A Stable Linear Adaptive Controller Applied to a Pneumatic Actuator System

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Abstract—This paper presents a stable linear adaptive control scheme for the position control of a pneumatic actuator system. The proposed controller does not require the experimental identification of the mass flow rate, which is really a novelty in the field of such applications. The controller synthesis is based on an uncertain non-affine nonlinear model of the pneumatic system. Firstly, the existence of an ideal controller which can achieve position control objectives is demonstrated using the implicit function theory. Nevertheless, even if the actuator model is well-known, this ideal controller cannot be known and computed. The aim of the proposed approach is to construct this unknown ideal controller using a simple linear controller with a stable adaptation mechanism. The stability of the closed-loop system is studied by using a Lyapunov approach. Finally, simulation results are provided to show the capabilities of the presented control method.

I. INTRODUCTION

Control design for pneumatic actuator systems has been a topic of active research in recent years due to their important potential applications. In [1], a feedback linearization control method with disturbance rejection is proposed. In [2], a combined robust differentiator and robust controller via high order sliding mode for an electropneumatic system is presented. In [3], a robust 3rd order sliding mode position controller is applied for an electropneumatic system. In [4], a nonlinear backstepping control and nonlinear sliding mode control laws are developed for this system. In [5], the authors developed a nonlinear passivity based control law for an electropneumatic system. In [6], an application of a robust sliding mode adaptive gain control law for the pneumatic system is presented. In [7], an adaptive twisting sliding mode control is proposed and applied to the pneumatic system. The two latter references have highlighted the interest to use adaptive gains, a consequence of such gains being that the evaluation of uncertainties/perturbations bounds is not required (or is very limited). Note also that all the previously cited works [1]-[7] are using control laws based on the model, and are requiring an adequate model of the mass flow rate. This latter, which plays a key-role in the dynamics of the pneumatic actuator, has been often written as an affine nonlinear function in the control input. However, the determination of such a function is a very hard task. There is then a practical interest to avoid this fastidious step in the identification of the experimental system.

In this paper, a stable linear adaptive controller is proposed using directly a non-affine model of the pneumatic system, which means that the mass flow rate is “weakly” identified. Within this scheme, a linear control with its adaptive law is used to estimate an unknown ideal controller that can achieve position control objectives for a pneumatic actuator. The existence of this ideal controller is shown by using the implicit function theory [8], [9]. The gain adaptation law is designed, based on the gradient descent method, by directly minimizing the error between the unknown implicit ideal controller and the used linear controller. The stability of the closed-loop system is studied using a Lyapunov method. The developed controller guarantees the boundedness of all the signals in the closed-loop system, and ensures the convergence of the tracking error to a vicinity of the origin.

Recall that, in the approach described in the sequel of the paper, the experimental identification of the mass flow rates is not necessary and the control system does not require the same level of accuracy in the pneumatic actuator model as in [1]-[7]. Furthermore, it does not require that the system reads as a system affine in the control input. It is really a novelty in the field of pneumatic actuator control with respect to previous robust controllers.

The paper is organized as follows. Section 2 presents the description of the used pneumatic actuator system and its model. The control problem formulation and the proposed linear adaptive controller are given in Section 3. Section 4 details simulations of the proposed control scheme applied to the pneumatic actuator system. Section 5 concludes this article.

II. DESCRIPTION AND MODELING OF THE PNEUMATIC SYSTEM

The considered electropneumatic system is displayed by Figure 1. This system consists of two actuators, respectively named the “main” actuator and the “perturbation” one [7]. In this work, only the control of the “main” actuator position is studied, the “perturbation” actuator being used to generate an external force (unknown for the “main” actuator). The “main” actuator is a double acting pneumatic actuator controlled by two servodistributors and is composed by two chambers denoted $P$ and $N$. The pneumatic jack horizontally moves a load carriage of mass $M$. This carriage is coupled to the “perturbation” actuator. The control objective is to drive the position of the “main” actuator to desired trajectory in

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Fig. 1. Description scheme of the electropneumatic system.

spite of the unknown perturbation force (which can be time-varying).

The pressure dynamics in the both chambers $N$ and $P$ of the “main” actuator reads as (for details, see [2], [4], [7])

\[ p_p = \frac{krT}{V_p(y)} \left( q_m(u, p_p) - \frac{S}{rT} p_p v \right) \]
\[ p_N = \frac{krT}{V_N(y)} \left( q_m(-u, p_N) + \frac{S}{rT} p_N v \right), \tag{1} \]

whereas the mechanical part dynamics is described by

\[ \dot{y} = v, \]
\[ \dot{v} = \frac{1}{M} \left( S(p_p - p_N) - b_v v - F_{ext} \right), \tag{2} \]

with $V_p(y) = V_0 + S \cdot y$ the volume of the $P$-chamber, $V_N(y) = V_0 - S \cdot y$ the volume of the $N$-chamber, $V_0$ being the half-cylinder volume, $p_p$ and $p_N$ the pressures in chambers $P$ and $N$ respectively, $u$ the input voltage bounded as $|u| \leq 10V$, $k$ the polytropic constant, $T$ the chamber temperature, $S$ the piston surface, $y$ the piston position, $v$ the piston velocity, $r$ the perfect gas constant, $b_v$ the viscous friction coefficient, and $F_{ext}$ the external force produced by the “perturbation” actuator.

From (1)-(2), it is clear that the actuator position $y$ has a relative degree equal to 3 versus the control input $u$. Then, the input-output representation of the “main” actuator reads as

\[ y^{(3)} = \frac{SkrT}{M} \left( \frac{q_m(u, p_p)}{V_p(y)} - \frac{q_m(-u, p_N)}{V_N(y)} \right) - \frac{kS^2}{M} \left( \frac{p_p}{V_p(y)} + \frac{p_N}{V_N(y)} \right) \dot{y} - \frac{1}{M} \left( b_v \ddot{y} + F_{ext} \right). \tag{3} \]

Let introduce the function $f(\cdot)$ defined as

\[ y^{(3)} = f(y, \dot{y}, \ddot{y}, p_p, p_N, u). \tag{4} \]

Denoting $X = [p_p, p_N, y, \dot{y}, \ddot{y}]^T$, let define $\Omega$ as the physical domain in which the system is evolving [6], [7]

\[ \Omega = \{ X \mid 1 \text{ bar} \leq p_p \leq 7 \text{ bar}, 1 \text{ bar} \leq p_N \leq 7 \text{ bar}, |y| \leq 72 \text{ mm}, |\dot{y}| \leq 1 \text{ m/s}, |\ddot{y}| \leq 15 \text{ m/s}^2 \}. \tag{5} \]

It yields that, for $X \in \Omega$, systems dynamics are bounded under a bounded control input $u$. Furthermore, $\forall X \in \Omega$, $V_p(y) > 0$ and $V_N(y) > 0$. Note that, given that the relative degree of $y$ equals 3 and the state dimension of the system equals 4, it means that, if the output of the system is $y$, the internal dynamics has a dimension equal to 1. In fact, this internal dynamics is one of the pressure and is stable in the operating domain [4].

The main difficulty for model (3) is to know the mass flow rates $q_m(u, p_p)$ and $q_m(-u, p_N)$. The identification is really a fastidious task (and needs to be done when the servodistributors are changed). However, in many previous works [4], [6], [7], this identification has been made; then, the mass flow rates have been experimentally identified for the servodistributors used in the experimental set-up (Servotronic from Joucomatic co.) and read as

\[ q_m(u, p_p) = \varphi_p + \psi_p u, \]
\[ q_m(-u, p_N) = \varphi_N - \psi_N u, \tag{6} \]

with $\varphi_j$ and $\psi_j$ ($j \in \{P, N\}$) fifth-order polynomials with respect to $p_j$.

In this work, it is worth to note that the identification of the flow rate $q_m(\cdot)$ is not necessary. So, the proposed control scheme will be developed directly based on the nonaffine nonlinear dynamic systems of the “main” actuator. It is really a novelty in the application field of pneumatic actuators.

### III. Problem Statement

Denoting $x = [x_1, x_2, x_3]^T = [y, \dot{y}, \ddot{y}]^T$ and $p = [p_p, p_N]^T$. From (4), one gets

\[ \dot{x}_1 = x_2, \]
\[ \dot{x}_2 = x_3, \]
\[ \dot{x}_3 = f(x, p, u), \tag{7} \]

with $p = [p_p, p_N]^T$ the pressure vector and $u$ the control input. The objective being to control the position $y$, the input-output representation

\[ y^{(3)} = f(x, p, u). \tag{8} \]

In this section, the goal is to design a control law $u(t)$ such that the position $y(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded. Throughout this paper, suppose that the following assumptions regarding (8) and the desired trajectory $y_d(t)$ are fulfilled

**Assumption 1:** The function $f_u(x, p, u) = \frac{\partial f(x, p, u)}{\partial u}$ is nonzero and bounded as $0 < \delta_0 < f_u(x, p, u) < \delta_1$ for all $(x, p, u) \in \Omega \times R$, $\delta_0$ and $\delta_1$ being positive constants.

**Assumption 2:** The desired trajectory $y_d(t)$ and its time derivatives $y_d^{(i)}(t), i = 1, 2, 3$, are smooth and bounded.
Remark 1: Since the mass flow rates \( q_m(u, p_P) \) and \( q_m(-u, p_N) \) are not well-known for the pneumatic system, it is assumed that \( f_u(x, p, u) > 0 \) and bounded. It is worthy of mentioning that, with the identified mass flow rates terms

\[
q_m(u, p_P) = \varphi_p + \psi_p u, \quad q_m(-u, p_N) = \varphi_N - \psi_N u, \tag{9}
\]

with \( \varphi_j > 0 \) and \( \psi_j > 0 \) over the physical domain, one has

\[
f_u(x, p, u) = \frac{Sk_rT}{M} (\psi_p + \psi_N) > 0,
\]

which implies that the Assumption 1 is fulfilled.

Define the tracking error as

\[
e(t) = y_d(t) - y(t), \tag{10}\]

and the filtered tracking error as

\[
s(t) = \left( \frac{d}{dt} + \lambda \right)^3 \int_0^t e(\tau) d\tau, \lambda > 0. \tag{11}\]

From (11), \( s(t) = 0 \) represent a linear differential equation whose solution implies that \( \int_0^t e(\tau) d\tau, e(t), \dot{e}(t), \ddot{e}(t) \) converge to zero [10].

Thus, the control objective becomes the design of a controller to force \( s(t) \) at zero: therefore, the original stabilizing problem of the (4)-dimensional vector \( \begin{bmatrix} f_u(t) e(t) \end{bmatrix} \) is reduced to the problem of keeping the scalar \( s(t) \) at zero. Moreover, bounds on \( s(t) \) can be directly translated into bounds on the tracking error [10].

Proposition 1: Consider the pneumatic actuator system defined in (7) with the filtered error \( s(t) \) given by (11). Then, there exists some ideal control \( \hat{u} \) which can achieve position control objectives such that \( \int_0^t e(\tau) d\tau, e(t), \dot{e}(t) \) converge to zero as \( t \to \infty \).

Proof. The time derivative of \( s(t) \) reads as

\[
\dot{s} = y_d^{(3)} + \beta_3 \dot{e}^{(2)} + \beta_2 e^{(1)} + \beta_1 e - \beta_1 e(0) - f(x, p, u), \tag{12}\]

with \( \beta_i = \frac{3^i}{(3+i+1)(i-1)!} \lambda^{n-i+1}, i = \{1, 2, 3\} \). Let a signal \( \omega \) be defined as

\[
\omega = y_d^{(3)} + \beta_3 \dot{e}^{(2)} + \beta_2 e^{(1)} + \beta_1 e - \beta_1 e(0) + Ks + K_0 \tanh(\varepsilon_0), \tag{13}\]

with \( K > 0 \) and \( K_0 > 0 \). The parameter \( \varepsilon_0 \) is a small positive constant, and \( \tanh(\cdot) \) is the hyperbolic tangent function. From this previous equation, one has

\[
y_d^{(3)} + \beta_3 \dot{e}^{(2)} + \beta_2 e^{(1)} + \beta_1 e - \beta_1 e(0) = \omega - Ks - K_0 \tanh(\varepsilon_0), \tag{14}\]

From (12)-(14), one gets

\[
\dot{s} = -Ks - K_0 \tanh(\varepsilon_0) - (f(x, p, u) - \omega). \tag{15}\]

From Assumption 1 and given that \( \omega \) defined in (13) does not explicitly depend on \( u \), the partial derivative of \( f(x, p, u) - \omega \) with respect to the input \( u \) satisfies

\[
\frac{\partial (f(x, p, u) - \omega)}{\partial u} = \frac{\partial f(x, p, u)}{\partial u} > 0. \tag{16}\]

Thus, based on the implicit function theorem [8], [9], the nonlinear algebraic equation \( f(x, p, u) - \omega = 0 \) is locally solvable with respect to the input \( u \) for each \( (x, p, \omega) \). Thus, there exists some “ideal” controller \( \hat{u}(x, p, \omega) \) satisfying the following equality for all \( (x, p, \omega) \in \Omega \times \mathbb{R} \)

\[
f(x, p, \hat{u}(x, p, \omega)) - \omega = 0, \tag{17}\]

Therefore, if the control input \( u \) is chosen as the “ideal” control law, i.e. \( u = \hat{u} \), the closed-loop error dynamic (15) reads as

\[
\dot{s} = -Ks - K_0 \tanh(\varepsilon_0), \tag{18}\]

given the dynamics of \( s \), it is obvious that \( s(t) \to 0 \) as \( t \to \infty \) and, therefore, \( \int_0^t e(\tau) d\tau, e(t), \dot{e}(t) \) and \( \ddot{e}(t) \) converge to zero [10].

According to the previous analysis, the “ideal” control law \( \hat{u}(x, p, \omega) \) can ensure the boundedness of all variables in the closed-loop system and guarantees output tracking of a specified desired trajectory \( y_d(t) \). Nevertheless, this control law cannot be formally obtained and implemented given that the function \( f(x, p, u) \) is not well-known. Then, the sequel of the paper displays a methodology which allows the control law \( \hat{u}(x, p, \omega) \) to be approximated by a linear adaptive controller.

IV. LINEAR ADAPTIVE CONTROLLER DESIGN

In the previous section, based on the implicit function theory, the existence of “ideal” control law that can achieve position control objectives is demonstrated. In this section, a linear controller is used to construct adaptively this unknown ideal implicit controller. The presented control scheme has been developed also for a class of uncertain continuoustime single-input single-output nonaffine nonlinear dynamic systems (see [11], for more details).

A. Control Law

From the mean value theorem [9], the “ideal” control law \( \hat{u}(x, p, \omega) \) can be written around the desired state vector \( \dot{x}_d = [y_d^{(1)}, y_d^{(2)}]^T \) as the following

\[
\hat{u} (x, p, \omega, x_d) = \hat{u}(x_d, p) = \frac{\partial \hat{u}(x_d, p, x_d)}{\partial x_{\alpha}} (x - x_d), \tag{19}\]

with \( \alpha \in [0, 1] \) and \( x_{\alpha} = \alpha x + (1 - \alpha) x_d \). Denote

\[
K_c(x, p, x_d) = \frac{\partial u^*(x_{\alpha}, p, x_d)}{\partial x_{\alpha}}, \tag{20}\]

then, equation (19) can be rewritten as

\[
\hat{u} (x, p, x_d) = K_c(x, p, x_d) (x - x_d) + \hat{u}(x_d, p), \tag{21}\]

with \( e = x - x_d = [e_1^{(1)}, e_2^{(2)}]^T \). Let assume that there exist optimal bounded time varying parameters \( K_c \in \mathbb{R}^3 \),

\[
\frac{\partial (f(x, p, u) - \omega)}{\partial u} = \frac{\partial f(x, p, u)}{\partial u} > 0. \tag{16}\]
\[ K_i \in \mathbb{R} \text{ and } K_0 \in \mathbb{R} \text{ with bounded time derivatives such that the ideal controller } u(x, p, x_d) \text{ fulfills} \]
\[ \dot{u}(x, p, x_d) = K_\varepsilon e + K_i \int_0^\varepsilon \varepsilon(\tau) d\tau + K_0 + \varepsilon(x, p, x_d), \]  
(22)
with \( \varepsilon(x, p, x_d) \) the approximation error. The gains \( \left( K_\varepsilon, K_i, K_0 \right) \) are the unknown optimal parameters which minimize the function \( |\varepsilon(x, p, x_d)| \). Denoting \( \Pi(e) = [e^1, e^2, \int_0^\varepsilon e(\tau) d\tau, 1]^T \) and \( \hat{\Theta} = [K_\varepsilon, K_i, K_0]^T \), equation (22) can be re-written as
\[ \dot{u} = \Pi^T(e) \hat{\Theta} + \varepsilon(x, p, x_d). \]  
(23)

**Assumption 3:** The approximation error \( \varepsilon(x, p, x_d) \) in (23) is bounded such that
\[ \varepsilon^2(x, p, x_d) \leq \tilde{\varepsilon}_0 s^2 + \tilde{\varepsilon}_1, \]  
(24)
where \( \tilde{\varepsilon}_0 \) and \( \tilde{\varepsilon}_1 \) are two positive constants.

Since the “ideal” parameter vector \( \hat{\Theta} \) is unknown, so it should be estimated by a suitable adaptation law. Let \( \Theta(t) \) be an estimation of the “ideal” vector \( \hat{\Theta} \); the problem consists now in defining the control law as an adaptive linear approximation of the ideal controller (23), i.e. the control law for system (7) reads as
\[ u = \Pi^T(e) \Theta(t), \]  
(25)
with \( \Theta(t) \) evolving with respect to an adaptation law defined in the next subsection.

**B. Adaptation Law for Linear Control**

Our goal in this subsection is to design an adaptation law for \( \Theta \) such that the linear controller (25) approximates the unknown “ideal” controller (23), i.e. the adaptation law should be designed in order to make the error between \( \dot{u} \) and \( u \) as small as possible. Furthermore, the adaptation law should guarantee the boundedness of the parameters estimates. Define the error between the controllers \( \dot{u} \) and \( u \) as
\[ e_u = \dot{u} - u. \]  
(26)
From (23) and (25)-(26), it yields
\[ e_u = \dot{u} - \Pi^T(e) \hat{\Theta} = \Pi^T(e) \hat{\Theta} + \varepsilon(x, p, x_d), \]  
(27)
with \( \hat{\Theta} = \hat{\Theta} - \Theta \). By invoking the mean value theorem [9], there exists a constant \( \alpha \) with \( 0 < \alpha < 1 \), such that the nonlinear function \( f(x, p, u) \) can be expressed around \( \dot{u} \) as
\[ f(x, p, u) = f\left(x, p, \dot{u}\right) + f_u(u) \cdot (u - \dot{u}), \]  
(28)
with \( f_u(u) = \frac{\partial f(x, p, u)}{\partial u} > 0 \) and \( u_\alpha = \alpha u + (1 - \alpha) \dot{u} \). By substituting (28) into the error equation (15), one gets
\[ \dot{s} = -Ks - K_0 \tanh\left(\frac{s}{\sigma u_\alpha}\right) - \left(f\left(x, p, \dot{u}\right) + f_u(u) \cdot (u - \dot{u}) - \omega\right). \]  
(29)
By using (17), one gets
\[ \dot{s} = -Ks - K_0 \tanh\left(\frac{s}{\sigma u_\alpha}\right) - f_{u,u} \cdot (u - u^*), \]  
(30)
which can be rewritten as
\[ \dot{s} = Ks + K_0 \tanh\left(\frac{s}{\sigma u_\alpha}\right) = f_u(u) \cdot (u - \dot{u}) = f_{u,u} \cdot e_u. \]  
(31)

Note that \( \dot{u} \) is an unknown quantity: the signal \( e_u \) in (26) is not available. Equation (31) is used to overcome this difficulty. Indeed, from (31), even if the signal \( e_u \) is not available for measurement, the quantity \( f_{u,u} \cdot e_u \) is measurable given that \( s \) and \( \dot{s} \) are available. This fact will be exploited in the design of the parameters adaptive law.

Consider now a quadratic cost function which evaluates the discrepancy between the implicit ideal controller \( \dot{u} \) and the actual linear controller \( u \), defined as
\[ J(\Theta) = \frac{1}{2} e_u^2 = \frac{1}{2} \left(\dot{u} - u\right)^2, \]  
(32)
The gradient descent method is used here to minimize the cost function (32). Hence, by applying the gradient descent method [10], one gets as an adaptation law for the gain \( \Theta \), the following first order differential equation
\[ \dot{\Theta} = -\eta(t) \nabla_{\Theta} J(\Theta), \]  
(33)
with \( \eta(t) \) a positive time-varying parameter. From (32), the gradient of \( J(\Theta) \) with respect to \( \Theta \) is
\[ \nabla_{\Theta} J(\Theta) = -\Pi(e) e_u. \]  
(34)
Therefore, the gradient descent algorithm becomes
\[ \dot{\Theta} = \eta(t) \Pi(e) e_u. \]  
(35)
The adaptation law (35) cannot be implemented since the signal \( e_u \) is not available. In order to make (35) computable, from (31), one selects the design parameter \( \eta(t) = \eta_0 f_{u,u} \), with \( \eta_0 \) a positive constant. Thus, (35) becomes
\[ \dot{\Theta} = \eta_0 \Pi(e) f_{u,u} \cdot e_u. \]  
(36)
By using (31), one gets
\[ \dot{\Theta} = \eta_0 \Pi(e) \left(\dot{s} + Ks + K_0 \tanh\left(\frac{s}{\sigma u_\alpha}\right)\right). \]  
(37)
As shown in [12], the adaptation law (37) cannot guarantee the boundedness of the parameters \( \hat{\Theta} \) in the presence of approximation errors that are unavoidable in such adaptive schemes. Then, in order to improve the robustness of the adaptation law (37) in the presence of approximation errors, one modifies it as follows [12]
\[ \dot{\Theta} = \eta_0 \Pi(e) \left(\dot{s} + Ks + K_0 \tanh\left(\frac{s}{\sigma u_\alpha}\right)\right) - \eta_0 \sigma \Theta, \]  
(38)
with \( \sigma \) a small positive constant.
C. Stability of the Closed-loop System

In order to analyze the tracking error convergence and the stability of the closed-loop system, consider the following Lyapunov candidate function

\[
V = \frac{1}{2} s^2 + \frac{1}{2 \eta_0} \dot{\Theta}^T \dot{\Theta}.
\]  
(39)

By using (31)-(38)-(27), the time derivative of (39) reads as

\[
\dot{V} = -K s^2 - K_0 s \tanh (\gamma e) + s f_{u_0} e_u - e_u f_{u_0} e_u + \varepsilon (x, x_d) f_{u_0} e_u + \sigma \dot{\Theta}^T \Theta + \frac{1}{\eta_0} \dot{\Theta}^T \dot{\Theta}.
\]
(40)

With the following inequalities

\[
\sigma \dot{\Theta}^T \Theta \leq -\frac{\sigma}{2} \| \dot{\Theta} \|^2 + \frac{\sigma}{2} \| \Theta \|^2,
\]

\[
\frac{1}{\eta_0} \dot{\Theta}^T \dot{\Theta} \leq \frac{1}{4} \| \dot{\Theta} \|^2 + \frac{1}{\sigma \eta_0} \| \Theta \|^2,
\]

\[
\varepsilon (x, p, x_d) f_{u_0} e_u \leq \frac{1}{4} f_{u_0} e_u^2 + f_{u_0} e_u^2 (x, p, x_d),
\]

\[
s f_{u_0} e_u \leq \frac{1}{4} f_{u_0} e_u^2 + f_{u_0} s^2,
\]

and the inequality (24), equation (40) can be bounded as

\[
\dot{V} \leq -\frac{1}{2} f_{u_0} e_u^2 - K_0 s \tanh (\gamma e) - (K - (1 + \varepsilon_0) f_{u_0}) s^2 - \frac{\sigma}{2} \| \dot{\Theta} \|^2 + \frac{\sigma}{2} \| \Theta \|^2 + \frac{1}{\sigma \eta_0} \| \Theta \|^2 + f_{u_0} \bar{e}_1.
\]
(41)

Since the parameters \( \dot{\Theta} \), its time derivative \( \dot{\Theta} \) and the function \( f_{u_0} \) are assumed bounded, then there exists a positive constant bound \( \Psi \) defined as

\[
\Psi = \sup_t \left( \frac{\sigma}{2} \| \dot{\Theta} \|^2 + \frac{1}{\sigma \eta_0} \| \Theta \|^2 + f_{u_0} \bar{e}_1 \right).
\]
(42)

Then, equation (41) can be simplified as

\[
\dot{V} \leq -\frac{1}{2} f_{u_0} e_u^2 - K_0 s \tanh (\gamma e) - (K - (1 + \varepsilon_0) f_{u_0}) s^2 - \frac{\sigma}{4} \| \dot{\Theta} \|^2 + \Psi.
\]
(43)

By assuming that the design parameter \( K \) is chosen such that \( K > \delta_1 \), the inequality (43) can be written as in (44) where \( \gamma = \min \left( 2 \times (K - (1 + \varepsilon_0) \delta_1), 0.5 \sigma \eta_0 \right) \)

\[
\dot{V} \leq -\gamma V + \Psi,
\]
(44)

**Theorem 1**: Consider the system (7). Suppose that Assumptions 1, 2 and 3 are satisfied. Then, the control law (25) with the adaptation law (38) guarantees that the closed-loop system is UUB stable and the output tracking error converges to a small neighborhood of the origin.

**Proof.** From (44), one has

\[
V(t) \leq V(0) e^{-\gamma t} + \frac{\Psi}{\gamma}.
\]
(45)

From (45), it can be shown that, for \( V \geq \frac{\Psi}{\gamma} \), \( \dot{V} < 0 \). According to a standard Lyapunov theorem, the signals \( s(t), \Theta(t) \) and \( u(t) \) in the closed-loop system are bounded. Moreover, from (39) and (45), one has \( |s(t)| \leq \sqrt{|s(0)|^2 + \frac{1}{\eta_0} |\Theta(0)|^2} e^{-0.5\gamma t} + \frac{2\sqrt{\Psi}}{\gamma} \). In order to achieve the convergence of a tracking error to a small neighborhood around zero, the parameters \( K, \sigma \) and \( \eta_0 \) should be chosen appropriately. Then, it is possible to make \( \sqrt{\frac{2\sqrt{\Psi}}{\gamma}} \) as small as desired. Denoting \( \Phi = \sqrt{\frac{2\sqrt{\Psi}}{\gamma}} \). Then, there exists a time \( T \) such that \( |s(t)| \leq \Phi \) for \( t > T \). It implies that the tracking errors converge to residual sets \( |z_i(t)| \leq 2^i \lambda^{(i-n)} \Phi \), \( i = 0, 1, 2, 3 \) with \( z = \int_0^t e(\tau) d\tau \). This completes the proof.

V. Simulation Results

This section is dedicated to verify thanks simulations, the efficiency of the previously proposed adaptive controller for the position control of the pneumatic system. These simulations have been made with Matlab-Simulink (The Mathworks co.) by using Runge-Kutta integration algorithm with a fixed step size equal to 0.001 sec.

The used parameters of the pneumatic actuator are the following

\[
M = 3.4 kg, k = 1.2, \quad b_v = 50, \quad S = 45 \times 10^{-4} m^2, \quad V_0 = 3.7 \times 10^{-4} m^3, \quad T = 293 K, \quad r = 287 J/kg/K, \quad \eta = 0.2 sin(0.2 \pi t) mm. \quad \text{The system initial conditions are} \quad y(0) = 10mm, \quad \dot{y}(0) = 0 m/s, \quad \ddot{y}(0) = 0 m/s^2, \quad p_p(0) = p_n(0) = 2 bar. \quad \text{The initial values of the parameter estimates are} \quad \Theta(0) = (0, 0, 0, 0, 0)^T. \quad \text{The design parameters used in simulation are} \quad \lambda = 0.1, \quad K = 15, \quad K_0 = 5, \quad \varepsilon_0 = 0.01, \quad \eta_0 = 5, \quad \sigma = 0.001.
\]

The simulation results appear in Figures 2-6. Figure 2 shows the actual position versus time whose the tracking error versus desired trajectory (Figure 3-Top) is maintained in a vicinity of 0. Note that the control input \( u \) (Figure 3-Bottom) is feasible by a practical point-of-view given that there is no saturation and no too much fast dynamics; furthermore, the control signal \( u \) is almost smooth. It means that the control solution is practically applicable.

The time evolution of the gain \( K_0 \) is shown in Figure 4-Top whereas the gain \( K_0 \) is displayed by Figure 4-Bottom and the gains \( K_{e-1}, K_{e-2}, K_{e-3} \) in Figure 5. It is clear that the gains are evolving versus perturbation signals: they are increasing when there is perturbation magnitude variation. The pressures \( p_p \) and \( p_n \) are given by Figure 6-Top.
This paper proposes a linear adaptive controller for a pneumatic actuator system. The developed controller guarantees the boundedness of the whole signals in the closed-loop system, and ensures the convergence of the position tracking error to a neighborhood of 0. The main interest of this controller is the very reduced identification process of the mass flow rate, whereas previous robust strategies require this identified function. Then, the proposed control scheme can be viewed as a stable and simple model free controller for the pneumatic actuator systems. Simulation results demonstrate the feasibility of the proposed controller. Future works will consist in applying the control solution on the real experimental set-up.

**REFERENCES**


