Optimal Control of Multi-Vehicle Systems with LTL Specifications

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Abstract—In this paper, optimal control of multi-vehicle systems is studied. When collision avoidance between vehicles and obstacle avoidance are imposed, state discretization is effective as one of the simplified approaches. Furthermore, using state discretization, cooperative actions such as rendezvous can be easily specified by linear temporal logic (LTL) formulas. However, it is not necessary to discretize all states, and partial states (e.g., the position of vehicles) should be discretized. From this viewpoint, a new control method for multi-vehicle systems is proposed in this paper. First, the system in which partial states are discretized is formulated. Next, the optimal control problem with constraints described by LTL formulas is formulated, and its solution method is proposed. Finally, numerical simulation is shown. The proposed method provides us a useful method in control of multi-vehicle systems.

I. INTRODUCTION

In recent years, there have been a lot of studies on control of multi-vehicle systems (see, e.g., [2], [6], [10], [17]). In multi-vehicle systems, collision avoidance between vehicles and obstacle avoidance must be considered. Furthermore, cooperative actions such as rendezvous must be also considered. These constraint conditions are complex, and it is difficult in a general setting to solve the optimal control problem with these constraints. In order to overcome this difficulty, it will be one of the natural approaches to approximately solve complex problems by simplification techniques. One of the typical methods in simplification techniques is that a continuous system is approximated by a discrete system (see, e.g., [5], [19], [20], [21]). By using the obtained discrete system, cooperative actions can be specified by linear temporal logic (LTL) formulas [4], which can be expressed by a set of linear inequalities with binary variables [9], [10].

However, it is not necessarily required to discretize all states, and discretization of partial states may be enough. In the obstacle avoidance problem, complex constraints are imposed for only the position of vehicles. Then for the velocity of vehicles, a simple constraint such as \( |\dot{x}_p| \leq \bar{\tau} (\dot{x}_p) \) is frequently imposed. In addition, by using partial state discretization, the difference between the original problem and the approximated problem becomes small. Thus the method that partial states are discretized has been proposed so far [8], [14]. In this method, the idea of waypoints on the state space are introduced. The problem of finding a piecewise-continuous state-feedback controller minimizing a given cost function is reduced to the problem of finding a waypoint at each sampling time.

Then, complex constraints such as obstacle avoidance are expressed as a directed graph, which is imposed for this problem. However, in [8], [14], multi-vehicle systems have not been considered, and it is important to consider a method for describing cooperative actions and a modeling method of directed graphs in the multi-vehicle case. To our knowledge, for multi-vehicle systems, a control method using partial state discretization has not been proposed so far.

In this paper, based on partial state discretization, we propose a control method of multi-vehicle systems with LTL specifications. First, the system in which partial states are discretized is formulated. Next, the optimal control problem with cooperative actions specified by LTL formulas is formulated. In this paper, the rendezvous problem is considered as one of the typical cooperative actions, and can be specified by the LTL formula. Third, as a solution method, the optimal control problem is reduced to a mixed integer quadratic programming (MIQP) problem. In addition, to efficiently solve the MIQP Problem, an efficient method to model directed graphs is also proposed. Finally, numerical simulation is shown. The proposed method provides us a useful method in control of multi-vehicle systems.

II. PRELIMINARIES

In this section, first, the outline of a modeling method using partial state discretization is explained. Next, we briefly review the optimal control problem and its solution method for single-agent systems.

A. Modeling Method Using Partial State Discretization

As an example, we consider the obstacle avoidance problem [3], [11], [12], [16], [22] of a single vehicle as shown in Fig. 1 (a). Suppose that the vehicle given by \( \dot{x}_p = -\dot{x}_p + u \), where \( x_p \in \mathcal{R} \) (the position), \( u \in \mathcal{R} \) (the driving force), will move along the \( x_p \)-axis so as to avoid the obstacles. However, it will be in general difficult to solve such a problem in real time because complex constraints such as time-varying and/or non-convex constraints for the state are imposed. So it will be indispensable to solve this kind of problem in an approximated way. One realistic approach will be to use a directed graph obtained by approximately discretizing it with respect to time axis and state space, as shown in Fig. 1.
where “•” and “square” express the waypoint candidate and the obstacle on the \((t, x_p)\)-space, respectively. Then we will consider the problem of simultaneously optimizing the waypoint sequence, i.e., the path on the directed graph, and the continuous-time trajectory between two waypoints. We remark that only the position \(x_p\) is discretized. The velocity \(\dot{x}_p\) is not discretized, because in many cases, a simple constraint such as \(|\dot{x}_p| \leq \tau\) (\(\tau\) is a constant) is imposed for \(\dot{x}_p\).

In addition, discretization methods for the state space have been already studied (see, e.g., [5], [19], [20], [21]). In this paper, we suppose that a part of the state space is discretized in advance.

### B. Optimal Control of Single-Vehicle Systems

Suppose that the sampling (discrete) time \(t_i, i = k, k + 1, \ldots\) is given. Suppose also that a finite set \(\mathcal{F}_i\) whose element takes a value in \(\mathbb{R}^{n_d}\) and a set-valued mapping \(\mathcal{E}_i(\cdot) : \mathcal{F}_i \rightarrow \mathcal{F}_i\) specifying a directed graph on \(\mathcal{F}_i\) at time \(t_i, i = k, k + 1, \ldots\) are given, where \(\mathcal{F}_i\) is a power set of \(\mathcal{F}_j\), i.e., a set of subsets of \(\mathcal{F}_j\). In addition, let \(x_i \in \mathbb{R}^n\) denote the waypoint for \(x(t_i)\). Then consider the following system:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t), \quad (1) \\
Cx(t) + Du(t) &\leq G, \quad (2) \\
x(t_i) &= x_i, \quad (3) \\
x_i &\in \mathcal{E}_i(x^d), \quad (4) \\
x_{i+1} &\in \mathcal{E}_i(x^d) \quad (5)
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the state, \(u \in \mathbb{R}^m\) is the input, \(x^d \in \mathcal{F}_i\) is the waypoint candidate at time \(t_i\), \(C \in \mathbb{R}^{p \times n}, D \in \mathbb{R}^{p \times m}, G \in \mathbb{R}^p\) are certain matrices/vectors. The dynamics of a vehicle is expressed as (1). We assume that the pair \((A, B)\) is controllable. The linear constraint at each sampling time is expressed by (2). The constraint (3) guarantees that the state passes through the waypoint at each sampling time. By (4), the state constrained by the directed graph (5) is specified.

Next, the following finite-time optimal control problem on \([t, t + T]\) \((T = \text{horizon length})\) is formulated as follows.

**Problem 1:** For the system of (1)-(5), suppose that the state at the current time \(t_k\) is given as \(x(t_k) = x_k\). Then find a state feedback controller \(u(t) \in \mathcal{P}C^m, t \in [t_k, t_{k+N}]\),

and waypoints of the state \(x_i, i = k + 1, k + 2, \ldots, k + N\), minimizing the cost function

\[
J(x_k, u(\cdot), \{x_i\}_{i=k+1, \ldots, k+N}) = \sum_{i=k}^{k+N-1} J_i(x_i, u(\cdot), x_{i+1})
\]

where \(J_i(\cdot)\) is the cost of the continuous evolution of the dynamics between discrete-states given by

\[
J_i(\cdot) = \int_{t_i}^{t_{i+1}} \left\{ (x(t) - x_{i+1})^TQ(x(t) - x_{i+1}) + u^T(t)Ru(t) \right\} dt
\]

with \(Q \geq 0\) and \(R > 0\).

Problem 1 can be reduced to the following optimal control problem of discrete-time linear system (see, e.g., [7], [8], [14]).

**Problem 2:**

\[
\min \sum_{i=k}^{k+N-1} v_i, \quad i = k, k + 1, \ldots, k + N - 1
\]

subject to \(x_{i+1} = x_i, x_i : \text{given}, \quad C\dot{x}_i + D_i u_i \leq G, \quad v_i = \begin{bmatrix} v^e_i & v^f_i \end{bmatrix} \in \mathbb{R}^{n_e} \times \mathcal{E}i(v^d_{i+1})
\]

Details of \(\mathcal{S}^i, \mathcal{C}_i, \mathcal{D}_i\), and the optimal state-feedback controller are omitted due to the limited space. In order to solve Problem 2, we next consider to reduce Problem 2 into a mixed integer quadratic programming (MIQP) problem. Then the directed graph \(v^d_i \in \mathcal{E}i(v^d_{i-1}), v^d_{k-1} = x_k\), \(x^d_k \in \mathcal{F}_k\), e.g., as shown in Fig. 1 (b), has to be expressed as a kind of linear form with respect to decision variables such as \(v^d_i\). For simplicity of discussion, the cardinality of \(\mathcal{F}_i\) (the number of discrete points) is the same for all \(i\), and is denoted by \(n_f\). That is, we suppose that

\[
\mathcal{F}_i = \{ \lambda^{i}_1, \lambda^{i}_2, \ldots, \lambda^{i}_{n_f} \}
\]

where \(\lambda^{i}_k \in \mathbb{R}^{n_d}\). Note here that \(\mathcal{E}i(v^d_{i-1}) \subseteq \mathcal{F}_i\) holds. Then \(x^d_i \in \mathcal{F}_i\) is expressed as

\[
x^d_i = \Lambda_i x^f_i, \quad \Lambda_i := [ \lambda^{i}_1 \lambda^{i}_2 \cdots \lambda^{i}_{n_f} ] \in \mathbb{R}^{n_d \times n_f}
\]

where \(x^f_i \in \{0, 1\}^{n_f}\) is a binary variable satisfying \(e^T_i x^f_i = 1\). In addition, \(x_i\) and \(v_i\) in Problem 2 can be expressed as

\[
\begin{bmatrix} x_i \\ v_i \end{bmatrix} = \begin{bmatrix} \Lambda_i & 0 \\ 0 & \Lambda_3 \end{bmatrix} \begin{bmatrix} x^f_i \\ \tilde{v}_i \end{bmatrix}
\]

where \(\Lambda_3 := \text{block-diag}(I_{n_v}, \Lambda_1)\), \(\Lambda_1 := \text{block-diag}(I_{n_v}, \Lambda_3)\), \(\tilde{v}_i := \begin{bmatrix} (v_i^e)^T \\ (v_i^f)^T \end{bmatrix} \in \mathbb{R}^{n_e} \times \{0, 1\}^{n_f}\), and \(x^f_i = u_i, x^f_i - D_i u_i \leq 0\) is given as

\[
x^f_{i+1} = u_i, \quad x^f_i - D_i u_i \leq 0
\]

where \(x^f_i, u_i \in \{ \eta \in \{0, 1\}^{n_f} | e^T_{n_f} \eta = 1 \}\) and \(D_i\) is the adjacency matrix expressing the state transition at \(t_i\) and \(t_{i+1}\). See [13] for further details. By substituting (8) into Problem 2, Problem 2 can be rewritten as an MIQP problem in which \(\tilde{v}_i, i = k, k+1, \ldots, k+N-1\), are decision variables.
III. PROBLEM FORMULATION

Now we consider multi-vehicle systems. For simplicity of discussion, we assume that the number of vehicles is two. The proposed method can be extend to multi-vehicle systems consisting of more than two vehicles.

For each vehicle, the same finite set $\mathcal{F}_i$ of (6) and the same $\Lambda_i$ of (7) are used, that is, the discrete state space is shared. In addition, the sampling time $t_i$ is also shared. For the vehicle $j \in \{1, 2\}$, consider the following system:

$$
\begin{align*}
  x_j(t) &= A_j x_j(t) + B_j u_j(t), \\
  C_j x_j(t_i) + D_j u_j(t_i) &\leq G_j, \\
  x_j(t_i) &= x_{i,j}, \\
  x_{i,j} &= \begin{bmatrix} x_{i,j}^c \\ x_{i,j}^d \end{bmatrix}, \\
  x_{i,j}^d(i_{i,j},) &\in \mathcal{E}_{i,j}(x_{i,j}^d),
\end{align*}
$$

where $x_j(t) \in \mathbb{R}^n$ and $u_j(t) \in \mathbb{R}^m$ are the state and the control input, respectively. We assume that the pair $(A_j, B_j)$ is controllable. For the vehicle $j$, the linear constraint (10) is imposed, and the waypoint at time $t_i$ is given as $x_{i,j}$.

For the system (9)-(13), we consider the following finite-time optimal control problem.

**Problem 3:** For the system of (9)-(13), suppose that the state at the current time $t_i$ is given as $x_j(t_k) = x_{k,j}$, $j = 1, 2$. Then find a state feedback controller $u_j(t) \in \mathcal{P}C^m$, $t \in [t_k, t_{k+N}]$, and waypoints of the state $x_{i,j}$, $i = k+1, k+2, \ldots, k+N$, minimizing the cost function

$$
J(x_{k,j}, u_j(\cdot), \{x_{i,j}\}_{i=k+1,k+2,\ldots,k+N}) = \sum_{j=1}^{2} \sum_{k+N-1}^{k+1} J_j(x_{i,j}, u_j(\cdot), \{x_{i,j}\}_{i=k+1,k+2,\ldots,k+N})
$$

where $J_j(\cdot)$ is the cost of the continuous evolution of the dynamics between discrete-states given by

$$
J_{i,j}(\cdot) = \int_{t_i}^{t_{i+1}} \left\{ \left( x_j(t) - x_{i+1,j} \right)^T Q_j (x_j(t) - x_{i+1,j}) + u_j^T(t) R_j u_j(t) \right\} dt
$$

under the constraint

$$
x_{i+1,j}^d \neq x_{i+2,j}^d, \quad i = k, k+1, \ldots, k+N - 1
$$

where $Q_j \geq 0$ and $R_j > 0$ are the weighting matrices given in advance.

By the constraint (15), collision avoidance between vehicles can be expressed. So obstacle avoidance and collision avoidance between vehicles can be achieved simultaneously by solving Problem 3. Furthermore, it is obvious that if the condition (15) can be expressed as a linear form, then Problem 3 can be reduced to an MIQP problem.

However, in Problem 3, only a collision avoidance is considered. In order to describe complicated cooperative actions, we introduce linear temporal logic (LTL) [4].

The outline of LTL is explained below. In LTL, logical operators and temporal operators are used. The logical operators usually consist of $\neg$, $\land$, $\lor$, $\rightarrow$, and $\leftrightarrow$. The temporal operators consists of path-specific quantifiers $\phi$ (eventually), $\phi$ (always), $\phi$ (next), $\phi$ (until). LTL formulas, state formulas, and path formulas are defined as follows:

- **Propositional variables and propositional constants** (true or false) are state formulas.
- **If $\phi, \psi$ are state formulas, then $\neg\phi, \phi \land \psi, \phi \lor \psi, \phi \rightarrow \psi, \phi \leftrightarrow \psi$ are also state formulas.**
- **If $\phi, \psi$ are state formulas, then $\phi \land \psi, \phi \lor \psi, \phi \rightarrow \psi, \phi \leftrightarrow \psi$ are path formulas.**
- **All state and path formulas consist of the above formulas, and all LTL formulas consist of state formulas.**

Suppose that $\phi, \psi$ are given as propositional variables. Then the meaning of each path-specific quantifier is also explained as follows:

- **$\phi$** eventually has to hold (somewhere on the subsequent path).
- **$\phi$** has to hold on the subsequent path.
- **$\phi$** has to hold at the next state.
- **$\phi\psi$** has to hold until at some position $\psi$ holds.

This implies that $\psi$ will be verified in the future.

It has been shown that LTL formulas can be expressed as linear inequalities with binary variables (see, e.g., [9], [10]). Furthermore, three symbols are defined. First, $\phi$ implies

$$
\phi := \circ \circ \cdots \circ \phi.
$$

Next, $\circ \phi$ implies that the propositional variable $\phi$ has to hold until $r$ steps. For example, $\circ \phi$ is given as

$$
\phi := \phi \lor \circ \phi \lor \circ \circ \phi.
$$

Finally, $\phi \circ \phi$ implies that the propositional variable $\phi$ has to hold one-time-only until $r$ steps. For example, $\phi \circ \phi$ is given as

$$
\phi := (\phi \land \neg \circ \phi \land \neg \circ \circ \phi) \lor (\neg \phi \land \circ \phi \land \circ \circ \phi) \lor (\neg \phi \land \neg \circ \phi \land \circ \circ \phi).
$$

By using LTL formulas, several cooperative actions can be described. As one of the typical actions, we consider the rendezvous problem, which is a class of the consensus problems [15]. In this problem, multiple vehicles must arrive at a target position simultaneously. More precisely, the notion of rendezvous in this paper is defined as follows.

**Definition 1:** It is said that two vehicles rendezvous if the following conditions hold:

$$
\begin{align*}
  x_{i,1}^d &\neq x_{i,2}^d, \\
  x_{i,l}^c &\neq x_{i,l+1}^c, \quad l \in \mathcal{L} \subseteq \{1, 2, \ldots, n^c\}
\end{align*}
$$

where $x_{i,j,l}^c$ is the $l$-th element of $x_{i,j}^c$, and $\alpha^d \in \mathcal{F}_i$ and $\alpha^c$ are a given constant or a decision variable in the finite-time optimal control problem, respectively. In addition, $\mathcal{L}$ is a given index set.

More precisely, the subsequent path in the finite time interval $k, k+1, \ldots, k+N$ is considered, because the finite-time optimal control problem is discussed in this paper.
By using (16) and (17), we can consider the rendezvous problem such that two vehicles arrive at a target position and those velocities are the same. The proposition variables $\varphi$ is assigned to rendezvous. That is, if two vehicles rendezvous, then $\varphi = 1$ (true), otherwise $\varphi = 0$ (false). Then we consider the following problem.

**Problem 4:** For the system of (9)–(13), suppose that the state at the current time $t_k$ is given as $x_j(t_k) = x_{k,j}$, $j = 1, 2$. In addition, suppose that $t_{k+1}, t_{k+2} \in \{ t_k, t_{k+1}, \ldots, t_{k+N} \}$ are given. Then find a state feedback controller $u_j(t) \in PC^m$, $t \in \{ t_k, t_{k+1}, \ldots, t_{k+N} \}$, and waypoints of the state $x_{i,j}$, $i = k + 1, k + 2, \ldots, k + N$, minimizing the cost function (14) under the constraint that the following LTL formula

$$\phi := \circ_{r_1-1} \neg \varphi \land \circ_{r_2} \varphi \land \circ_{r_2+1} \square \neg \varphi$$  \hfill (18)

which can be equivalently transformed into the following linear inequalities:

$$\left\{ \begin{array}{l}
x_{i,1,s}^f + x_{i,2,s}^f + \alpha_s^f - \delta_{i,s} \leq 2, \\
-x_{i,1,s}^f - x_{i,2,s}^f - \alpha_s^f + 3\delta_{i,s} \leq 0.
\end{array} \right.$$

See [1] for further details. Note here that if for some $\delta \in \{ 1, 2, \ldots, n_f \}$, $\delta_{i,s} = 1$ holds, then $\delta_{i,s} = 0$, $s \in \{ 1, 2, \ldots, n_f \} \backslash \{ \delta \}$ holds. By using $\delta_{i,s}$, the condition (17) can be expressed as the following linear inequalities:

$$(1 - \delta_{i,s})x_{i,j}^f + \delta_{i,s} \alpha_s^f \leq x_{i,j,l}^f \leq (1 - \delta_{i,s})x_{i,j}^f + \delta_{i,s} \alpha_s^f,$$

$$j \in \{ 1, 2, \ldots, l \}, \ l \in L$$

where $x_{i,j,l}^f, l \in L$ are lower and upper bounds given in advance, respectively. If $\delta_{i,s} = 1$ holds in (20), then (17) holds, otherwise no constraint is imposed. In addition, from (19) and (20), we see that $\alpha^f$ and $\alpha_s^f$ may be either a decision variable or a constant. Thus the propositional variable $\varphi$ can be expressed as

$$\varphi = \delta_{i,1} \lor \delta_{i,1} \lor \cdots \lor \delta_{i,n_f}.$$  \hfill (21)

Consider time evolution of $\varphi$. Let $P_i^\varphi \in \{ 0, 1 \}$ denote a binary variable such that if $\varphi$ is true at time $t_i$, then $P_i^\varphi = 1$, otherwise $P_i^\varphi = 0$. Then from (21), $P_i^{\varphi,1}$, $i = 0, 1, \ldots, N$ can be expressed as the following linear inequalities:

$$\left\{ \begin{array}{l}
P_i^{\varphi,1} \leq \sum_{s=1}^{n_f} \delta_{i,s}, \\
P_i^{\varphi,1} \geq \delta_{i,s}, \ s = 1, 2, \ldots, n_f.
\end{array} \right.$$

Next, consider expressing (18) as linear inequalities. The proposition variables $\psi_1, \psi_2$, and $\psi_3$ are defined as

$$\psi_1 := \circ_{r_1-1} \neg \varphi,$$

$$\psi_2 := \circ_{r_2} \varphi,$$

$$\psi_3 := \circ_{r_2+1} \square \neg \varphi,$$

respectively. First, consider $\psi_1$. The proposition variable $\psi_1$ can be expressed as

$$\psi_1 = \neg \varphi \land \circ \neg \varphi \land \cdots \land \circ_{r_1-1} \neg \varphi.$$  \hfill (22)

Then $P_i^{\psi_1}$ can be expressed as

$$P_i^{\psi_1} = P_i^{\varphi,1} \land P_i^{\varphi,2} \land \cdots \land P_i^{\varphi,n_f}.$$  \hfill (23)

Noting that $\neg \varphi$ is transformed into $1 - \varphi$ (i.e., $P_i^{\varphi,1}$ is equivalent to $1 - P_i^{\varphi,1}$), the propositional variable $P_i^{\psi_1}$ can be expressed as the following linear inequalities:

$$\left\{ \begin{array}{l}
\sum_{i=0}^{r_1} (1 - P_i^{\varphi,1}) - P_i^{\psi_1} \leq r_1, \\
\sum_{i=0}^{r_1} (1 - P_i^{\varphi,1}) + (r_1 + 1)P_i^{\psi_1} \leq 0.
\end{array} \right.$$

Next, consider $\psi_2$. The proposition variable $\psi_2$ can be expressed as

$$\psi_2 = \psi_{2,1} \lor \psi_{2,2} \lor \cdots \lor \psi_{2,r_2}.$$  \hfill (24)

where $\psi_{2,i}, i \in \{ 1, 2, \ldots, r_2 \}$ is defined as

$$\psi_{2,i} := \neg \varphi \land \circ \neg \varphi \land \cdots \land \circ_{r_2-1} \neg \varphi.$$  \hfill (25)
(24) can be expressed as the following linear inequalities:

$$
\begin{align*}
\frac{P_k}{\psi_2} &\leq \sum_{i=1}^{r_2} \frac{P_k}{\psi_2,i}, \\
\frac{P_k}{\psi_2} &\geq \frac{P_k}{\psi_2,i}, \quad i = 1, 2, \ldots, r_2.
\end{align*}
$$

(25) can be expressed as the following linear inequalities:

$$
\begin{align*}
\sum_{j=0,j\neq i}^{r_2} (1 - \frac{P_k}{\phi}) + \frac{P_k}{\phi} - \frac{P_k}{\phi_2,i} &\leq r_2, \\
- \sum_{j=0,j\neq i}^{r_2} (1 - \frac{P_k}{\phi}) - \frac{P_k}{\phi} + (r_2 + 1) \frac{P_k}{\phi_2,i} &\leq 0.
\end{align*}
$$

Thus $\frac{P_k}{\psi_2}$ can be transformed into (26) and (27). Third, the propositional variable $\psi_3$ can be expressed as

$$\psi_3 = \ominus \frac{r_2+1}{\varphi} \wedge \cdots \wedge \ominus N \ominus \varphi,$$

which can be transformed into

$$
\begin{align*}
\sum_{i=r_2+1}^{N} \frac{P_k}{\phi} - \frac{P_k}{\phi_3,i} &\leq N - r_2 - 1, \\
- \sum_{i=r_2+1}^{N} \frac{P_k}{\phi} + (N - r_2) \frac{P_k}{\phi_3,i} &\leq 0.
\end{align*}
$$

Finally, $\phi = \psi_1 \vee \psi_2 \vee \psi_3$ can be transformed into the following linear inequalities:

$$
\begin{align*}
\frac{P_k}{\phi} &\leq \sum_{i=1}^{3} \frac{P_k}{\psi_i}, \\
\frac{P_k}{\phi} &\geq \frac{P_k}{\psi_i}, \quad i = 1, 2, 3.
\end{align*}
$$

Thus, the propositional variable $\phi$ in (18) can be transformed into a set of linear inequalities (19), (20), (22), (23), (26), (27), (28) and (29). That is, Problem 4 can be reduced to an MIQP problem.

C. Efficient Modeling Method of Directed Graphs

The computation time for solving the MIQP problem may be frequently long. Here, we focus on directed graphs such as Fig 1 (a), because redundant edges are included. By eliminating redundant edges before solving the MIQP problem, decreasing the computation time will be achieved. In this section, the outline is explained.

First, consider the case of single-vehicle systems. By focusing on the current state and the candidates of terminal states, redundant edges can be found. In [14], only the current state is focused. The method in this paper is a more sophisticated version of the method in [14].

Next, consider the case of multi-vehicle systems. Owing to obstacles and the current state, timings and discrete states that two vehicles can rendezvous are limited, and can be computed from directed graphs. According to timings and discrete states obtained, edges can be eliminated. In the case where a target point is given, edges can be further eliminated.

In the above two methods, redundant edges can be eliminated based on adjacency matrices between $t_i$ and $t_{i+1}$, and the computation time for eliminating edges is relatively smaller than that of the MIQP problem.

V. NUMERICAL EXAMPLE

As an example, the dynamics of the vehicle are given as

$$\ddot{x}_p = -3 \dot{x}_p + x_d, \quad \dot{x}_d = -5 x_d + u$$

where $x_p$, $x_d$, and $u$ are the position, the driving force, and the control input, respectively. So for $x := [x_d \dot{x}_p x_p]^T$, we have $A, B$ of the system (9) as follows:

$$A_j = \begin{bmatrix} -5 & 0 & 0 \\ 1 & -3 & 0 \end{bmatrix}, \quad B_j = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$
VI. Conclusion

In this paper, using partial state discretization, we have proposed a control method for multi-vehicle systems with linear temporal logic (LTL) specifications. By discretizing elements of states with complex constraints such as obstacle avoidance, we can easily consider both collision avoidance between vehicles and obstacle avoidance. In addition, cooperative actions can be specified by linear temporal logic (LTL) formulas. As one of the examples of LTL formulas, we have considered the rendezvous problem. Furthermore, we have also proposed an efficient modeling method of directed graphs expressing complex constraints. Finally, we have shown numerical simulation.

In order to solve the problem faster, it is important to develop a distributed algorithm. Then the problem formulation and the MIQP problem obtained in this paper will give us several suggestions. Furthermore, in numerical simulation, we have considered simple dynamics of vehicles. It is also important to consider complex dynamics of vehicles.

From these examples, we see that elimination of redundant edges is effective for decreasing the computation time. In addition, for all cases, the computation time for eliminating edges was less than 2 [msec]. From this result, we see that the computation time for eliminating edges is sufficiently smaller than that of the MIQP problem appeared in this paper.

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