tilde{f}_{-i}}) + \epsilon_{\tilde{g}_i}, 
\]
where the NN weight vector is \( W_{\tilde{g}_i} \in R^{l_{\tilde{g}}}, \epsilon_{\tilde{g}_i} \) is the NN estimation error, and \( \Phi_{\tilde{g}_i} \in R^{l_{\tilde{g}}} \). The estimations of weight vectors are denoted as \( \hat{W}_{\tilde{g}_i} \). The terms \( r_{-i}, x_{-i}, \hat{W}_{\tilde{f}_{-i}}^T, \hat{W}_{\tilde{f}_{-i}} \) are the sliding mode error, states, and the NN estimated weight vectors of the neighbors of agent \( i \) on the communication digraph, respectively. The error of the NN weights is defined as 
\[
\tilde{W}_{\tilde{g}_i} = W_{\tilde{g}_i} - \hat{W}_{\tilde{g}_i}. 
\]

**Remark 1:** Considering Stone-Weierstrass approximation theorem [22], positive numbers \( W_{\tilde{f}_i}^M, \epsilon_{\tilde{f}_i}^M, W_{\tilde{g}_i}^M, \epsilon_{\tilde{g}_i}^M \) exist such that 
\[
|\tilde{W}_{\tilde{f}_i}| \leq W_{\tilde{f}_i}^M, \quad |\epsilon_{\tilde{f}_i}| \leq \epsilon_{\tilde{f}_i}^M, \quad |\tilde{W}_{\tilde{g}_i}| \leq W_{\tilde{g}_i}^M, \quad \text{and} \quad |\epsilon_{\tilde{g}_i}| \leq \epsilon_{\tilde{g}_i}^M. 
\]
Definition 2: The $Y_i$ are cooperative uniformly ultimately bounded with respect to $Y_0$ if there exists a compact set $\Omega \subset \mathbb{R}^\ell$ so that $\forall (Y_i(t_0) - Y_0(t_0)) \in \Omega$ there exists a bound $B$ and a time $t_f(B,Y_i(t_0) - Y_0(t_0))$, both independent of $t_0$, such that $\|Y_i(t_0) - Y_0(t_0)\| \leq B, \forall t > t_0 + t_f$ [16].

Theorem 1: The commensurate reformulated dynamics of the agents in (1) and the leader node in (2) are considered. Let the digraph $G_r$ contain a spanning tree and $b_i \neq 0$ for at least one root node. It is assumed that the internal dynamics in (6) are asymptotically stable. Supposed that the control inputs are chosen as

$$u_i = -c_i r_i - \frac{\dot{W}_{\ell_i} \phi_{\ell_i}}{d_i + b_i} - \frac{W_{\ell_i} \phi_{\ell_i}}{d_i + b_i},$$

where $c_i$ is the control gain, and the tuning laws are chosen as

$$\dot{W}_{\ell_i} = F_{\ell_i} \phi_{\ell_i} - \kappa_{\ell_i} F_{\ell_i} \hat{W}_{\ell_i},$$

$$\dot{W}_{\ell_i} = F_{\ell_i} \phi_{\ell_i} - \kappa_{\ell_i} F_{\ell_i} \hat{W}_{\ell_i},$$

where the arbitrary positive definite matrices $F_{\ell_i} \in \mathbb{R}^{\ell_i \times \ell_i}$ and $F_{\ell_i} \in \mathbb{R}^{\ell_i \times \ell_i}$ and the coefficients $\kappa_{\ell_i}$ and $\kappa_{\ell_i} > 0$ are the design parameters. Then $Y_i$ is cooperative UUB with respect to $Y_0$ and all nodes synchronize to $Y_0$ if $c_i$ is chosen as

$$c_i > \frac{\sigma^2(P_i)}{2\beta(d_i + b_i)}.$$  

Proof: The Lyapunov function candidate for each agent is considered as

$$V_i = \frac{1}{2} \frac{r_i^2}{g_i} + \frac{1}{2} \dot{W}_{\ell_i} F_{\ell_i}^{-1} \dot{W}_{\ell_i} + \frac{1}{2} W_{\ell_i} F_{\ell_i}^{-1} \dot{W}_{\ell_i} + \frac{1}{2} E_{\ell_i} P E_{\ell_i}^T.$$  

The derivative of $V_i$ is written as

$$\dot{V}_i = \frac{1}{2} \left(-g_{i0} + \frac{g_{i0}}{g_i} \right) r_i^2 + n_i r_i + r_i (d_i + b_i) u_i$$

$$+ \dot{W}_{\ell_i} F_{\ell_i}^{-1} \dot{W}_{\ell_i} + \hat{W}_{\ell_i} F_{\ell_i}^{-1} \hat{W}_{\ell_i} + E_{\ell_i} P E_{\ell_i}^T.$$  

Placing (17), (27), and (28) into (31) yields

$$\dot{V}_i = \frac{1}{2} \left(-g_{i0} + \frac{g_{i0}}{g_i} \right) r_i^2 - c_i (d_i + b_i) r_i^2$$

$$- \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i} - \kappa_{\ell_i} \hat{W}_{\ell_i} W_{\ell_i} \right|^2 - \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i} \right|^2$$

$$- \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i} + E_{\ell_i} P E_{\ell_i}^T + r_i P E_{\ell_i}^T + e_{\ell_i}^T + e_{\ell_i} \right|^2.$$  

Placing (18) into (32) yields

$$\dot{V}_i = \frac{1}{2} \left(-g_{i0} + \frac{g_{i0}}{g_i} \right) r_i^2 - c_i (d_i + b_i) r_i^2 - \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i} - \kappa_{\ell_i} \hat{W}_{\ell_i} W_{\ell_i} \right|^2$$

$$- \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i} - \kappa_{\ell_i} \hat{W}_{\ell_i} W_{\ell_i} \right|^2.$$  

According to Assumption 2 and Remark 1

$$\dot{V} \leq -c_i (d_i + b_i) r_i^2 + \kappa_{\ell_i} \left| \dot{W}_{\ell_i} W_{\ell_i}^T \right|^2$$

$$- \kappa_{\ell_i} \left| \dot{W}_{\ell_i} \right|^2 + \kappa_{\ell_i} \left| \hat{W}_{\ell_i} \right|^2 W_{\ell_i}^T - \kappa_{\ell_i} \left| \dot{W}_{\ell_i} \right|^2$$

$$- \beta \left| \hat{W}_{\ell_i} \right|^2 E_{\ell_i}^T E_{\ell_i}^T + r_i P E_{\ell_i}^T + e_{\ell_i}^T + e_{\ell_i}.$$  

Equation (34) can be written as

$$\dot{V} \leq -H^T SH + G^T H,$$

where

$$H = \begin{bmatrix} E_{\ell_i} \hat{W}_{\ell_i} W_{\ell_i} \hat{W}_{\ell_i}^T \end{bmatrix},$$

$$S = \begin{bmatrix} 0 & -\kappa_{\ell_i} & -\kappa_{\ell_i} \\ \frac{\beta}{2} & 0 & 0 \\ 0 & 0 & 0 \\ \sigma(P_i) & 0 & 0 \\ 0 & 0 & \kappa_{\ell_i} \end{bmatrix},$$

$$G = \begin{bmatrix} 0 & \kappa_{\ell_i} W_{\ell_i}^T & \kappa_{\ell_i} W_{\ell_i}^T \end{bmatrix}^T.$$  

If the following conditions hold,

1) $S$ is positive definite, and

2) $\|H\| > \frac{\|G\|}{\sigma(S)},$

then $\dot{V} < 0$. According to Sylvester's criterion, $S$ is positive definite if

$$c_i > \frac{\sigma^2(P_i)}{2\beta(d_i + b_i)}.$$  

Since $\|G\|_2 \geq \|G\|_1 \geq \cdots \geq \|G\|_\infty$, the second condition holds if

$$\begin{bmatrix} E_{\ell_i} \hat{W}_{\ell_i} W_{\ell_i} \hat{W}_{\ell_i}^T \end{bmatrix} > \frac{\|G\|}{\sigma(S)},$$

$$\hat{W}_{\ell_i} > \frac{\|G\|}{\sigma(S)},$$

$$\hat{W}_{\ell_i} > \frac{\|G\|}{\sigma(S)},$$

$$|r_i| > \frac{\|G\|}{\sigma(S)}.$$  

Therefore, the sliding mode error and the NN weights approximation errors are ultimately bounded by $\frac{\|G\|}{\sigma(S)}$. Since the sliding mode errors are ultimately bounded, the local neighborhood tracking errors in (8) are also bounded [16]. According to Lemma 1, $\delta_i$ are also ultimately bounded and, hence, all $Y_i$ are cooperative uniformly ultimately bounded with respect to $Y_0$ and all nodes synchronize to the leader node. If the zero dynamics are asymptotically stable, then (5) , (6), and (26) are asymptotically stable. This completes the proof. □
V. SECONDARY VOLTAGE CONTROL OF MICROGRIDS

In this section, the adaptive feedback linearization-based tracking synchronization method presented in Section IV is used to implement the secondary voltage control of microgrids. Figure 1 shows the block diagram of an inverter-based DG. Note that nonlinear dynamics of each DG in a microgrid are formulated on its own d-q (direct-quadrature) reference frame. The reference frame of microgrid is considered as the common reference frame, and the dynamics of other DGs are transformed to the common reference frame. The angular frequency of this common reference frame is denoted by \( \omega_{com} \).

The nonlinear dynamics of the \( i^{th} \) DG by neglecting the fast dynamics of voltage and current controllers can be written as

\[
\begin{align*}
\dot{x}_1 &= \omega_{ni} - m_{pi} x_2 - \omega_{com}, \\
\dot{x}_2 &= \omega_{xi} (x_6 x_8 + x_7 x_9 - x_2), \\
\dot{x}_3 &= \omega_{xi} (x_6 x_9 - x_7 x_8 - x_3), \\
\dot{x}_4 &= \frac{-r_{fi}}{L_{fi}} x_4 + \omega_{com} x_5 + \frac{V_{ni} - n_{Qi} x_3 - x_6}{L_{fi}}, \\
\dot{x}_5 &= \frac{-r_{fi}}{L_{fi}} x_5 - \omega_{com} x_4 - \frac{x_7}{L_{fi}}, \\
\dot{x}_6 &= \omega_{com} x_7 + \frac{x_4 - x_8}{C_{fi}}, \\
\dot{x}_7 &= -\omega_{com} x_6 + \frac{x_5 - x_9}{C_{fi}}, \\
\dot{x}_8 &= \frac{-r_{ci}}{L_{ci}} x_8 + \omega_{com} x_9 + \frac{x_6 - y_{bdi}}{L_{ci}}, \\
\dot{x}_9 &= \frac{-r_{ci}}{L_{ci}} x_9 - \omega_{com} x_8 - \frac{x_7 - y_{bqi}}{L_{ci}},
\end{align*}
\]

(41)

where \( x_1 = [\delta_i P_i Q_i i_{ldi} i_{lqi} v_{odi} v_{odi} i_{odi} i_{lqi}]^T \).

The primary voltage control is usually implemented as a local controller at each DG using the droop technique. Droop technique prescribes a desired relation between the voltage amplitude and the reactive power. The primary voltage control is

\[
v_{o,magi}^* = V_{ni} - n_{Qi} Q_i,
\]

(42)

where \( V_{o,magi}^* \) is the voltage set point provided for the internal voltage control, \( Q_i \) is the filtered reactive power at the DG’s terminal, \( n_{Qi} \) is the droop coefficient that is chosen based on the reactive power ratings of DGs, and \( V_{ni} \) is the primary control reference [18].

The secondary voltage control chooses \( V_{ni} \) such that the output voltage amplitude of each DG synchronizes to its nominal value, i.e., \( v_{o,magi} \rightarrow v_{ref} \). For secondary voltage control, the input and output in (1) are \( u_i = V_{ni} \) and \( y_i = v_{o,magi} \), respectively. Considering the nonlinear dynamics of each DG in (41), the input \( V_{ni} \) appears in the second derivate of \( v_{o,magi} \), i.e., \( r = 2 \) in (4).

VI. SIMULATION RESULTS

The effectiveness of the proposed adaptive and robust distributed control is verified by simulation on an islanded microgrid shown in Fig. 2. This microgrid consists of four DGs. The lines between buses are modeled as series RL branches. The specifications of the lines and loads are summarized in Table I. Due to the adaptive nature of the proposed methodology, the specifications of DGs are not required for the secondary control implementation. However, these specifications are required to model DGs in the simulations and are summarized in Table 1.

It is assumed that the DGs communicate with each other through the communication network shown in Fig. 2. DG 1 is the only digraph that is connected to the leader node with \( g_1 = 1 \). The coupling gain in (26) is \( c_r = 10 \) which satisfies (29).

The NN parameters are set to \( F_{i} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \), \( F_{\theta} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \), \( \kappa_{i} = 10 \), and \( \kappa_{\theta} = 10 \). It is assumed that the microgrid is isolated from the main grid at \( t = 0 \), and the secondary control is applied at \( t = 1.5 \) s. Figure 3 shows the simulation results when the reference voltage value is set to 380 V. As seen in Fig. 3, while the primary control keeps the voltage amplitudes stable, the secondary control establishes all the terminal voltage amplitudes to the pre-specified reference values after 0.3 seconds.

VII. CONCLUSION

In this paper, a fully distributed and adaptive control scheme has been presented for the synchronization problem of multi-agent systems with unknown nonlinear and non-identical dynamics. The proposed control scheme has been used for the design of the secondary control of microgrids with unknown DG parameters for they can be taken as multi agents. A microgrid test system is used to verify the effectiveness of the proposed controller. The simulation results show that the proposed controller provides the synchronization for the output voltage of DGs.
Fig 2. The microgrid test system and implemented communication network.

Table I

<table>
<thead>
<tr>
<th>SPECIFICATIONS OF THE MICROGRID TEST SYSTEM</th>
<th>DG 1 &amp; 2</th>
<th>DG 3 &amp; 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_P</td>
<td>9.4×10⁻³</td>
<td>12.5×10⁻³</td>
</tr>
<tr>
<td>n_P</td>
<td>1.3×10⁻³</td>
<td>1.5×10⁻³</td>
</tr>
<tr>
<td>R_c</td>
<td>0.03 Ω</td>
<td>0.03 Ω</td>
</tr>
<tr>
<td>L_c</td>
<td>0.35 mH</td>
<td>0.35 mH</td>
</tr>
<tr>
<td>R_t</td>
<td>0.1 Ω</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>L_t</td>
<td>1.35 mH</td>
<td>1.35 mH</td>
</tr>
<tr>
<td>C_t</td>
<td>50 µF</td>
<td>50 µF</td>
</tr>
<tr>
<td>Line 1</td>
<td>R_l1</td>
<td>0.23 Ω</td>
</tr>
<tr>
<td>Line 2</td>
<td>R_l2</td>
<td>0.35 Ω</td>
</tr>
<tr>
<td>Line 3</td>
<td>R_l3</td>
<td>0.23 Ω</td>
</tr>
<tr>
<td>Load 1</td>
<td>L_l0</td>
<td>318 µH</td>
</tr>
<tr>
<td>Load 2</td>
<td>L_l1</td>
<td>1847 µH</td>
</tr>
<tr>
<td>Loads</td>
<td>P_l1</td>
<td>12 kW</td>
</tr>
<tr>
<td></td>
<td>P_l2</td>
<td>15.3 kW</td>
</tr>
<tr>
<td></td>
<td>Q_l1</td>
<td>12 kVAr</td>
</tr>
<tr>
<td></td>
<td>Q_l2</td>
<td>7.6 kVAr</td>
</tr>
</tbody>
</table>

Fig 3. DG output voltage magnitudes before and after applying the adaptive distributed control.

REFERENCES


