On the Simultaneous Realization of Virtually Zero-power and Zero-compliance Controls

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Abstract—A virtually zero-power and zero-compliance mechanism is presented in this paper. A zero-compliance system using an integral control accompanies power consumption to cancel disturbances on the system. On the other hand, systems using a zero-power control can avoid power consumption at steady-state. However, the zero-power control causes a negative deflection of the system in the presence of disturbance. Therefore, in a conventional controlled system, zero-power and zero-compliance characteristics simultaneously are incompatible to each other. This study proposes a new mechanism to obtain zero-power and zero-compliance states simultaneously in a controlled system. The proposed mechanism comprises a zero-power control magnetic suspension system connected with a normal spring (positive stiffness) in series, where the stiffness of the magnetic suspension and the normal spring are equal in absolute magnitude.

I. INTRODUCTION

Presently, the control with less power consumption has been a focus of industrial researches. A zero-power control is one of the means of reducing power consumption, which was originally invented to use in active magnetic bearings [1]-[2]. A zero-power control behaves like a negative stiffness control.

An integral control is a common approach for realizing zero-compliance. However, an integral control usually accompanies power consumption even in steady-states. In some applications, such power consumption should be avoided. Therefore, in this study, a suspension mechanism is proposed to make zero-compliance and zero-power characteristics compatible. Such idea has been already implemented in the field of vibration isolation [3-5]. This paper discusses the idea outlined above in a more general manner for applying this idea in various fields.

The proposed mechanism consists of positive stiffness and negative stiffness suspensions which are connected in series. The negative stiffness and positive stiffness suspensions are achieved by the voice coil motor (VCM) guided by zero-power control and spring-mass system, respectively. Inherently, the proposed mechanism consumes zero power at steady-state, and simultaneously can realize zero compliance when the absolute negative stiffness and positive stiffness of the suspensions are equal in magnitude. On the other hand, if an integral control is applied instead of the zero-power control to obtain the proposed mechanism, in principle, it would cause zero compliance independently of equal stiffness, but there is power consumption. This power consumption becomes less when the stiffnesses are equal in absolute magnitude [6].

In the original concept of the proposed mechanism, the positive suspension is supposed to be achieved by a normal spring; but in the experimental set up, the voice coil motor (VCM) guided by PD (proportional derivative) control, providing varied positive stiffness, is used instead for finding the responses corresponded to unequal stiffness.

In practical practice, however, it is difficult to maintain the exactly same absolute stiffness of the series connected suspensions used in a controlled system [7]. In such a case, either zero compliance or zero power characteristic of the system is inevitably lost. This study emphasizes on the zero-power characteristic rather than the zero-compliance characteristic, for that why a zero-power control is used to realize the proposed mechanism. The relation between difference in stiffness and respective steady-state deflection is studied experimentally. In addition, the cost function minimizing of the system, utilizing the proposed mechanism, is carried out to find the optimal operating point.

II. ZERO COMPLIANCE AND ZERO POWER

A suspension system with infinite stiffness provides zero compliance to direct disturbance. In this study, a series combination of negative and positive stiffness suspensions is considered to acquire an infinite stiffness system. The concept of infinite stiffness by a series combination of positive and negative stiffness is presented in below.

The combined stiffness of two springs connected in series (Fig. 1) can be expressed as

\[ k_c = \frac{k_1 k_2}{k_1 + k_2}, \]  

where \( k_1 \) and \( k_2 \) denote the stiffnesses of the springs. If one of the springs has negative stiffness and both of them are equal in absolute magnitude of stiffness, i.e., \( |k_1| = |k_2| \), then the combined stiffness \( k_c \) becomes infinite.
\[ k_x = \frac{(-k_2)k_s}{-k_2 + k_s} = \infty \]  

(2)

The relative displacement of the upper table (Fig. 1) against a direct disturbance can be obtained as follows:

\[ |k_1| = | - k_2 | \Rightarrow \frac{\text{Force}}{x_1 - x_0} = - \frac{\text{Force}}{x_2 - x_1} \Rightarrow x_2 - x_0 = 0. \]  

(3)

Therefore, even if a direct disturbance acts on the upper table, it has no steady-state displacement independently of whatever low absolute equal magnitude of stiffness of the springs. This concept is applied to realize the proposed mechanism, where the negative stiffness and positive stiffness are realized using a zero-power control and a normal spring, respectively. Because a zero-power system consumes zero power and behaves as it has negative stiffness, the proposed mechanism can hold the zero-power and zero-compliance properties simultaneously.

### III. BASIC SYSTEM

To realize the negative stiffness suspension using a zero-power control, a horizontal magnetic suspension system was fabricated (Fig. 2). In the suspension system, a ferromagnetic positioning stage with mass \( m_n \) between two permanent magnets that provide bias force at steady-states moves along horizontal direction (translation motion). A VCM (voice coil motor) is used to drive the positioning stage. The motion equation of the positioning stage is presented as follows:

\[ m_n \ddot{x}_n(t) = k_s x_n(t) + k_i i(t) + f_d(t), \]  

(4)

where \( x_n \): negative displacement of the positioning stage, \( k_s \): gap-force coefficient of the permanent magnets, \( k_i \): coefficient of the VCM, \( i \): control current, \( f_d \): disturbance acting on the positioning stage.

The state space equation of the positioning stage can be written as follows:

\[ \dot{x}_n(t) = A_n x_n(t) + B_n i(t) + D_n f_d(t), \]  

(5)

where

\[ x_n = [x_n \quad \dot{x}_n] \], \( A_n = \begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix}, \ B_n = \begin{bmatrix} b \\ 0 \end{bmatrix}, \ D_n = \begin{bmatrix} 0 \\ d \end{bmatrix}, \]

\[ a = \frac{k_s}{m_n}, \ b = \frac{k_i}{m_n}, \ d = \frac{1}{m_n}. \]

### A. Series Connection with a Positive-Stiffness Suspension

To obtain the zero-power and zero-compliance mechanism utilizing a zero-power control (section II), the positioning stage shown in Fig. 2 is connected with a positive-stiffness suspension, and shown in Fig. 3 where the positive-stiffness suspension system is simply modeled by a normal spring-mass system. The motion equations of the series connected suspensions, together representing the proposed mechanism, are given by

\[ m_p \ddot{x}_p(t) = -k_p x_p(t) - k_i i(t), \]  

(7)

where \( m_p \): mass of the spring-mass system, \( x \): absolute displacement of the positioning stage in the series connection, \( x_n \): positive displacement of the spring-mass system, \( k_p \): spring constant. The absolute displacement of the positioning stage can be defined as follows;

\[ x(t) = x_n(t) + x_p(t), \]  

(8)

where \( x_n \): denotes the negative displacement of the positioning stage respect to the middle mass. The state space equation of the system shown by Fig. 3 can be written as follows:

\[ \ddot{x}(t) = A \dot{x}(t) + B i(t) + D f_d(t), \]  

(9)

where

\[ x = [x \quad \dot{x}], \ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a & 0 & -a & 0 \end{bmatrix}, \ B = \begin{bmatrix} b \\ 0 \end{bmatrix}, \ D = \begin{bmatrix} 0 & d \end{bmatrix}, \]

\[ c = \frac{k_i}{m_p}, \ e = \frac{k_p}{m_p}. \]
It is assumed that all the coefficients (a to e) are positive.

IV. ZERO-POWER CONTROL

There are several methods of realizing zero-power control [8]:

1. Velocity feedback,
2. Integral local current feedback,
3. Observer-based controlling,
4. Low-pass filter and
5. Integral voltage feedback.

In this study, the second method is applied with a normal state feedback control. The zero-power control is applied to the positioning stage so that it behaves as a negative stiffness suspension; consequently, the state space equation shown by Eq. (9) can be rewritten as follows:

\[
\dot{x}_c(t) = A_x x_c(t) + B_c i(t) + D_c f_d(t),
\]

where

\[
x_c = \begin{bmatrix} x \end{bmatrix},
A_x = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},
B_c = \begin{bmatrix} B \\ 1 \end{bmatrix},
D_c = \begin{bmatrix} D \end{bmatrix},
\]

\[K = \begin{bmatrix} P_{d1} \\ P_{v1} \\ P_{d2} \\ P_{v2} \\ P_1 \end{bmatrix},
\]

and \(P_d, P_v, P_1\) are proportional, derivative and integral gains, respectively.

A. Controller Design by Pole Assignment

The Laplace transform of the state space equation (13) can be written as follows

\[sX_c(s) = (A_x + B_c K)X_c(s) + D_c F_d(s)
\]

\[X_c(s) = (sI - A_x - B_c K)^{-1} D_c F_d(s),
\]

where \(I\) denotes an identity matrix. The left-hand side coefficients of Eq. (15) represent the state transition matrix of the system. Moreover, the determinant of the state transition matrix defines the characteristic polynomial of the system and is given in below

\[
\det(sI - A_x - B_c K) = s^5 + \gamma_4 s^4 + \gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0,
\]

where

\[
\gamma_4 = \left(-bP_{v1} + cP_{v2} - P_1\right),
\gamma_3 = \left(-bP_{d1} + cP_{d2} - a + e\right),
\gamma_2 = \left((-be - ac)P_{v1} - acP_{v2} + (a - e)P_1\right),
\gamma_1 = \left((-be - ac)P_{d1} - acP_{d2} - ae\right),
\gamma_0 = aP_1.
\]

The poles are assumed to be given by “\(\lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}\)”, and the respective characteristic polynomial is expressed as follows:

\[s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 = 0,
\]

where coefficient are the function of poles.

The state feedback gains \(K\) respect to the poles \(\lambda\) are obtained by comparing the coefficients of Eqs. (16) and (17), and given as follows:

\[
\begin{bmatrix}
0 & -b & 0 & c & -1 \\
b & 0 & c & 0 & 0 \\
0 & -be - ac & 0 & -ac & a - e \\
-b e - ac & 0 & -ac & 0 & 0 \\
0 & 0 & 0 & 0 & ae \\
\end{bmatrix}
\begin{bmatrix}
p_{d1} \\
p_{v1} \\
p_{d2} \\
p_{v2} \\
p_1 \\
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_4 \\
\alpha_5 + a - e \\
\alpha_2 \\
\alpha_1 + ae \\
\alpha_0 \\
\end{bmatrix}
\]

B. Current Regulation

To achieve the zero-power characteristic, the control current has to be zero in steady-states. A transfer function representation is used to examine the zero-power behavior. The dynamics of the positioning stage, spring-mass system and the controller (zero-power) used for the proposed mechanism are given in Eqs. (19), (20), and (20), respectively as follows:

\[X(s) = \frac{hs^2 + ac + be}{(s^2 - a)(s^2 + e)} I(s) + \frac{d(s^2 + e)}{(s^2 - a)(s^2 + e)} F_d(s),
\]

\[\text{Photograph of the experimental setup.}
\]

Fig. 3. Positioning stage with positive stiffness suspension in series.
\[ X_p(s) = -\frac{c}{s^2 + \epsilon} I(s), \quad (20) \]

\[ I(s) = -(P_d^{(1)} + sP_v^{(1)})X_p + (P_d^{(2)} + sP_v^{(2)})X_p + \frac{P_I}{s} I(s), \quad (21) \]

where the initial values are assumed to be zero for simplicity. Substituting Eqs. (19) and (20) into Eq. (21) yields

\[ I(s) = \frac{(\beta_4 s^3 + \beta_3 s^2 + \beta_2 s + \beta_1) s}{s^5 + \gamma_4 s^4 + \gamma_3 s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0} F_d(s), \quad (22) \]

where

\[ \beta_4 = -dP_v^{(1)}, \quad \beta_3 = -dP_d^{(1)}, \quad \beta_2 = -dP_v^{(1)} I, \quad \beta_1 = -dP_d^{(1)} I \]

The disturbances is considered to be stepwise and it can be modeled as

\[ F_d(s) = \frac{F_n}{s}, \quad F_G: \text{(const)}. \quad (23) \]

When the controller parameters are selected to stabilize the closed-loop system, the control current in the steady-states \( I(\infty) \) becomes as follows:

\[ \lim_{s \to \infty} i(t) = \lim_{s \to \infty} I(s) = \lim_{s \to 0} I(s) = 0. \quad (24) \]

Therefore, the zero-power performance can be achieved by the proposed mechanism.

V. EXPERIMENT

A. Experimental Setup

A photograph of the experimental setup which was fabricated for examining the concept of the proposed mechanism is shown in Fig. 4. In the experimental setup, the positioning stage is connected in series with a middle mass (stage), where the both stages are driven by VCMs. In fact, the middle mass is driven by the VCM guided with a PD control rather than a mechanical spring for realizing variable positive stiffness. The controller gains are selected so that the VCMs belonged to the upper and middle stages can maintain the same magnitude of absolute stiffness. Eddy-current gap sensors are used to detect the relative displacements of the stages. The power amplifiers manufactured by Takasago Co. Ltd. are used to supply current to the VCMs according to the command signals. The designed control algorithm is implemented with a digital signal processor DS 1105 manufactured by dSPACE™.

B. Experimental Result

Figure 5 shows the step responses of the positioning stage and middle mass of the experimental setup utilizing the proposed mechanism. The relative displacements of these moving stages are measured against a step-wise disturbance. It is observed that the positioning stage can cancel the step-wise disturbance applied on it and returns almost to its original position and maintain this zero-compliance at steady-state. Concurrently, it is also confirmed that the control current converge to zero at steady-states. Hence, the zero-compliance and zero-power characteristics are simultaneously achieved.

Meanwhile, theoretically it is shown that the proposed mechanism solely can maintain zero-compliance when the

![Fig. 5 Step responses of the positioning stage and middle mass with zero-power control and control current.](image)

![Fig. 6 Positioning stage and control current for different percentage of deviation in stiffness with zero-power control.](image)
absolute magnitudes of the stiffnesses are equal \("(|k_s| = |k_p|)\) [3], although in actual practice, often it cannot hold the ideal condition \("(|k_s| = |k_p|)\).\) The behaviors of the experimental setup under the condition of unequal stiffness, i.e., \((|k_s| \neq |k_p|)\) are investigated as well and presented in the following paragraphs.

The displacements of the positioning stage, involving the different deviations between the two stiffnesses, are measured against the same step-wise disturbance and shown in Fig 6. It is noticed that the zero-compliance of the positioning stage is lost for unequal stiffness and the displacement increases with the increase of difference between the two stiffnesses. However, the control current converges to zero in steady-state independently of these differences.

Furthermore, with respect to the priority of achieving zero-compliance, the displacement cancellation control (integral control) is implemented instead of the zero-power control. The step responses of the positioning stage and middle mass, using proposed mechanism with displacement cancellation control, are measured against the same step-wise disturbance on the positioning stage, and the corresponding results are shown in Fig. 7. It is observed that, at steady-state, the positioning stage can maintain zero compliance independently of the magnitude of difference between stiffnesses, but there is power consumption. Moreover, this power consumption increases with the increase of difference between the magnitudes of stiffnesses.

VI. INTERMEDIATE CONTROL

The results and discussions part given above showed that the proposed mechanism based on (i) zero-power control and (ii) displacement cancellation control, respectively gives priority to achieve zero-power and zero-compliance characteristics. Moreover, the proposed mechanism loses zero-compliance and zero-power characteristics, respectively when the condition \("(k_s \neq k_p)\) exists. To improve this situation, an intermediate control is proposed, which comprises a mixture of the zero-power and displacement cancellation controls. To find out the optimal control point of the intermediate control with respect to the operating cost, a cost function analysis is conducted and presented in below.

Let us consider the negative stiffness part (positioning stage) for analyzing the cost function as the operating cost is
solely associated with this part (positive stiffness part by normal spring). The force balance equation against the disturbance \( f_d \) can be written as follows:

\[
f_d = k_s x_n + k_i i.
\]  

(25)

The force balance equation (25) gives the control feasibility line of the system which is shown in Fig. 8(a). The slope of the control feasibility line depends on the magnitude of \( k_s \) and \( k_i \). For a particular permanent magnet and actuator, the coefficients \( k_s \) and \( k_i \) are constant.

Meanwhile, the cost function of the system is the function of absolute displacement of the positioning stage \( x \) and control current \( i \). The absolute displacement against the disturbance \( f_d \) can be defined by the displacements of the negative stiffness part \( (X_{n0}) \) and the positive stiffness part \( (X_{p0}) \), and shown as follows:

\[
x = -X_{n0} + X_{p0},
\]  

(26)

where \( X_{n0} = \frac{f_d}{k_s} \), \( X_{p0} = \frac{f_d}{k_p} \).

Because \( k_s \) is constant, the negative displacement \( (X_{n0}) \) is constant for a certain disturbance. Inherently, the displacement \( x \) varies due to varying of positive displacement \( (X_{p0}) \). Moreover, the absolute displacement \( x \) becomes zero when \( X_{p0} \) is equal to \( X_{n0} \). Therefore, the cost function with regards to control cost can be expressed as follows:

\[
n = q_s \left( x - \left( X_{p0} - X_{n0} \right) \right)^2 + q_i i^2,
\]  

(27)

where \( q_s \) and \( q_i \), respectively represent the cost weighted function for displacement and control current. The contour of the cost function whose value is assumed equal to \( u^2 \) is an ellipse given by

\[
\frac{x - \left( X_{p0} - X_{n0} \right)}{q_s}^2 + \frac{i^2}{q_i} = 1,
\]  

(28)

which is shown in Fig. 8(b).

The cost function (elliptical shape) and the control feasibility line are drawn in a single plane which is shown in Fig. 8(c). The intersection points of the control feasibility line and the axes of the ellipse (cost function) confine the intermediate control region. The intermediate control becomes optimal respect to operating cost at that point where cost function and control feasibility line intersect each other, i.e., it indicates a specific set of control current and displacement which satisfies both cost function and control feasibility line. It is noticed [Eq. (27)] that the optimal control point depends on the weighted functions \( q_s \), \( q_i \), and \( f_d \).

Figure 9 depicts the approximate structure of the proposed intermediate control where gains \( r_1 \) and \( r_2 \) are the shear of displacement cancellation control and zero-power control, respectively. The gains \( r_1 \) and \( r_2 \), corresponding to the optimal operating point, are determined as follows:

\[
r_1 = \left(1 - \frac{Ax}{X_{p0} + X_{n0}} \right), \quad r_2 = \left(\frac{Ax}{X_{n0}} \right).
\]

VII. CONCLUSIONS

The mechanism corresponded to simultaneous zero-power and zero-compliance is studied in both analytically and experimentally. A Zero-power control and a displacement cancellation control both are implemented individually to achieve the proposed zero-power and zero-compliance mechanism. It is observed that the proposed mechanism can be obtained by the both controllers under a single identical condition \( k_s \neq k_p \). The cost function analysis shows that the optimal control for the proposed mechanism is an intermediate control between zero-power control and displacement cancellation control.

REFERENCES


