Robust Adaptive Control for Spacecraft Cooperative Rendezvous and Docking

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Abstract—The relative motion control for spacecraft cooperative rendezvous and docking is investigated. The relative position dynamics described in the target spacecraft’s line-of-sight (LOS) frame and attitude tracking dynamics of the chaser spacecraft are all formulated in Euler-Lagrange forms. A simple robust adaptive controller is designed for the chaser with modeling uncertainties. This controller utilizes the relative motion information and the orbit & attitude information of the target spacecraft in real time, and it can be proved via the Lyapunov theory that the closed-loop system is uniformly ultimately bounded stable. Numerical simulation results demonstrate the stability and robustness of the controlled closed-loop system.

I. INTRODUCTION

The spacecraft cooperative autonomous rendezvous and docking control has become an important research area in recent years due to rapidly growing space activities [1]. In this scenario, the orbit and attitude of the target spacecraft are controlled before the rendezvous and docking operations, and the orbit and attitude information of target is precisely priori known by the chaser. Thus, the relative motion and attitude dynamics are simplified to facilitate the integrated 6 degrees-of-freedom controller design for spacecraft rendezvous and docking.

Some research works have been published with a focus on dynamics and controls in the rendezvous and docking problem. Kluever [2] proposed a continuous feedback control law that guides a chaser to dock with a target with a desired approaching direction and speed. However, it used simple Clohessy-Wiltshire [3] equations, which is valid only for a circular orbit and also uses the assumption that the chaser is on the target orbit plane. More recently, Karlgaard [4] proposed a continuous feedback controller for rendezvous navigation in elliptical orbit. In this work, the full equations of relative motion were converted into full nonlinear equations in a spherical coordinate system, and then a navigation control law was derived with the simple feedback linearization method. Singla et al. [5] also proposed an adaptive output feedback control law based on full nonlinear equations of relative motion. Their control law did not use velocity feedback, and thus was less sensitive to the measurement noise. It was also adaptive to the unknown mass of the chaser spacecraft. In [6], for the capture phase of the rendezvous and docking mission, the authors proposed an attitude-synchronization approach combined with relative position tracking, which provides an effective method for developing a control system to ensure a successful capture.

Since spacecraft relative motion model is a typical Euler-Lagrange mechanical system as rigid-link robot manipulator, so the dynamics of autonomous rendezvous and docking can be expressed in a second-order differential equation. The main contribution of this paper can be described as follows. First, the relative motion dynamics model with modeling uncertainties is expressed in Euler-Lagrange form. Second, a simple integrated 6 degrees-of-freedom robust adaptive controller enlightened by the rigid-link robot manipulator control is designed for spacecraft cooperative rendezvous and docking.

The rest of the paper is organized as follows. The relative motion model is formulated with standing assumptions in the next section. Following, the control objective and a robust adaptive controller design methodology are described. Then a stability proof of the closed-loop system is given under the proposed controller. Finally, simulations results are provided to demonstrate the effectiveness of the designed controller.

II. RELATIVE MOTION EQUATIONS

Consider the relative motion of a chaser spacecraft to a cooperative target spacecraft in a general Keplerian orbit. The Local-Vertical-Local-Horizontal (LVLH) frame with \( (x, y, z) \) axes and origin fixed to the target spacecraft is used to describe the relative position motion. The \( +z \) axis directs to the center of the Earth, the \( +y \) axis points to the negative orbit normal, and the \( +x \) axis is along the velocity vector of the target spacecraft. The scalar \( \tau > 0 \) is the distance between the mass center of target and the center of the Earth, and \( \rho = \sqrt{x^2 + y^2 + z^2} \) is the distance between the chaser mass center and the target mass center. It is assumed that distance between these two spacecrafts is small compared with the target orbit radius, that is \( \tau \gg \rho \), and bounded disturbances and control forces are applied to the two spacecrafts [7]. These assumptions are generally made in the literature on the autonomous rendezvous and docking control for a short time period. Then, the relative motion equations can be described in LVLH frame as follows [8]:

\[
\begin{align*}
\dot{x} - \dot{v}z + 2\dot{v} \dot{z} - (v^2 - \frac{\mu}{\tau^2})x &= \frac{f^x}{m} + \frac{d^x}{m} \\
\dot{y} + \frac{\mu}{\tau^2} y &= \frac{f^y}{m} + \frac{d^y}{m} \\
\dot{z} + \ddot{v} x + 2\dot{v} \dot{x} - (v^2 - \frac{2\mu}{\tau^2})z &= \frac{f^z}{m} + \frac{d^z}{m}
\end{align*}
\]
where \( m \) is the mass of chaser; \( f^o = [f^o_x, f^o_y, f^o_z]^T \) and \( d^o = [d^o_x, d^o_y, d^o_z]^T \) are control and disturbance force vectors of the chaser spacecraft, respectively; \( \mu = 3.986 \times 10^{14} (m^3/s^2) \) is the Earth gravitational coefficient; \( \nu \) is the true anomaly of the target orbit. The evolution of \( \mathbf{r} \) and \( \mathbf{v} \) is governed by [9]

\[
\dot{\mathbf{r}} = -\frac{\mu \mathbf{v}}{2\mathbf{v}^2}, \quad \dot{\mathbf{v}} = \frac{n(1 + e_k \cos \nu)^2}{(1 - e_k^2)^{3/2}} \nu - \frac{2n^2(1 + e_k \cos \nu)^3 e_k \sin \nu}{(1 - e_k^2)^{3/2}}
\]

where \( e_k \) is the target orbital eccentricity, \( n = \sqrt{\frac{\mu}{a_k^3}} \) is the target mean orbital angular velocity, \( a_k = \frac{r_k}{1 - e_k} \) is the target orbital semi-major axis, \( r_k \) is the target perigee altitude.

The LOS frame with \((x_s, y_s, z_s)\) axis and origin located the mass center of the target spacecraft is also used to describe the relative position motion as the LVLH frame. Its \(x_s\)-axis points from the center of target to the center of chaser, \(z_s\)-axis is vertical with \(x_s\)-axis in \(oxz\) plane of the LVLH frame, and finally the direction of \(y_s\)-axis completes a right handed orthogonal frame. Then, we use \( \alpha \) to denote the elevation angle between the LOS and its projection in \(oxz\) plane of the LVLH frame, and use \( \beta \) to denote the azimuth angle between the \(x\)-axis of the LVLH frame and the LOS projection in \(oxz\) plane of the LVLH frame as shown in Figure 1.

![Fig. 1. Coordinate systems](image)

Define a spherical coordinate vector \( \mathbf{r}_s = [\rho, \alpha, \beta]^T \) which satisfies

\[
x = \rho \cos \alpha \cos \beta, \quad y = \rho \sin \alpha, \quad z = -\rho \cos \alpha \sin \beta
\]

where \( \rho \) is the distance between the chaser mass center and the target mass center. With the transformation (2), the relative position motion equation (1) can be described in the LOS frame as [7]:

\[
M_1(\mathbf{r}) \dot{\mathbf{r}}_s + N_1(\mathbf{r}_s, \dot{\mathbf{r}}_s) + g_1(\mathbf{r}_s) = E(f^o + d^o)
\]

where \( M_1(\mathbf{r}) = mH \), \( N_1(\mathbf{r}_s, \dot{\mathbf{r}}_s) = mA \). \( g_1(\mathbf{r}_s) = mA \).

\[
H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^2 \cos^2 \alpha \end{bmatrix},
\]

\[
A = \begin{bmatrix} 0 & -\rho \alpha & -c_1 \cos^2 \alpha \\ \rho \alpha & \rho \beta & c_2 \sin \alpha \cos \alpha \\ c_1 \cos^2 \alpha & c_2 \sin \alpha \cos \alpha & c_3 \end{bmatrix},
\]

\[
c_1 = \rho \beta - 2 \nu \dot{\nu}, \quad c_2 = \rho^2 \beta - 2 \nu^2, \quad c_3 = \rho \rho^2 \alpha - \rho^2 \alpha \sin \alpha \cos \alpha; \quad E = \text{diag}\{1, \rho, -\rho \cos \alpha\} \}
\]

The LOS frame with \( \mathbf{r} \) and \( \mathbf{v} \) are control and disturbance force vectors of the chaser spacecraft, respectively; \( \mathbf{r}_o \) is the rigid spacecraft with respect to the Earth-Centered Inertial (ECI) frame can be described in the modified Rodrigues parameters (MRP) as

\[
\sigma \triangleq \zeta \tan \left( \frac{\theta}{4} \right)
\]

where \( \theta \in (-2\pi, 2\pi) \) is the rotation angle about the Euler axis, and \( \zeta \) is the unit vector of the Euler axis. Then the chaser attitude dynamics in terms of the MRP take in the form [8]

\[
\begin{cases}
\dot{\sigma} = G(\sigma) \omega \\
J \dot{\omega} + S(\omega) J \omega = \tau_u + \tau_d
\end{cases}
\]

with

\[
G(\sigma) = \frac{1}{4} [(1 - \sigma^T \sigma) I_3 + 2 \sigma \sigma^T]
\]

and for arbitrary vector \( \phi = [\phi_1, \phi_2, \phi_3]^T \), \( S(\phi) \) denotes a skew-symmetric matrix which is given by

\[
S(\phi) \triangleq \begin{bmatrix} 0 & -\phi_1 & \phi_2 \\ \phi_1 & 0 & -\phi_3 \\ -\phi_2 & \phi_3 & 0 \end{bmatrix}
\]

and satisfies \( \phi^T S(\phi) = 0 \), also for the other arbitrary vector \( \eta \in \mathbb{R}^3 \), it follows that \( S(\phi) \eta = -S(\eta) \phi \) and \( \phi^T S(\eta) \phi = 0 \).

As noted in [8], \( G(\sigma) \) has following properties:

\[
G^{-1}(\sigma) = \frac{16}{(1 + \sigma^T \sigma)^2} G^T(\sigma)
\]

\[
\|G(\sigma)\| = \frac{1 + \sigma^T \sigma}{4}
\]

Denote \( P = G^{-1}(\sigma) \), we can rewritten the chaser attitude motion equation (4) as

\[
M_2(\sigma) \dot{\sigma} + N_2(\sigma, \dot{\sigma}) \sigma = p^T \tau_u + p^T \tau_d
\]
where \(M_2(\sigma) = P^TJP, \ N_2(\sigma, \dot{\sigma}) = P^TJP - P^TSP(\dot{\sigma})P\).

Thus, using (3) and (7), and defining system state \(q = [r^T, \sigma^T]^T\), we derive the integrated dynamics for relative position and attitude between chaser and target spacecrafts as

\[
M(q)\ddot{q} + N(q, \dot{q})q + g(q) = R(u + w)
\]

where

\[
M(q) = \begin{bmatrix}
M_1(r) & 0 \\
0 & M_2(\sigma)
\end{bmatrix}, \quad
N(q, \dot{q}) = \begin{bmatrix}
N_1(r, \dot{r}) & 0 \\
0 & N_2(\sigma, \dot{\sigma})
\end{bmatrix}, \quad
R = \begin{bmatrix}
ER_{\omega} & 0 \\
0 & P^T
\end{bmatrix},
\]

\[
u = [f^T, \tau_o^T]^T, \quad w = [d^T, \tau_o]^T, \quad f^c \quad \text{and} \quad d^c \quad \text{are the actual control forces and disturbance forces on the chaser spacecraft expressed in the chaser body frame, respectively.}
\]

\[
R_{sc} = R_{ct}R_{ci}^T \quad \text{is the coordinate transformation matrix from the chaser body frame to the LVLH frame,} \quad R_{ct} = R_{co}R_{oi}, \quad \text{and} \quad R_{ci} = R(\gamma_1)R(\gamma_2)R(\gamma_3) \quad \text{is the rotation matrix from the ECI frame to the LVLH frame and can be computed with the target orbital elements as follows:}
\]

\[
\begin{align*}
R(\gamma_1) &= \begin{bmatrix}
\cos \gamma_1 & \sin \gamma_1 & 0 \\
-\sin \gamma_1 & \cos \gamma_1 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
R(\gamma_2) &= \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \gamma_2 & \sin \gamma_2 \\
0 & -\sin \gamma_2 & \cos \gamma_2
\end{bmatrix} \\
R(\gamma_3) &= \begin{bmatrix}
\cos \gamma_3 & \sin \gamma_3 & 0 \\
-\sin \gamma_3 & \cos \gamma_3 & 0 \\
0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

and \(\gamma_1\) is right ascension of ascending node, \(\gamma_2\) is inclination, \(\gamma_3\) is argument of perigee.

It is easy to show that the dynamical equation (8) shares similar properties with the rigid-link robot manipulator dynamics. Specifically, the matrix \(M(q)\) is positive-definite and symmetric, and can be bounded as follows

\[
\Lambda(M)\|\xi\|^2 \leq \xi^TM(q)\xi \leq \overline{\Lambda}(M)\|\xi\|^2, \quad \forall \xi \in \mathbb{R}^6
\]

where \(\Lambda(M)\) and \(\overline{\Lambda}(M)\) are the minimum and maximum eigenvalue of matrix \(M(q)\). Moreover, \(M(q)\) and \(N(q, \dot{q})\) satisfy the following skew-symmetric relationship

\[
\xi^T[M(q) - 2N(q, \dot{q})]\xi = 0, \quad \forall \xi \in \mathbb{R}^6
\]

Remark 1: Note that the information about chaser attitude \(R_{ct}\) is necessary for computing \(f^c\) and \(d^c\), so the spacecraft relative position dynamics and attitude dynamics are coupling with each other.

Remark 2: Using the properties of the skew-symmetric matrix, equation (10) can be calculated explicitly as

\[
\xi^T[M(q) - 2N(q, \dot{q})]\xi = \xi^T\begin{bmatrix}
2P^TSP(\dot{\sigma})P & 0 \\
0 & -2S(n)
\end{bmatrix}\xi = 0
\]

where \(n = [-c_2 \sin \alpha \cos \alpha, -c_1 \cos^2 \alpha, \rho \alpha]^T\).

### III. Problem Statement

A. Standing Assumptions

Generally, in the motion equations (1) and (4), mass \(m\), inertia matrix \(J\), disturbance forces \(\tau_d\), and the disturbance torques \(\tau_d\) are not precisely known. In this paper, following assumptions are exploited in the subsequent development.

**Assumption 1:** \(m = m_0 + m_\Delta\) is a positive scalar and \(J = J_0 + J_\Delta\) is a symmetric positive-definite matrix, where nominal value \(m_0\) is a known positive constant and \(J_0 = \text{diag}\{J_{11}, J_{22}, J_{33}\}\) is a known positive diagonal matrix; \(m_\Delta\) is a unknown positive value and there exists a known constant \(m_\Delta\) such that \(m_\Delta \leq m_\Delta\). \(J_\Delta = [\Delta_{ij}]\) is a unknown symmetric matrix and there exists a known constant matrix \(\mathcal{J}_\Delta = [\Delta_{ij}]\) and such that \(|\Delta_{ij}| \leq \mathcal{J}_\Delta(i, j = 1, 2, 3)\).

**Assumption 2:** \(d^c = d_0^c + d^c_\Delta\) and \(\tau_d = \tau_{d0} + \tau_{d\Delta}\) are assumed to be continuously differentiable, where nominal value \(d_0^c\) and \(\tau_{d0}\) are known vectors. The unknown vectors \(d^c_\Delta\) and \(\tau_{d\Delta}\) satisfy that there exists known constants \(d^c_\Delta\) and \(\tau_{d\Delta}\) such that \(|d^c_\Delta| \leq d^c_\Delta\) and \(|\tau_{d\Delta}| \leq \tau_{d\Delta}\), respectively.

**Assumption 3:** The chaser docking port is toward the direction of chaser body frame \(+x\) axis, and the target docking port is toward the direction of target body frame \(-x\) axis. The cooperative target attitude is stabilized in the LVLH frame, and the chaser attitude only requires to track the target attitude trajectory.

B. Control Objective

Under assumption 3, we know that desired attitude trajectory of the chaser is the target attitude trajectory, so the autonomous rendezvous and docking control objective is to position the chaser at a certain safe distance from the target while keep its docking port facing the docking port of the target. The attitude trajectory of two spacecrafts must be kept the same during the chaser maneuver so that subsequent operations, for instance capturing or docking, can be carried out safely. Therefore, by defining the desired attitude trajectory \(\sigma_d, \dot{\sigma}_d\), the relative position trajectory \(r_1, \dot{r}_1\), and the desired system state \(q_d = [r^T, \sigma^T]^T, \dot{q}_d = [\dot{r}^T, \dot{\sigma}^T]^T\), we can conclude that the control objective under the assumptions 1~3 in this paper is to design a controller to guarantee that the system state \((\dot{q}, \ddot{q})\) can uniformly ultimately bounded track the desired trajectory \((\dot{q}_d, \ddot{q}_d)\).

### IV. Controller Design and Stability Analysis

To accomplish the control objective, we define tracking error as \(e = q - \dot{q}\) and filtered tracking error as

\[
s = \dot{e} + \Lambda \ddot{e}
\]

where \(\Lambda = \begin{bmatrix}
\Lambda_1 & 0_{3 \times 3} \\
0_{3 \times 3} & \Lambda_2
\end{bmatrix}\) is a diagonal positive definite gain matrix. Using (8) and (11), we get system error equation

\[
Ms + Ns = h(q, \dot{q}, q_d, \dot{q}_d) - R(u + w)
\]

where \(h = Mq_d + Nq_d + g\) and \(\dot{q}_d = q_d + \Delta \dot{e}\). Furthermore, from Assumption 1 and 2, we know

\[
h = h_0 + h_\Delta, \quad w = w_0 + w_\Delta
\]
where $h_0 = M_0 \ddot{q}_r + N_0 \dot{q}_r + g_0$, $h_\Delta = M_\Delta \ddot{q}_r + N_\Delta \dot{q}_r + g_\Delta$

$$M_0 = \begin{bmatrix} m_0H & 0 \\ 0 & \rho_1J_0P \end{bmatrix}, M_\Delta = \begin{bmatrix} m_\Delta H & 0 \\ 0 & \rho_1J_0P \end{bmatrix},$$

$$N_0 = \begin{bmatrix} m_0A & 0 \\ 0 & \rho_1J_0P \end{bmatrix}, N_\Delta = \begin{bmatrix} m_\Delta A & 0 \\ 0 & \rho_1J_0P \end{bmatrix},$$

$$g_0 = \begin{bmatrix} m_0a \\ 0 \end{bmatrix}, g_\Delta = \begin{bmatrix} m_\Delta a \\ 0 \end{bmatrix},$$

$$w_0 = \left[ \begin{array}{c} R_1^T \Gamma, r_0, d_0^T \\ \tau_d \end{array} \right], w_\Delta = \left[ \begin{array}{c} R_1^T \Gamma, r_0, d_\Delta^T \\ \tau_d \end{array} \right].$$

Therefore, the error equation (12) can be rewritten as

$$M_s + N_s = h_0 + h_\Delta - R(u + w_0 + w_\Delta) \quad (13)$$

For convenience of dealing with modeling uncertainties in controller designing procedure, a linear operator for any vector $\chi = [\chi_1, \chi_2, \chi_3, \chi_4]^T$ is introduced as

$$L(\chi) = \begin{bmatrix} \chi_1 & 0 & 0 & 0 \\ 0 & \chi_2 & 0 & 3 \\ 0 & 0 & \chi_3 & 2 \\ 0 & \chi_4 & \chi_5 & \chi_6 \end{bmatrix}$$

Then, defining $\theta_\Delta = [\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6]^T$, we have

$$M_s + N_s = h_0 + Y(q, q, q, q, q) \theta_\Delta - R(u + w_0 + w_\Delta) \quad (14)$$

where $Y(q, q, q, q, q) = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}$, $Y_1 = Hr + Ar + A_1(r_1 - r)$, $Y_2 = \rho^T \Gamma L(\Delta)$, $\rho = \rho^T \Gamma L(\Delta)$, $\Delta = \begin{bmatrix} \Delta_1, \Delta_2 \end{bmatrix}$, and $\theta_\Delta = \begin{bmatrix} \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6 \end{bmatrix}$.

Since uncertain inertia parameters are unknown, we use $\hat{\theta}_\Delta$ to denote the estimation of $\theta_\Delta$, and define $\Omega = \{ \theta_\Delta : ||\theta_\Delta|| \leq ||\Delta|| \}$, where $\Delta = [\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6]^T$, so we design the robust adaptive control input for (14) as

$$u = R^{-1} (Ks + h_0 - w_0 + Y \hat{\theta}_\Delta) \quad (15)$$

$$\hat{\Delta}_\Delta = \begin{cases} -\Gamma Y^T s & ||\Delta_\Delta|| < ||\Delta||, \text{ or } \hat{\Delta}_\Delta = \frac{\Delta_\Delta}{||\Delta||} Y^T s \geq 0. \quad (16) \\
-\Gamma (Y^T s - \psi) & ||\Delta_\Delta|| = ||\Delta||, \text{ or } \hat{\Delta}_\Delta = \frac{\Delta_\Delta}{||\Delta||} Y^T s < 0. \end{cases}$$

where $\psi = \frac{\theta_\Delta T \Delta_\Delta Y^T s}{||\Delta||^2}$, $K$ and $\Gamma$ are diagonal positive definite gain matrices.

Substituting controller (15) into (13) gives the closed-loop dynamics

$$M_s + (N + K)s = Y \hat{\theta}_\Delta - R\Delta_\Delta \quad (17)$$

where $\Delta_\Delta \triangleq \hat{\theta}_\Delta - \Delta_\Delta$.

From (5), (6), $||R_{so}|| = 1$, $\delta \triangleq \max \{1, \rho\}$, and

$$||P^T|| = ||G^{-T}(\sigma_e)|| = \frac{4}{1 + \sigma_e^T \sigma_e} \leq 4$$

we know the uncertain term $R\Delta_\Delta$ in (17) satisfies

$$||R\Delta_\Delta|| = \left\| \left[ \begin{array}{c} ER_{so} \Delta_\Delta^T \\ \rho_1 J_0 P \end{array} \right] \right\| \leq \delta d^o + 4 \tau_d \Delta_\Delta \leq w_m \quad (18)$$

Then, following theorem can be concluded.

**Theorem 1:** Consider the cooperative spacecraft autonomous rendezvous and docking dynamics (8) under Assumptions 1~3, the adaptive controller composed of (15) and (16) ensures the estimate parameters satisfy $\hat{\theta}_\Delta \in \Omega$, and tracking errors of the closed-loop system are uniformly bounded to tracking errors of model parameters.

**Proof.** For proving $\hat{\theta}_\Delta \in \Omega$, we chose

$$V_\theta(t) = \frac{1}{2} \hat{\theta}_\Delta^T \hat{\Delta}_\Delta$$

when $||\Delta|| < ||\Delta||$, or $||\Delta|| = ||\Delta||$ and $\hat{\theta}_\Delta^T Y^T s \geq 0$, then

$$V_\theta(t) = \hat{\theta}_\Delta^T \hat{\Delta}_\Delta = -\Gamma \hat{\theta}_\Delta^T Y^T s \leq 0$$

that is $||\theta_\Delta|| \leq ||\hat{\theta}_\Delta(0)|| \leq ||\hat{\Delta}||$. When $||\Delta|| = ||\Delta||$ and $\hat{\theta}_\Delta^T Y^T s < 0$, then

$$\hat{\theta}_\Delta = \hat{\theta}_\Delta$$

hence $||\hat{\theta}_\Delta|| \leq ||\hat{\Delta}||$, this means the estimate parameters are bounded.

Consider the Lyapunov function candidate

$$V = \frac{1}{2} s^T M_s + \frac{1}{2} \hat{\theta}_\Delta^T \Gamma^{-1} \hat{\theta}_\Delta \quad (19)$$

when $||\Delta|| < ||\Delta||$, or $||\Delta|| = ||\Delta||$ and $\hat{\theta}_\Delta^T Y^T s \geq 0$, differentiating (19) and using (17), (10) yields

$$V = -s^T K s - s^T R\Delta_\Delta \quad (20)$$

When $||\Delta|| = ||\Delta||$ and $\hat{\theta}_\Delta^T Y^T s < 0$, then differentiating (19) yields

$$V = -s^T K s - s^T R\Delta_\Delta + \frac{\hat{\theta}_\Delta^T \Delta_\Delta Y^T s}{||\Delta||^2} \quad (19)$$

Since

$$\hat{\theta}_\Delta^T \Delta_\Delta = \frac{1}{2} (||\Delta||^2 - ||\Delta_\Delta||^2 + ||\hat{\Delta}_\Delta - \Delta_\Delta||^2) \geq 0$$

and $\hat{\theta}_\Delta^T Y^T s < 0$, then $||\Delta||^2 - ||\Delta_\Delta||^2 \leq 0$, hence

$$V \leq -s^T K s - s^T R\Delta_\Delta \quad (21)$$

Thus, according to (20), (21) and (18), we have

$$V \leq -\frac{\lambda(T)}{2} ||s||^2 + \frac{\delta}{\kappa} ||s|| \leq -\frac{\lambda(T)}{2} ||s||^2 - \kappa ||s||^2 + \frac{\delta}{\kappa} ||s|| \leq -\frac{\lambda(T)}{2} ||s||^2 - \frac{\delta}{\kappa} ||s||^2 + \frac{\delta}{\kappa} ||s||$$

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where $0 < \kappa < \lambda(K)$, $\lambda(K)$ is the minimum eigenvalue of matrix $K$. Moreover, from (9) and $\| \theta \| \leq 2\| \bar{\theta} \|$, we know

$$V \leq \frac{\lambda(M)}{2} \| \theta \|^2 + 2\lambda(\Gamma^{-1})\| \bar{\theta} \|^2$$

where $\lambda(\Gamma^{-1})$ is the maximum eigenvalue of matrix $\Gamma^{-1}$. Then

$$V \leq -2aV + b$$

with $a = \frac{\lambda(K) - \kappa}{\lambda(M)}$, $b = 4a\lambda(\Gamma^{-1})\| \bar{\theta} \|^2 + \frac{w_m}{4\kappa}$, thus

$$V \leq \left( V(0) - \frac{b}{2a} \right) e^{-2at} + \frac{b}{2a}$$

Since $V \geq \frac{\lambda(M)}{2} \| \theta \|^2$ from (9) and (19), this implies that the ultimate bound of $\| \theta \|$ can be given by

$$\| \theta \| \leq \sqrt{2V \lambda(M)} \leq \sqrt{2\lambda(M)} \left( 2V(0) - \frac{b}{a} \right) e^{-2at} + \frac{b}{a}$$

Since the solution for linear time-invariant system (11) is

$$\theta(t) = \theta(0)e^{-\lambda t} + \int_0^t e^{-\lambda(t-\tau)} \theta(\tau) d\tau, \quad \forall t \geq 0$$

and from the definition of matrix $\lambda$ in (11), we know

$$\| e^{-\lambda t} \| \leq e^{-\lambda t} \lambda(t)$$

where $\lambda(\Lambda)$ is the minimum eigenvalue of matrix $\Lambda$, then

$$\| e(t) \| \leq \| e(0) \| e^{-\lambda(\Lambda)t} + \int_0^t e^{-\lambda(\Lambda)(t-\tau)} \| \theta(\tau) \| d\tau$$

$$\leq \| e(0) \| e^{-\lambda(\Lambda)t} + \int_0^t e^{-\lambda(\Lambda)(t-\tau)} \| \theta(\tau) \| d\tau$$

$$= \| e(0) \| e^{-\lambda(\Lambda)t} + \pi(1 - e^{-\lambda(\Lambda)t})$$

$$\leq \| e(0) \| e^{-\lambda(\Lambda)t} + \frac{\pi}{\lambda(\Lambda)}$$

Thus, with $\epsilon = 2V(0) + 4\lambda(\Gamma^{-1})\| \bar{\theta} \|^2$, we have

$$\| e(\infty) \| \leq \frac{1}{\lambda(\Lambda)} \sqrt{\frac{\epsilon}{\lambda(M)}} + \frac{w_m^2}{4k\lambda(\Lambda) - \kappa \lambda^2(M)}$$

It means the closed-loop system tracking errors are uniformly ultimately bounded stable as $t \to \infty$. \qed

Remark 3: The ultimate bounds for tracking errors can be tuned by choosing appropriate controller parameter matrix $K$, $\Gamma$ and $\Lambda$. Larger $\Lambda$, $K$ and $\Gamma$ would result in smaller ultimate bounds of tracking errors. Moreover, larger $K$ would contribute to faster convergence rate of tracking errors and larger control efforts.

V. SIMULATION STUDIES

In this section, the simulation scenario describes an example of the spacecraft cooperative autonomous in-orbit rendezvous and docking mission, where the target spacecraft has stabilized at the given attitude in the LVLH frame. The target is assumed to be in a low earth orbit with $r_e = 400$ km, perigee altitude. Eccentricity $e_k = 0.1375$. Orbit inclination is set to $\gamma_l = 45^\circ$. Argument of perigee and right ascension of ascending node are all set to $\gamma_0 = 0^\circ$.

We know the nominal values of the chaser inertia and mass are $J_o = \text{diag}(400, 250, 600) \text{kg m}^2$ and $m_0 = 50 \text{kg}$, respectively. $\theta_\Lambda = [10, 25, 15, 10, 30, 55, 25]^T$. Moreover, the nominal values of the external disturbance are $\tau_{d0} = [1, 1, 1]^T \times 10^{-2} \text{N m}$ and $d_{o0} = [1, 1, 1]^T \times 10^{-2} \text{N}$, respectively. We use the proposed controller (15) and (16) to achieve the spacecraft rendezvous and docking and the controller parameters are chosen as $K = \text{diag}(400, 400, 400, 1000, 1000, 1000, 1000)$, $\Lambda = \text{diag}(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$, $\Gamma = \text{diag}(0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2)$, and the initial conditions for the adjustable parameters is chosen as $\theta_\Lambda(0) = [0, 0, 0, 0, 0, 0, 0, 0]^T$. The simulation aim to control system state from $q(0) = [50, \pi/6, -\pi/6, 0, 0, 0]^T$ to $q_i = [5, 0, 0, 0, 3, 0, 4, 0, 0]^T$, $\dot{q}_i = [0, 0, 0, 0, 0, 0, 0]^T$.

Suppose the simulation parameters of the chaser are $m = 58.2 \text{kg}$.

$$J = \begin{bmatrix} 424.4 & 22.5 & -51.5 \\ 22.5 & 263.6 & -27 \\ -51.5 & -27 & 598.3 \end{bmatrix} \text{(kg m}^2\text{)},$$

$$\tau_d = \begin{bmatrix} 1 + \sin(\frac{\pi t}{125}) + \sin(\frac{\pi t}{250}) \\ 1 + \sin(\frac{\pi t}{125}) + \sin(\frac{\pi t}{250}) \\ 1 + \cos(\frac{\pi t}{125}) + \cos(\frac{\pi t}{250}) \end{bmatrix} \times 10^{-2} \text{N m},$$

$$d = \begin{bmatrix} 1 + \sin(\frac{\pi t}{125}) + \sin(\frac{\pi t}{250}) \\ 1 + \sin(\frac{\pi t}{125}) + \sin(\frac{\pi t}{250}) \\ 1 + \cos(\frac{\pi t}{125}) + \cos(\frac{\pi t}{250}) \end{bmatrix} \times 10^{-2} \text{N}.$$
VI. CONCLUSIONS

In this paper, a simple robust adaptive controller is proposed for spacecraft cooperative autonomous rendezvous and docking problem. The dynamical coupling, uncertain inertia parameters and bounded external disturbances are considered in the 6 degrees-of-freedom motion model. With the proposed controller, it is proved that the chaser can track both the target attitude and desired relative position trajectory, and the tracking errors converge to the small neighborhood of the origin. Simulation results show the effectiveness of the proposed method.

REFERENCES