An Identification Method for Individual Driver Steering Behaviour Modelled by Switched Affine Systems

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Abstract—This paper addresses the issue of modelling and identification of individual driver steering behaviour from a new point of view, incorporating the idea of human motion being built up by an individual and limited repertoire of learned patterns. We introduce a switched affine model structure to explain a measurable motion alphabet in the driving context and show that this leads to a new identification problem that differs from general hybrid identification issues. To solve this problem, we derive a multi-step model output error criterion and propose an algorithm to simultaneously identify switching times and subsystem parameters out of measurable movement data. We show that this algorithm is capable of identifying the true parameters of known systems as well as fitting real movement trajectories even though no a priori information is given about the true system complexity.

I. INTRODUCTION

The understanding of human motion is currently of great interest in various scientific disciplines. Especially in automotive research focussing on advanced driver assistance systems (ADAS), there is much effort put in the generation of appropriate driver models which may help to design such systems. Many accidents are at least partly caused by mental overload [1]. Therefore the aim is either to detect distraction or fatigue and warn the driver [2] or to support him in performing the driving task without entirely suspending him from the control loop [3], [4]. In recent years, the latter approach led to the ideas of haptic guidance by active driver-car-interfaces, introduced as haptic gas pedal for longitudinal car following support (see e.g. [5], [6]) and Lane Departure Warning Systems [7]. Meanwhile the development of steer-by-wire systems with haptic steering wheels is in focus [8], [9], [10]. But all driver models suffer from the lack of individuality. They focus on the common properties of driver behaviour and do not take into account that human beings perform movements in an individual way as shown in various experiments [11], [12]. Therefore general model parameterizations do not fit very well to measurements from different subjects. To receive good behavioural models for the actual driver in the loop it is thus necessary to identify the individual model parameters. In this paper, we propose a way to identify individual driver behaviour online.

State of the art driver models range from qualitative descriptive models including psychological aspects [13], [14] to more or less physically motivated mathematical control models [15], [16], depending on the research focus. In recent years there is an obvious trend towards more and more physical/biological inspired models of the human driver. These consist mostly of a neuromuscular dynamics submodel followed up by the kinematics of the limbs and the human-machine-interface [17], [18], [19]. The former includes well understood mechanisms of the locomotion system as active muscle stiffness and the reflex feedback loop [17], [20]. However, the driving task on a maneuvering level is mainly performed by a leading controller that calculates the necessary control inputs to the car - thus the necessary limb movements - and commands the setpoints to the neuromuscular dynamics (see Fig. 1). This part of the driver behaviour is not that well understood as it happens on a higher neural/mental level. For this reason most of the driver models use rather abstract leading controllers without a biological reference. Albeit some models include visual or auditory perception [21], [22]. In this paper we derive a driver model focusing on learned human motion vocabulary, decomposed of elementary building blocks from a limited and individual repertoire.

There is much evidence for the existence of learned elementary movement blocks - sometimes called motor primitives - on different abstraction levels. While experiments have shown kinematic primitives on a neuromuscular level [23], [24], [25], [26], more complex movements seem to consist of some kind of dynamic submovement primitives. Those represent the mapping of cognitive movement goals to an actual movement trajectory and they can be recalled quickly and subconscious [27], [28], [29], [30], [31], [32], [33], [34], [35]. This leads to a driver model as depicted in Fig. 2, wherein the leading controller, the neuromuscular dynamics and eventually the limb kinematics are substituted by a switched system. Inspired by Del Vecchio et. al, we call the set of parameterizations of a switched system the alphabet.
of *movemes* [30]. Our aim is generating the driver’s alphabet of movement primitives out of measurement data which finally means the identification of the subsystem parameters of a switched system from its output data. The superimposed mechanism, which triggers the switched system is considered in [36].

The identification of switched systems is closely related to the field of identification of hybrid systems. Herein different approaches have been proposed during the last years including algebraic methods [37], [38], clustering methods [39], mixed-integer programming [40] and others. An overview can be found in [41]. Recently sparse optimization methods have been discussed for the identification of switched systems in [42], [43]. All these approaches use some one-step prediction function. This works well as long as the systems under consideration are not autonomous. In our case, in which the output depends solely on its previous output values, the performance of the multi-step model output trajectory is of tremendous importance. The main aim of this paper is to propose a new method to identify the subsystem parameters of autonomous switched systems as well as the unknown switching times.

The paper is structured as follows: In section II, we formally specify the identification problem. In section III we prove that under certain assumptions on the measurement noise, the switched system identification problem can be solved by combining a least-squares estimation with a prediction-error based stop-criterion that results in an optimal multi-step model output performance. This leads to an algorithm, which is presented in section IV. Finally, in section V we give some identification results for an illustrative example as well as real steering wheel measurement data. We conclude with a summary of our findings in section VI.

II. PROBLEM STATEMENT

Although we need not to further specify the model structure of the switched system in general we will focus on the case of piecewise affine autoregressive systems without exogenous input in the following sections.

**Preliminaries:** Let there exist $T$ samples of some system output data $\{w_k\}_{k=1}^T$, $w_k \in \mathbb{R}$ representing the output trajectory produced by a stable switched system of the form

$$w_k = \sum_{j=1}^{n_a(\lambda_k)} a_j(\lambda_k)w_{k-j} + c(\lambda_k)$$

with $\lambda_k \in [1, N]$ as the number of the currently active discrete subsystem at time $k$. The number $n_a(\lambda_k)$ denotes the order of the currently active subsystem. Although this notation allows different model orders at different times in general, we assume $n_a(\lambda_k) = \text{const.}$ in this paper. As long as $\lambda_k = \lambda_{k+1}$ the system stays in the same subsystem. For $\lambda_k \neq \lambda_{k+1}$ the systems changes the currently active subsystem. We call $\tau_i = k + 1$ the start time and $\tau'_i = \tau_i + 1$ the end time of the interval $i$. The sequence $\{\tau'_j\}_{j=1}^M$ is called a switching sequence consisting of $M$ switches. Resulting from the switching sequence we can define a mode sequence $\{\lambda_j\}_{j=1}^M$ representing the active subsystem at each interval $i$. Referring to the overall identification goal we call the set of subsystem parameterizations $\{[a_1(\lambda_j) \cdots a_{n_a(\lambda_j)}(\lambda_j) c(\lambda_j)]^T\}_{j=1}^N$ the individual movement alphabet of the driver.

We assume that the output of the switched system is influenced by some arbitrary noise $n_k$ and call

$$y_k = w_k + n_k \quad (2)$$

measurement data. Special cases of $n_k$ will be discussed in section III. If there are estimated parameters

$$\hat{\Theta} = [a_1(\lambda_j) \cdots a_{n_a(\lambda_j)}(\lambda_j) c(\lambda_j)]^T$$

and measurement values $r_{k-1} = [y_{k-1} \cdots y_{k-n_a(\lambda_j)} 1]$ of the underlying system available we call

$$\{\hat{y}_k(\hat{\Theta})\} = \{\hat{y}_k | \hat{y}_k = r_{k-1}\hat{\Theta} \forall k > n_a\} \quad (3)$$

the one-step model output and

$$\{\hat{y}_k(\hat{\Theta})\} = \{\hat{y}_k | \hat{y}_k = [y_{k-1} \cdots y_{k-n_a(\lambda_j)} 1]\hat{\Theta} \forall k > n_a\} \quad (4)$$

the multi-step model output respectively. Note that the one-step model output is calculated using the past $n_a$ measurement values $\{y_j\}_{j=k-1}^{k-n_a(\lambda_j)}$ to calculate the respective output values whereas the multi-step output model is calculated using the past $n_a$ multi-step values $\{\hat{y}_j\}_{j=k-1}^{k-n_a(\lambda_j)}$. Both model outputs use the measured start values $\{\hat{y}_k(\hat{\Theta})\} = \{\hat{y}_k(\hat{\Theta})\} = \{y_k\} \forall k \in [1, n_a]$. \n
**Problem 1:** Given some measured output data $\{y_k\}_{k=1}^T$ and the respective model order $n_a$, calculate the switching sequence $\{\tau'_j\}_{j=1}^M$ and the coefficients $\{[a_1(\lambda_j) \cdots a_{n_a(\lambda_j)}(\lambda_j) c(\lambda_j)]^T\}_{j=1}^N$ such that the resulting multi-step model output $\{\hat{y}_k(\hat{\Theta})\}_{k=1}^T$ tracks the measured trajectory $\{y_k\}_{k=1}^T$ at the best in a mean absolute error sense

$$\min_{\hat{\Theta}} \frac{1}{T} \sum_{k=1}^T |y_k - \hat{y}_k| \quad (5)$$

To solve this problem, we use a combination of a least-squares (LS) estimation $\hat{\Theta}$ depending on the first $k_{\text{est}}$ measurement values and an algorithm, trying to push $k_{\text{est}}$ as far as possible. To clarify the importance of using the multi-step model output $\{\hat{y}_k\}$ in problem 1 we give the following illustrative example:

Let there be an undisturbed ($\{n_k\} = \{0\}$) jump linear system consisting of second order subsystems only. The resulting measured trajectory is shown in Fig. 3 (left). Subsystem 1 is given by $y_k = 2y_{k-1} - y_{k-2} + 0.1$ and starts
with \( y_1 = 1 \) and \( y_2 = 2 \). The system switches at \( \tau_2 = 11 \).
Afterwards subsystem 2 is applied: \( y_k = y_{k-1} - 0.5y_{k-2} + 9 \)
and starts with \( y_{11} = 16 \) and \( y_{12} = 16.5 \).

Fig. 3 (middle) shows the situation for \( k_{est} = 10 \).
The LS estimation calculates the true parameters \( \hat{\Theta}_{10} = [2 - 1 0.1]^T \).
Hence both the one-step model output \( \{\tilde{y}_k\}_{k=1}^{10} \) and the multi-step model output \( \{\tilde{y}_k\}_{k=1}^{10} \)
match the true system output \( \{y_k\}_{k=1}^{10} \) perfectly.

The situation changes when \( k_{est} = 11 \). The usage of
one single value from system 2 causes the estimation to
change into \( \hat{\Theta}_{est} = [1 -0.5 0.1]^T \).

Definition 1 (Noise): We assume the disturbed model as
the typical regressor model [44]

\[
y_k = \sum_{j=1}^{n_{W,k}} a_j(\lambda_i)y_{k-j} + c(\lambda_i) + n_{W,k},
\]

with \( n_{W,k} \) being zero-mean white gaussian noise. It follows
directly from (6) that \( n_{W} \) and \( y \) are uncorrelated.

Lemma 1 is a straight forward result of the application of
[45].

Lemma 1 (Local parameter identifiability): Let there exist
measurement data \( \{y_k\}_{k=1}^{n_{y}} \) generated by a system (1)
affected by white noise \( n_{W} \) according to definition 1
and a known switching sequence \( \{\tau'_i\}_{i=1}^M \). If the estimation
data used for estimation belongs to the same interval \( i \) given by
\( k_{est} \in [\delta_i, \tau'_i] \) with \( \delta_i = \tau_i + n_{a} \) the estimation \( \hat{\Theta}_{k_{est}} \) equals
the real parameters \( \Theta^{*}_{\lambda_i} \).

Proof: The expectation of the estimated parameters is
given by

\[
E[\hat{\Theta}_{\lambda_i}] = \Theta^{*}_{\lambda_i} + [R(y)^TR(y)]^{-1}R(y)^TE[n] \tag{7}
\]

iff the regressor matrix \( R(y) = [r_{\tau-i}^T \cdots r_{k_{est}-1}^T]^T \) and the
noise vector \( n \) are not corrrelated. Hence \( \hat{\Theta}_{k_{est}} \) will be equal
\( \Theta^{*}_{\lambda_i} \) iff \( n \) is zero mean [44]. For an arbitrary also finite
interval length \( |\tau'_i - \tau_i| \) the parameter estimation \( \hat{\Theta}_{k_{est}} \) equals
the true values \( \Theta^{*}_{\lambda_i} \) [45].

Lemma 2 (Bounded multi-step error): Let there exist
measurement data \( \{y_k\}_{k=1}^{n_{y}} \) generated by a system (1)
affected by white noise \( n_{W} \) and a known switching sequence \( \{\tau'_i\}_{i=1}^M \). If the estimation
\( \hat{\Theta}_{k_{est}} \) equals the real parameters \( \Theta^{*}_{\lambda_i} \) for all \( k_{est} \in [\delta_i, \tau'_i] \) and the
additional condition of vanishing noise for the \( n_{a} \) start values of
the intervall holds, then the expectation \( E[\epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}})] \) of
the mean absolute error of the multi-step model output

\[
\epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) = \frac{1}{k_{est} - \tau_i + 1} \sum_{k=\tau_i}^{k_{est}} |y_k - \tilde{y}_k(\hat{\Theta}_{k_{est}})| \tag{8}
\]
is bounded.

Proof: Because the estimated parameters \( \hat{\Theta}_{k_{est}} \) at time
\( k_{est} \) equal the true values \( \Theta^{*}_{\lambda_i} \), the multi-step model output
based upon the noisefree start values \( \{\tilde{y}_k\}_{k=1}^{n_{y}} = \{y_k\}_{k=1}^{n_{y}} = \{w_k\}_{k=1}^{n_{a}} \) will exactly track the undisturbed system output.

\[
\{\tilde{y}_k(\hat{\Theta}_{k_{est}})\} = \{w_k\} \forall k \in [\tau_i, k_{est}] \tag{9}
\]
If we substitute this result in (8), the expectation of the mean absolute error of the multi-step model output turns to

\[ E \{ \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \} = E \left\{ \frac{1}{k_{est} - \tau_i + 1} \sum_{k=\tau_i}^{k_{est}} |y_k - w_k| \right\} \]

\[ = \frac{1}{k_{est} - \tau_i + 1} \sum_{k=\tau_i}^{k_{est}} E \{|\Delta_k|\} \]

(10)

with the difference \( \{ \Delta \} = \{ y \} - \{ w \} \) between the noisy measurement values and the undisturbed output values. In general \( \{ \Delta \} \) is neither zero-mean nor white, but gaussian distributed. Hence, all of the expectations \( E \{ |\Delta_k| \} \) are bounded to finite values. Consequently the expectation of \( \epsilon_{MS} \) is lower than a finite upper bound

\[ E \{ \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \} \leq \bar{\epsilon}. \]

(11)

**Theorem 1 (switching time environment):** Let there exist measurement data \( \{ y_k \}_{k=1}^{T} \) generated by a system (1) affected by white noise \( n_{W} \) and an unknown switching sequence \( \{ \tau_i \}_{i=1}^{M} \). If the mean absolute error of the multi-step model output \( \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \) is bounded by \( \bar{\epsilon} \) for every \( k_{est} \in \{ \delta_i, \tau_i' \} \) and each intervall \( i \in [1, M] \), it is possible to find an environment around the true switching times \( \tau_i' \) and the true parameters of the subsystems \( \Theta^*_i \).

**Proof:** We prove theorem 1 in four steps.

**A)** We can detect the switching time \( \tau_i' \) of intervall \( i \) for growing \( k_{est} \) by using \( \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) > \bar{\epsilon} \) as the stop-criterion for our algorithm. The mean absolute error of the multi-step model output \( \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \) is bounded by \( \bar{\epsilon} \) for every \( k_{est} \in \{ \delta_i, \tau_i' \} \). Hence the calculated switching time is

\[ \tau_i' = \min(k_{est} | \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) > \bar{\epsilon} ) \]

which is at least \( \tau_i' \geq \tau_i' + 1 = \tau_{i+1} \). Thus \( \tau_i' \) will never underestimate the true switching time.

**B)** Additionally we assume every pair of two consecutive subsystems to be theoretically identifiable. This means that there is no set of parameters \( \Theta^\# \) whose multi-step model error \( \epsilon_{MS}(\tau_{i+1}, \Theta^\#) \leq \bar{\epsilon} \) for the intervall \( [\tau_i, \tau_i'+1] \). Therefore in each intervall \( i \) the multi-step model output error \( \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \) will exceed \( \bar{\epsilon} \) at the latest at the end of the subsequent intervall \( i + 1 \), which implies \( \tau_i' < \tau_{i+1} \).

**C)** Given an algorithm that calculates a possible switching time \( \tau_i' \) for each intervall \( i \) with

\[ \tau_{i+1} \leq \tau_i' < \tau_{i+1}' \]

(13)

According to **A** and **B**, the true number of switching times \( M \) will be found as (13) holds for all \( i \in [1, M - 1] \) and the last switching time \( \tau_M = K \) as the end of the measurement. Furthermore, we can embrace the true switching points \( \tau_i' \) by running the algorithm once forwards and once backwards. We get the forward limit \( \hat{\tau}_i^{FW} \geq \tau_i' \) \( \forall \ i \in [1, M] \) as well as the backward limit \( \hat{\tau}_i^{BW} \leq \tau_{M-i}' \) \( \forall \ i \in [1, M - 1] \) respectively and \( \hat{\tau}_0^{FW} = 1 \) as the start time and \( \hat{\tau}_M^{BW} = \hat{\tau}_0^{FW} = K \) as the end time of the measurement.

**D)** The calculated sequences \( \{ \hat{\tau}_i^{FW} \}_{i=1}^{M} \) and \( \{ \hat{\tau}_i^{BW} \}_{i=1}^{M} \) lead to upper and lower bounds \( \{ \hat{\tau}_i^{FW}, \hat{\tau}_i^{BW} \}_{i=1}^{M} \) for the switching time of each intervall \( i \), whereas the lower bound belongs certainly to \( [\tau_i, \tau_i'] \). Using the interval \( [\hat{\tau}_i^{FW}, \hat{\tau}_i^{BW}] \) for the parameter estimation of the subsystem \( \lambda_i \) yields the correct parameter estimation \( \hat{\Theta} = \Theta^*_i \) according to lemma 1.

**Remark 1 (Solution for more general noise):** Theorem 1 is still valid under relaxed noise conditions if additional assumptions are made. For an arbitrary noise which in general is neither zero mean nor uncorrelated with \( y \) the calculated parameters \( \hat{\Theta}_{k_{est}} = \Theta^*_i + b \) will be affected by a bias

\[ b = E \{ (R(y)^{T}R(y))^{-1}R(y)^{T}n \}. \]

(14)

If the consequences of this bias are such that the mean absolute error of the multi-step model output \( \epsilon_{MS}(k_{est}, \hat{\Theta}_{k_{est}}) \) does not diverge for all \( k_{est,i} \in [\delta_i, \tau_i'] \) and \( i \in [1, M] \), theorem 1 holds as well.

**IV. IDENTIFICATION ALGORITHM**

It is reasonable to design an algorithm as a combination of parameter estimation and extension of the estimation interval. By using an alternating structure we generate the over estimation of the switches as given in theorem 1. This leads to the algorithm given in listing 1. For online identification purposes the implemented algorithm only uses the forward iteration and omits the backward execution.

Starting with the minimal estimation length \( \delta_i + n_a + 1 \) of the current interval \( i \) the parameters \( \hat{\Theta}_{k_{est}} \) of the currently active subsystem are obtained by LS estimation. Based on this estimate the trajectory error between the multi-step model output and the measured data is calculated for the estimation interval \( [\delta_i, k_{est}] \) and compared to a predefined error bound \( \epsilon_{max} \). Remember that the multi-step model output is based on the first \( n_a \) measured values. As long as \( \epsilon_{k_{est}} < \epsilon_{max} \), the estimation interval is expanded. The first time the error bound is exceeded, the switching time \( \tau_i' \) is detected and the algorithm restarts in the next interval \( i + 1 \). The following \( n_a \) measurement values are interpreted as start values.

The main point of the introduced algorithm is the calculation of the multi-step estimation error \( \epsilon_{k_{est}} \). As can be seen in the example in section II the one-step model output error is not sufficient. This is due to the weak dependency on regressor values belonging to a new interval. In other words, using the one-step model output error as a stop criterion to detect switches leads to poor multi-step model output trajectories. In contrast the multi-step model output error leads to a good identification of the switching times and therefore the parameter estimate \( \hat{\Theta}_{k_{est}} \) is not influenced by too much data points from the next interval. Nevertheless, under the above mentioned conditions the estimation is affected by a bias due to the calculation of the estimation border

3550
Initialize algorithm
while \( k < T \)
  for \( k_{est} = \delta_i + na + 1 : T \)
    build regressor matrix \( R(y) \)
    build measurement vector \( y \)
    estimate current parameter via LS estimation:
    \[
    \hat{\Theta}_{k_{est}} = (R(y)^T R(y))^{-1} R(y)^T y
    \]
    calculate multi-step model output \( \{\hat{y}_j\}_j^{k_{est}} \)
    calculate trajectory error \( \epsilon(k_{est}, \hat{\Theta}_{k_{est}}) \) using (8)
    if \( \epsilon_{k_{est}} < \epsilon_{\text{max}} \)
      continue
    else
      end of interval reached: \( \tau_{FW,i} = k_{est} - 1 \)
      save parameters for current interval: \( \hat{\Theta}_{\tau_{FW,i}} = \hat{\Theta}_{k_{est} - 1} \)
      save current start values
    end
    adjust \( k \) and \( i \)
end

Listing 1: Algorithm

\( k_{est} \) solely by the forward iteration which means that the algorithm uses the estimation interval \([\tau_{FW,i}, \tau_{FW,i+1}]\).

V. RESULTS

In this section we demonstrate the performance of the introduced algorithm.

First we present an artificial example with known ground truth (see Fig. 4). The bold line denotes an output trajectory which is produced by four different second-order affine subsystems whose parameters are given by the dark bars in Fig. 5. The generating subsystems switch at \( k = 2, 3, 5 \) and 5 seconds respectively. There is a white noise added to the measurement with a SNR of 70dB. It can be seen in Fig. 4 that the identification algorithm calculates the true number of subsystems. The first two switches (dotted vertical lines) are found very close to the real parameters, while the third and fourth are quite close to the real parameters, which can be seen in Fig. 5. Therefore, the measurement trajectory is nearly perfectly met by the model (dashed line). In contrast, the last switch is widely overestimated, due to the used error bound of \( \epsilon_{\text{max}} = 3^\circ \). Thus, the first values of subsystem 4 are interpreted as noisy measurements of subsystem 3. If the noise increases it is necessary to raise the error bound \( \epsilon_{\text{max}} \) as well. Therefore a greater overestimation of the switching times will lead to worse subsystem parameter estimations.

The identification method is now applied to real measurement trajectories. To identify the driver’s movement alphabet, we measured the steering wheel angle \( \delta_{SW} \), which is directly related to the driver’s arm positions. The measurements have been recorded in a driving simulator equipped with an active steering wheel and the CarMaker realtime simulation environment by IPG automotive GmbH. There were a total of 21 testruns with different double lane change maneuvers at different speeds for one subject. The steering wheel angle was recorded by an incremental encoder of 40000 increments per full rotation at a sampling frequency of 100 Hz. Three measured trajectories are exemplarily depicted by the bold lines in Figs. 6, 7 and 8. It is assumed that the model order is \( n_a(\lambda_i) = 2 \) \( \forall i \). The identified switching times are illustrated by the vertical dotted lines. The results show that the algorithm is able to find subsystem parameters and switching times such that the multi-step model output fits well to the measurement data and an acceptable number of intervals is obtained. The switched system structure is able to explain such multidynamic movement trajectories.

We used MATLAB R2011c running on an INTEL i7–2600@3.40Ghz core with 8GB RAM for computing the shown solution. The factor computation time to duration of the signal is \( f = 0,089 \) in mean and was calculated by executing the algorithm for all 21 real measurement

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Subsystems</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>( \lambda_2 )</td>
</tr>
<tr>
<td>( c )</td>
<td>( \lambda_3 )</td>
</tr>
<tr>
<td>( c \cdot a_2 )</td>
<td>( \lambda_4 )</td>
</tr>
</tbody>
</table>

Fig. 4: Trajectory of order \( n_a = 2 \) and multi-step model output trajectory of identified subsystems for error bound \( \epsilon_{\text{max}} = 3^\circ \), noisy measurement with SNR=70dB, sampling frequency of 100 Hz

Fig. 5: Comparison of the true parameters and the identified estimates for all four subsystems. Note that the \( c \)-Parameter of subsystem 1 and 4 is multiplied by a factor \( c_\ast = 10 \cdot c \) for better visibility.
trjectories. The implementation is hence fast enough to allow possible online computation in the future.

VI. SUMMARY

We have shown that under some assumptions on the noise, the switching sequence and the coefficients of an unknown switched affine system can be calculated solely from its measured output data. The true switching times are proven to lie within the identified limits in a stochastic sense. Therefore the estimated subsystem parameters are guaranteed to equal the true parameters. The forward part of the introduced method was implemented in an identification algorithm and applied to artificial and real measurement data.

Our results show that it is possible to identify movement primitives as parametrizations of switched systems from measurements of motion data. Therefore the proposed algorithm is able to find individual driver properties that can help to improve driver models. An application of the identified subsystems for a steering task can be found in [36]. In future work, we will investigate unknown and possibly different model orders of the used subsystems and the extension to possible input values.

REFERENCES
