Adaptive nonlinear control of braking in railway vehicles.

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Abstract—In this work a new technique for anti-slip control in railway vehicles is proposed. This technique is based on a new parametrization of the unknown adherence force. The control philosophy is alternative to the ones currently implemented in the industry and found in the literature, and aims at enhancing vehicle performances by estimating the maximum available adhesion force, which is then exploited via a nonlinear controller. The effects of the implemented control scheme are the use of the maximum available deceleration in case of poor adhesion conditions and, consequently, a reduction in the wear rate of the rolling stock. The results are illustrated via simulation.

I. INTRODUCTION

One of the most important problems in railway vehicles control is the control of the traction/braking effort. Due to the nature of the contact forces between the wheels and the rails, both made of steel, the friction coefficient, which is responsible for the longitudinal (and lateral) forces exchange, is usually low in nominal conditions (dry surfaces). The presence of contaminants on the railway furthermore reduces the available friction coefficient, and with that the maximum traction/braking force achievable for the motion of the vehicle. When the wheel exerts an excessive force with respect to the maximum available one, two behaviors can be distinguished. During a braking maneuver, the wheel tends to lock, which means that the wheel stops rotating and slips on the rail, with consequential damages for the wheel, which is flattened in the point of contact, as well as the rail. The same in a traction maneuver, when giving excessive force to the wheel produces heating (even sparkles if the motion is uncontrolled) which can damage both the wheel and the rail, and most important of all the train might not accelerate at all. A typical situation where this effect can be seen is when a tram is stopped and water or leaves or both are present on the railway, which can also be slightly in rise. The tram will not be able to start unless it goes backward, accelerates on a previous part of the railway and then overcomes the low adhesion point of the railroad. This causes delays and inefficiency in the energy consumption. Thus anti-slip control strategies have been studied and various solutions have been proposed in literature. A review of these is available in [1]. It is important to underline that this problems are, from a mathematical point of view, similar to the ones which one has in automobiles or motorcicles or airplanes in what, with opportune approximations, all these systems may be described with the same dynamical model, which is given in [2].

The estimation of the maximum adherence slip is very delicate for many reasons, all relying on the fact that a good knowledge of both the adherence force and the slip must be available at every time instant. Although many approaches have been studied, nowadays still fuzzy logics are the most implemented ones in railway vehicles’ anti-slip controllers. In this paper we propose a new control strategy based on modern nonlinear control techniques which exploits the characteristics of the nonlinear model describing the system.

The paper is organized as follows. In section [II] we present the mathematical models and the problems typically encountered when simulating or designing an anti-slip controller. The main focus is on braking dynamic. Results provided can be easily transferred to accelerating dynamic. In section [III] we present the adopted solution to the problem and give proof of the stability of the closed loop system. In section [IV] simulations are shown with the proposed control algorithm. Finally we give a resume of the achieved goals in the conclusion.

II. SYSTEM MODEL

Given a train bogie as sketched in Figures 1 and 2, the independent coordinates are: the bogie longitudinal speed $v(t)$ and the angular wheel speed $\omega(t)$.

Fig. 1. Sketch of the mechanical system modeled

The relative slip in a braking maneuver belongs to the interval $[0, 1]$, see e.g. [3], and is defined as follows:

$$\sigma(t) = \frac{v(t) - R\omega(t)}{v(t)}$$

(1)

In the latter $R$ is the wheel radius and is a constant parameter. The inputs for the system are the contact
force, \( F_a(\sigma(t)) \), which is a function of \( \sigma(t) \), and the motor torque \( T(t) \). Other constant parameters for the model are the bogie mass \( M \), the wheel inertia \( J \). The damping for the rotation of the axle is \( b_\omega \), while \( b_v \) and \( b_\sigma \) take into account respectively the damping due to the motion of the train and the air drag. A model for the system, considering the two wheels as a whole and neglecting the transmission dynamics, is:

\[
\begin{align*}
J \ddot{\omega} &= -b_\omega \omega + RF_a(\sigma) - gT \\
M \ddot{v} &= -b_v v - b_\sigma v^2 - F_a(\sigma)
\end{align*}
\]  
(2)

If one neglects the friction forces different from the contact one, the model is simplified and becomes:

\[
\begin{align*}
J \ddot{\omega}(t) &= RF_a(t) - T(t) \\
M \ddot{v}(t) &= -F_a(t)
\end{align*}
\]  
(3)

describing the system of Figure 2. In this equation \( T(t) \) represents the positive braking torque which is the control variable, \( F_a(t) \) the contact force between the wheel and the rail, which is an unknown disturbance. It is well known [4], [5], [6] that the adherence force is a function of the relative speed between the contacting surfaces of the wheel and the rail. It also depends on a number of other factors, e.g. the presence of contaminants on the railway, the geometry of the wheel and the rail to cite the most important ones; for an in depth survey the interested reader is referred to [4], [5], [6] and the bibliography therein.

One validated model for railway applications is given by Polach in [6] and is reported here for the sake of completeness:

\[
\begin{align*}
\dot{\mu} &= \mu_0 \left[ (1 - A) e^{-B(v - R_\omega)} + A \right] \\
\varepsilon &= \frac{2 C \pi a^2 b \sigma}{F_a} \\
F_a &= \frac{2 F_a \dot{\mu}}{\pi} \left( \frac{k_A}{1 + (k_A \varepsilon)^2} + \arctan(k_S \varepsilon) \right)
\end{align*}
\]  
(4)

Note that \( F_a \) is the vertical force acting on the contact surface. In these equations there are a number of parameters difficult to know a priori or to estimate accurately from measures available in real time: \( \mu_0 \) is the maximum friction coefficient at zero slip velocity; \( A, B \) influence the dependency of the friction on the absolute slip velocity \( v - R_\omega \); \( C \) is a proportionality coefficient characterising the contact shear stiffness; \( a \) and \( b \) are the diameters of the elliptical contact patch; \( k_A \) and \( k_S \) shape the adherence curve in different rail conditions (wet or dry). See [6] for more details about these parameters. Considering \( C a^2 b \) as one coefficient, there are 6 unknown parameters in Eq. 4.

This model, as well as others found in literature, is quite complicated for the use in control algorithm design, thus simpler curves are usually taken into consideration for the description of the adherence force. A common choice is to characterize it as the product of two terms as:

\[
F_a(t) = F_z(t) \mu(\sigma(t), \theta)
\]  
(5)

where \( \mu(\sigma(t), \theta) \) is called adherence coefficient, and \( F_z(t) = Mg + \Delta F_z \) the vertical load acting on the contact surface (as in Eq. 4), which can be considered constant on a first approximation. The shape of the adherence curve for different tram speeds is given in Figure 3 (solid lines).

![Adherence force at different tram speeds](image)

In order to achieve the maximum adherence force a different and simpler model can be used. This function has two local optima but only one at positive \( \sigma \), which is the one used in this application.

\[
\mu(\sigma, \theta) = \frac{\sqrt{\sigma}}{a + b \sigma + \sigma^2} = \frac{\sqrt{\sigma}}{\theta \Phi(\sigma)}
\]  
(6)

Equation 6 is defined for \( 0 \leq \sigma \leq 1 \), as we are dealing with braking dynamics, and is positive whenever \( a, b, c > 0 \), while \( \Phi(\sigma) = [1, \sigma, \sigma^2] \) is the regressor vector.

Parameters for the fitted curves are shown in Table I, where it is evident that the unknown parameters depend on the velocity. This model has the advantage of describing all the operating conditions with only 3 parameters, but depends nonlinearly on them. Also note that emphasis has
TABLE I
PARAMETRIZATION OF THE ADHESION MODEL FOR DIFFERENT SPEEDS

<table>
<thead>
<tr>
<th>Speed [km/h]</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.3203</td>
<td>3.61</td>
<td>9.4e-10</td>
</tr>
<tr>
<td>20</td>
<td>0.3018</td>
<td>4.456</td>
<td>1.68e-10</td>
</tr>
<tr>
<td>30</td>
<td>0.2891</td>
<td>5.197</td>
<td>0.2513</td>
</tr>
<tr>
<td>50</td>
<td>0.2789</td>
<td>6.232</td>
<td>2.371</td>
</tr>
</tbody>
</table>

been given to the description of the leftmost part of the adherence curve, where the peak is located, while in the rightmost part at least the sign of the derivative of the fitted curve is coherent with the model. As will be shown later, increasing the number of parameters to properly fit the curve over the whole range $\sigma \in [0,1]$ does not give any advantage, as the dynamic matrix of the estimator error is structurally singular.

Now we introduce the nonlinear system used for the controller design, which is obtained from Equations (3) with the new coordinate $\sigma$ from Equation (1), as shown in [2].

The parameters $\nu = \frac{M_2 g^2}{\beta}$ and $\Gamma = \frac{g}{\beta}$ in Equation (7) are constant and known.

In this model the state variable $v$ is slowly varying with respect to $\sigma$, thus it is considered as a slowly varying parameter in the dynamic law of $\sigma$.

III. SLIP ADAPTIVE ESTIMATION

With the model of Equation (7) an estimator of the adherence force based on the Immersion and Invariance technique [7] can be applied as follows.

Define:

$$z := \dot{\theta} - \theta + \beta(v, \sigma)$$

This variable is the parameter estimation error plus some function which is a designer’s choice.

Letting $\frac{\partial \beta}{\partial \sigma} = 0$ and differentiating $z$ with respect to time one obtains, in the hypothesis of constant parameters $\theta$:

$$\dot{z} = (\dot{\theta} + \frac{\partial \beta}{\partial \sigma} \dot{\sigma})$$

(9)

Substituting the expression of $\dot{\sigma}$ the equation suggest the following choice of $\dot{\theta}$:

$$\dot{\theta} = -\frac{g}{\nu} \frac{\partial \beta}{\partial \sigma} \frac{\nu}{v} [(\sigma - 1 - \nu)\mu(\sigma, z + \theta) + \Gamma T]$$

(10)

where $z + \theta = \dot{\theta} + \beta(v, \sigma)$ is the computed estimation of the parameters.

Substituting the expressions of $\mu$ and rearranging the equations one obtains the following dynamics for $z$:

$$\dot{z} = \frac{g\sqrt{\sigma}}{\Phi(\sigma)'\theta\Phi(\sigma)'(z + \theta)} \left[\frac{(\sigma - 1 - \nu) \frac{\partial \beta}{\partial \sigma} \Phi(\sigma)'}{v} z\right]$$

(11)

The term between squared parenthesis has rank 1 for any choice of $\beta(v, \sigma)$. Notice however that we are only interested in the convergence of $\phi(\sigma)'z$ to zero, in order to capture the value of the adherence coefficient. We define:

$$\beta(v, \sigma) = k_\beta \left[\frac{\sigma^2}{2}, \frac{\sigma^3}{3}\right]'$$

(12)

where $k_\beta$ is a positive real parameter useful to tune the convergence rate of the estimation. With this choice of $\beta$ the $z$ variable will converge to the iperplane $\Phi(\sigma)'z = 0$.

Lemma 1: Consider system (11) with function $\beta$ defined in (12) and $\theta$ is the set of constant parameters. Moreover, assume that the input $T$ in (7) is bounded. Then, for each $\sigma(0) \neq 0$ and $\sigma(0) \neq 1$, it follows

$$\lim_{t \to \infty} \Phi(\sigma)'z = 0$$

Proof: Consider the function $W = z'z$ and observe that $\dot{W} = -w'\Phi(\sigma)'z^2$, where $w$ is a positive bounded function of $\sigma, v$ and $z$. Hence $\Phi(\sigma)'z$ is a square integrable function. Moreover, noticing that the derivative of $\Phi(\sigma)'z$ is also bounded, the conclusion easily follows.

The previous result guarantees that we have a good estimate $\hat{\mu} = \mu(\sigma, \dot{\theta} + \beta(v, \sigma))$ of the function $\mu(\sigma, \theta)$, and that the function $(\Phi(\sigma)'(\hat{\theta} + \beta(v, \sigma)) - \Phi(\sigma)'\theta)$ converges to zero when the adherence curve parametrization $\theta$ is constant.

Once $\hat{\mu}$ is known at every time instant one can use this information to compensate the real adherence force. This is done with the control law of Equation (13), which is a feedback linearizing controller where the term $\rho$ is a new input.

$$T(t) = -\frac{1}{\Gamma} \left[\frac{(\sigma(t) - 1 - \nu) \hat{\mu}(\sigma, \dot{\theta} + \beta(\sigma)) + v \rho}{g}\right]$$

(13)

With this control law the new system dynamic is as follows

$$\dot{\sigma}(t) = \rho(\sigma, \delta) + n$$

(14)

where $n = \frac{2}{v} (\sigma - 1 - \nu)(\mu(\sigma, \theta) - \hat{\mu})$ is a signal that goes asymptotically to zero.

The function $\rho$ will be designed later. It depends on $\sigma$ and an additional input $\delta$. At this point one can give a dynamic law to $\sigma(t)$ in order to guarantee that the equilibrium point $\sigma^*$ (the value of $\sigma$ maximizing $\mu(\sigma, \theta)$ is asymptotically stable for all initial conditions of $\sigma$ in the open interval $(0,1)$. To do this a knowledge of the maximum operating point is needed, which means that the quantity $\frac{2\mu(v, \sigma)}{v}$ should be known at every time instant. Therefore, consider the first of Equation (7)
and the parametrization given for $\mu(\sigma, \theta)$. Differentiating with respect to time and with little rearrangements one obtains:

$$\ddot{v}(t) = -\frac{\partial \mu(\sigma, \theta)}{\partial \sigma} \dot{\sigma}$$  \hspace{1cm} (15)

where

$$\frac{\partial \mu(\sigma, \theta)}{\partial \sigma} = -\frac{\Phi(\sigma)'H\theta}{2\sqrt{\sigma} (\Phi(\sigma)')^2} \hspace{1cm} (16)$$

and $H = \text{diag}([1, -1, -3])$. We write again the equations of the model, with this new equation added.

The latter can be used to compute an estimation of $D\mu(\sigma, \theta) = \frac{\partial \mu(\sigma, \theta)}{\partial \sigma}$ with the same Immersion and Invariance technique used for $\mu$.

With a similar approach as before, we define a new variable:

$$\zeta := \dot{\sigma} - \alpha + \gamma(\dot{v}, \sigma, \delta)$$  \hspace{1cm} (17)

Differentiating $\zeta$ over time, with the hypothesis of constant parameters $\alpha$, one obtains:

$$\ddot{\zeta} = \dot{\alpha} + \frac{\partial \gamma}{\partial \dot{v}} \ddot{v} + \frac{\partial \gamma}{\partial \sigma} \ddot{\sigma} + \frac{\partial \gamma}{\partial \delta} \ddot{\delta}$$  \hspace{1cm} (18)

For this system one may choose the adaptation law for $\dot{\alpha}$ as in Equation (19):

$$\dot{\alpha} = -\frac{\partial \gamma}{\partial \dot{v}} \left[ -\frac{g\dot{\sigma}}{2\sqrt{\sigma} \left( \Phi(\sigma)'(\dot{\theta} + \beta) \right)^2} \right] [\Phi(\sigma)'H] \left( \zeta + \alpha \right)$$

$$- \frac{\partial \gamma}{\partial \sigma} \ddot{\sigma} - \frac{\partial \gamma}{\partial \delta} \ddot{\delta}$$  \hspace{1cm} (19)

which in turn gives the dynamics of $\zeta$ as in Equation (20):

$$\dot{\zeta} = \frac{\partial \gamma}{\partial \dot{v}} \left[ -\frac{g\dot{\sigma}}{2\sqrt{\sigma} \left( \Phi(\sigma)'(\dot{\theta} + \beta) \right)^2} \right] [\Phi(\sigma)'H] \zeta$$  \hspace{1cm} (20)

Notice that the implementation of the filter (19) requires the knowledge of $\dot{\sigma}$, given by equation (7), and the knowledge of $\mu(\sigma, \theta)$. However, one can replace $\mu(\sigma, \theta)$ with $\tilde{\mu}$, since we know that $\tilde{\mu} - \mu$ converges to zero. Moreover, as $\gamma$ depends on $\dot{v}$, the knowledge of this signal is necessary for the implementation of the filter. Again notice that, from equation (7), a good estimate of $\dot{v}$ is given by $-g\dot{\mu}$, since $\tilde{\mu} - \mu$ converges to zero.

As before we face now the fact that the variable $\zeta$ does not converge to zero, but towards the hyperplane $\Phi(\sigma)'H\zeta = 0$, which is the maximum loci. The function $\gamma$ can be selected as follows:

$$\gamma(\dot{v}, \sigma, \delta) = -\rho(\sigma, \delta)H\Phi(\sigma)\dot{v}$$  \hspace{1cm} (21)

We are ready to state the convergence result.

**Lemma 2:** Consider system (20) with function $\gamma$ defined in (21) and $\rho$ given by (14) with $n = 0$. Moreover, assume that the new input $\delta$ is bounded. Then, for each $\sigma(0) \neq 0$ and $\sigma(0) \neq 1$, it follows

$$\lim_{t \to \infty} \Phi(\sigma)'H\theta = 0, \quad \lim_{t \to \infty} \delta = 0$$

**Proof:** Consider the function $\tilde{W} = \zeta'\zeta$ and observe that $\tilde{W} = -\rho^2\dot{\tilde{w}}^2|\Phi(\sigma)'H\zeta|^2$, where $\tilde{w}$ is a positive bounded function of $\sigma$ and $\zeta$. Hence $\Phi(\sigma)'H\zeta$ is a square integrable function. Moreover, noticing that the derivative of $\Phi(\sigma)'H\zeta$ is also bounded, the conclusion easily follows.

The previous result says that, despite a perfect parameter estimation is not achieved, still an estimation of the value of the derivative of $\mu$ can be known at every instant. As a matter of fact, we have a good estimate $\hat{D}\mu = D\mu(\sigma, \dot{\sigma} + \gamma)$ of $D\mu(\sigma, \theta)$ and the function $(\Phi(\sigma)'H(\dot{\sigma} + \gamma(\dot{v}, \sigma, \delta)) - \Phi(\sigma)'H\theta)$ converges to zero.

Nonetheless, the velocity of convergence is a designer’s choice, via the function $\gamma(\dot{v}, \sigma, \delta)$. It is important to remember that the dynamics of $\sigma$ is assigned with the term $\delta$, which is a control variable, so that it is not affected by measurement noise at least in this analysis. Also remind that the previous result relies on the assumptions of Lemma 1, in particular $\dot{\theta} = 0$.

A proper choice of $\rho$ is:

$$\rho(\sigma, \delta) = -k_\delta \sigma(\sigma - 1)$$  \hspace{1cm} (22)

where $k_\delta$ is a positive parameter.

The dynamics of $\delta$ can be imposed so that the overall system has in $\sigma \in (0, 1)$ just one attractive equilibrium wherein the adherence curve has a peak, namely $\Phi(\sigma)'H\theta = 0$. Hence, we arrive to the following system

$$\dot{\sigma} = -k_\sigma \delta \sigma(\sigma - 1)$$  \hspace{1cm} (23)

$$\dot{\delta} = k_{d1} \Phi(\sigma)'H(\dot{\sigma} + \gamma(\dot{v}, \sigma, \delta)) - k_{d2}\delta$$  \hspace{1cm} (24)

The phase portrait for the system of Equations (23) and (24) is in Figure 4.

The equilibrium in the interval $\sigma \in (0, 1)$ and $\delta = 0$ is asymptotically stable for $k_\sigma > 0$, $k_{d1} > 0$, $k_{d2} > 0$, while other equilibrium points in $(\sigma, \delta) = (0, 0)$ and $(\sigma, \delta) = (1, 0)$ are saddle points. Thus the idea is to steer the slip towards the peak of the adhesion curve, which guarantees the maximum braking effort, and the minimum amount of control energy is lost in non-effective maneuvers.

**Lemma 3:** Consider (23), (24) and replace in (24) the function $\Phi(\sigma)'H(\dot{\sigma} + \gamma(\dot{v}, \sigma, \delta))$ with $\Phi(\sigma)'H\theta$. Then for each $\sigma(0) \neq 0$ and $\sigma(0) \neq 1$, it follows that $\sigma(t)$ and $\delta(t)$ are bounded signals and

$$\lim_{t \to \infty} \Phi(\sigma)'H\theta = 0, \quad \lim_{t \to \infty} \delta = 0$$
\[ x' = -K_1 y (x) (x - 1) \]
\[ y' = K_2 (A - B x - 3 C x^2) - K_3 y \]

\( C = 0.251 \)
\( K_3 = 1000 \)
\( B = 5.197 \)
\( K_2 = 1000 \)
\( A = 0.2891 \)
\( K_1 = 1000 \)

The robustness of this control scheme with respect to measurement errors, which have been neglected here, is of the proposed control algorithm. A quantification of the robustness of this control scheme with respect to measurement errors, which have been neglected here, is

**IV. SIMULATION RESULTS**

Simulations for the braking maneuver have been done with the model of Equation 7. We consider a constant parametrization for the adherence curve (e.g. does not change with train speed) to show the results of convergence as derived in the previous section. In particular simulation for the characteristic of 30km/h is shown, with the parameters of Table I, in Figure 3. The starting speed is 30km/h and the simulation time is 1s. The gains used in simulation are \( k_\beta = k_\sigma = k_\gamma = k_{d1} = k_{d2} = 1000 \). This value has been chosen to show the rate of convergence achievable, in principle, with the proposed method.

**V. CONCLUSIONS**

The purpose of this work was to study a new technique for the anti-slip control in railway vehicles. The results show that it is possible to stabilize the system in the optimal operating point by using nonlinear adaptive control. Further studies are being made to consider improvements of the proposed control algorithm. A quantification of the robustness of this control scheme with respect to measurement errors, which have been neglected here, is
necessary. From the theoretical point of view extensions to the traction case arise quite naturally and were not reported here for clarity of exposition. Practical limitations due to the actuators saturation should be investigated too but, with some minor modifications, the saturations bound can be included in the control. Once the optimal operating point is shown to be exactly achievable, one may ask what kind of cooperative dynamics in a long convey may arise, where each axle is capable of exploiting the maximum adherence force available and the latter varies from axle to axle, typically increasing as one moves from the ones in front of the convey to the ones in the rear. One remark that can be made is that this control system only uses typical measures available today on trains, so that in principle no new instrumentation is needed.

REFERENCES


