Torque allocation in electric vehicles with in-wheel motors: a performance-oriented approach

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Abstract—The coordinated control of vehicle actuators is gaining more importance as new platforms are becoming available, with chassis endowed with many different actuators that may help controlling the vehicle motion. Further, in-wheel motors (IWMs) allow using a single system to apply both positive and negative torques at the wheels, which can be actuated independently one from the other. In electric vehicles (EVs), moreover, such a freedom in the actuation mechanisms opens the way to the combined optimization of performance and energy consumption issues. In this paper, the problem of torque allocation for maximizing the vehicle performance in EVs is addressed. The proposed strategy is compared against a benchmark, a-causal optimal solution showing that only a negligible loss of performance is experienced.

I. INTRODUCTION AND MOTIVATION

Torque allocation for over-actuated vehicles is a very interesting and challenging research topic. The most modern vehicle architectures, in fact, offer a plurality of actuators that allow actively shaping the vehicle dynamic response. This active shaping can be performed by properly setting up hierarchical control problems that regulate the vehicle motion by allocating the torques at the wheels and then solving constrained control allocation problems that optimize different cost functions, that can be either safety, or energy, or performance-oriented (or, most interestingly, combinations of all these aspects), see e.g.,[1]. Usually, the high-level motion objectives are expressed in terms of imposing a desired yaw moment to the vehicle, which must then be transformed in wheel torque reference signals. When the actuators are installed in the same axle (e.g., with torque vectoring devices or with 2 IWMs), the allocation is easy to perform because there is a one-to-one mapping between the yaw-moment and the torque difference, see e.g., [2]. On the other hand, if 4 IWMs are present there is redundancy in the moment generation, which can be explored to minimize the energy consumption [1] or other performance metrics.

The approaches developed so far are, undoubtedly, very attractive and can ensure a safe and energy efficient operation of vehicle when the accelerations are moderate. However, as the vehicle approaches its operation limits, one may question if the existing allocation schemes will still be able to extract the maximum performance (e.g., to operate near the friction limits when one has combined steering and acceleration/braking).

In this context, the main goal of this work is to investigate torque allocation strategies that allow the driver to extract the maximum performance from the EV, i.e., minimize the “lap-time”, without trying to correct the trajectory or stabilize the vehicle motion. To this aim, we will start by formulating an optimal problem to find the torque distribution among the 4 IWMs that will enable the EV perform a given maneuver in minimum time, according to the minimum-time manoeuvring problem, see [3], which has emerged in recent years as a key tool for determining the optimal trajectory and driver inputs in the racing context, [4]. In the present work, this framework will be revisited to devise a benchmark, a-causal torque allocation solution, against which the online strategies can be objectively evaluated. Next, a causal, sub-optimal allocation strategy that uses only the current and past values of the driver’s inputs (steer, throttle and brake pedal) and the vehicle state (longitudinal and lateral acceleration) to establish the torque distribution is proposed.

II. VEHICLE MODEL

In this section, a two-track, non-linear vehicle model will be introduced, which will serve as a basis for the torque allocation strategies. To make the model tractable, the roll and pitch dynamics of the EV are neglected, as commonly done in this context, see e.g., [5].

Consider the vehicle represented in Fig. 1, as well as the vector $\mathbf{p} = [X \ Y \ \psi]^T$, which characterizes the position and orientation of vehicle’s center of gravity (CoG) in the XY axis system, fixed with earth. The dynamic evolution of $\mathbf{p}$ can be defined as

$$\mathbf{\dot{p}} = \mathbf{M}^{-1} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} - \Delta,$$

where $m$ is the vehicle mass, $I_z$ the yaw inertia, $\Delta \in \mathbb{R}^3$ a vector of disturbances, $(F_x, F_y)$ are the generalized forces applied to the vehicle CoG and $M_z$ the generalized yaw-moment; the matrix $\mathbf{T}(\mathbf{p})$ represents a change of coordinates between the $xy$ frame (integral to the CoG) and the XY coordinates. The equations of vehicle motion can also be compactly represented as

$$\mathbf{M} \mathbf{\ddot{p}} = \mathbf{T}(\mathbf{p})(\mathbf{F} - \Delta),$$

(1)
where $\mathcal{F} = \begin{bmatrix} F_x & F_y & M_x \end{bmatrix}^T$ is the generalized force/moment applied to the CoG, which can also be regarded as a pseudo-control input.

One now needs to define $\mathcal{F}$, which depends on complex nonlinear friction forces generated in the tire-road interface. To this end, it is convenient to first introduce some auxiliary variables: the linear velocities of the car $(v_x, v_y)$, specified in the $xy$ frames, and its yaw-rate ($\dot{\psi}$), can be obtained applying a change of coordinates between the $xy$ and $XY$ frames as

$$
\begin{bmatrix} v_x \\ v_y \\ \dot{\psi} \end{bmatrix}^T = T^{-1}(p) \tilde{p}.
$$

Similarly, given that the wheels speeds are the consequence of the superposition of $(v_x, v_y)$ and rotational motion (yaw-rate), one can write [6]:

$$
\begin{bmatrix} v_{Li} \\ v_{Ci} \end{bmatrix} = W^{-1}(\delta_i) \begin{bmatrix} 1 \\ 0 \\ \chi_{Li} \\ 0 \\ 1 \\ \chi_{Ci} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \dot{\psi} \end{bmatrix}, \quad i \in \{1l, 1r, 2l, 2r\}
$$

$$
\begin{bmatrix} \chi_{Li} & \chi_{Li}r \\ \chi_{Ci} & \chi_{Ci}r \end{bmatrix} = \begin{bmatrix} -c & c & a & b \\ -c & c & a & b \end{bmatrix} = \begin{bmatrix} -\frac{c}{2} & \frac{c}{2} & a & b \\ -\frac{c}{2} & \frac{c}{2} & a & b \end{bmatrix},
$$

where $(a, b)$ is the CoG position, $c$ is the track-width, $\delta_i$ is the steering angle of each wheel, and $W(\delta)$ denotes a change of coordinates between $xy$ and $LC$ frames, i.e.,

$$
W(\delta) = \begin{bmatrix} \cos(\delta) & -\sin(\delta) \\ \sin(\delta) & \cos(\delta) \end{bmatrix}.
$$

To express the load transfer between front-rear axle, and left-right wheels, a quasi-static mapping is used

$$
F_z = F_{z}^0 + A_x a_x + A_y a_y
$$

where $F_{z}^0$ is the static force distribution, $h$ the height of the CoG, $g$ the gravitational acceleration and $a_x$, $a_y$ the longitudinal and lateral acceleration. The generalized forces/moments $(\mathcal{F})$ applied to the CoG is the aggregated result of the individual friction forces generated in the tire-road interface, namely

$$
\mathcal{F} = BF^{xy}
$$

Given that the friction forces are usually represented in the $LC$ local coordinate frame [7], it is also helpful to decompose $F^{xy}$ in the $L$ and $C$ components

$$
F^{xy} = \mathcal{W}(\delta)F^{LC}
$$

$$
\mathcal{W}(\delta) = \begin{bmatrix} W(\delta_{1l}) & 0 & 0 & 0 \\ 0 & W(\delta_{1r}) & 0 & 0 \\ 0 & 0 & W(\delta_{2l}) & 0 \\ 0 & 0 & 0 & W(\delta_{2r}) \end{bmatrix},
$$

with $F^{LC} = [F_{L1l} F_{C1l} F_{L1r} F_{C1r} F_{L2l} F_{C2l} F_{L2r} F_{C2r}]^T$, and $\delta = [\delta_{1l} \delta_{2l} \delta_{1r} \delta_{2r}]^T$ represent the steering angles. To model the friction forces $F^{LC}$, a simplified version of the well known magic tire formula will be used, [7]. The main inputs for this model are: $i$) the longitudinal tire slip $\kappa_i$, a normalized difference between the wheel rotation speed ($\omega_i$) and the wheel linear speed ($v_{Li}$); and $ii$) the tire side-slip angle $\alpha_i$. Formally, these are defined as

$$
\kappa_i = \frac{\omega_i r_i - v_{Li}}{v_{Li}}, \quad \tan \alpha_i = -\frac{v_{Ci}}{v_{Li}},
$$

where $r_i$ its effective radius, $i \in \{1l, 1r, 2l, 2r\}$ and $v_{Li}, v_{Ci}$ the longitudinal and cornering speed of the tire. To incorporate the combined slip conditions, the so-called “theoretical slips” [7] are considered, i.e.,

$$
\sigma_{Li} = \frac{\kappa_i}{1 + \kappa_i}, \quad \sigma_{Ci} = \frac{\tan \alpha_i}{1 + \sigma_{Li}}, \sigma_i = \sqrt{\sigma_{Li}^2 + \sigma_{Ci}^2},
$$

which represents the main factors in generating the longitudinal ($F_{Li}$) and cornering forces ($F_{Ci}$)

$$
F_{Li} = \frac{\sigma_{Li}}{\sigma_i} F_i(\sigma_i, F_{zi}) \quad F_{Ci} = \frac{\sigma_{Ci}}{\sigma_i} F_i(\sigma_i, F_{zi})
$$

$$
F_i(\sigma_i, F_{zi}) = F_{zi} D \sin(C \tan(B \sigma_i)),
$$

Fig. 1. Representation of the vehicle model. Notation: $XY$ axis is fixed with the earth, $xy$ with the vehicle’s CoG, $LC$ with the wheels. The subscript $1j, j \in \{l, r\}$ refers to the vehicle’s front wheels, while $2j$ to the rears.
where $D, C, B$ are parameters of the model. Notice that, for simplicity, the representation of the tire-road friction forces assumes an isotropic model. Furthermore, from (10) and (11), it can be readily verified that the longitudinal and lateral forces must satisfy
\[ F_{Li} + F_{C_i}^2 \leq F_i^2 \leq D^2 F_{zi}^2, \tag{12} \]
which is known as the friction circle constraint. Finally, the wheel rotational dynamics is given by
\[ J\dot{\omega}_i = T_i - r_i F_{Li}, \tag{13} \]
where $J$ is the wheel inertia, and $T_i$ the wheel torque. Since the wheel rotational dynamics are, in general, much faster than the vehicle dynamics, the previous relations will be approximated with their steady-state value, i.e.,
\[ F_{Li} \approx rT_i. \tag{13} \]
To model the drivers' inputs, it is assumed that the vehicle has front steer only, controlled by the driver, and the front steer angles are approximately equal, that is
\[ \delta_{1r} \approx \delta_1, \delta_{2l} = \delta_{2r} = 0. \tag{14} \]
Finally, it is assumed that the torque applied to each wheel of the EV can be manipulated independently, and the summation of the wheels torques ($u_T$)
\[ T_{1l} + T_{1r} + T_{2l} + T_{2r} = u_T \tag{15} \]
is imposed by the driver.

### III. Baseline Optimal Solution

The definition of the benchmark optimal problem for the torque distribution strategy is now studied, the solution of which will enable us to extract the maximum performance from the IWMs. To this aim, an optimal control problem will be formulated, intended to move, in minimum time, the EV from an initial point $p_0 = (X_0, Y_0, \psi_0)$ to a final point $p_f = (X_f, Y_f, \psi_f)$, while fulfilling the physical constraints given by the road grip capabilities.

#### A. Time-Distance Transformation

Consider the equation of motion re-written in the state-space format
\[
\begin{align*}
\frac{dz_1}{dt} &= \dot{z}_2 \\
M \frac{dz_2}{dt} &= T(z_1)(F - \Delta)
\end{align*}
\tag{16a}
\tag{16b}
\]
where $z_1 = p$ is the position, $z_2 = dp/dt$ the velocity, and $t$ the independent variable. A common technique when dealing with minimum-time problems is to introduce the (normalized) spatial coordinate $s$ as independent variable. There are several advantages in adopting this change of variable. First, unlike the time, the use of the distance $s$ as independent variable allows us to fix its final value, which is particularly useful when discretizing the optimization problem [8]. Secondly, the distance is also a natural variable to parameterize the road boundaries (as will be shown in the following), and the incorporation of the road constraints in the problem is greatly simplified using $s$ as independent variable. Finally, if long routes need to be considered, the problem can be decomposed in shorter sectors, and this, albeit yielding sub-optimal results, is normally much easier to solve than the original global problem.

In the sequel, $s$ will represent the normalized distance between the vehicle position and the beginning of the road. The time and the (normalized) space increments are related as
\[ \dot{s} \, dt = ds, \tag{17} \]
where $\dot{s} = \dot{f}_s(z_1, z_2, s)$ is a normalized speed, the definition of which is postponed to Section III-C. Inserting the previous relation into (16), one has
\[
\begin{align*}
\frac{dz_1}{ds} &= \frac{1}{s} z_2 \\
M \frac{dz_2}{ds} &= \frac{1}{s} T(z_1)(F - \Delta),
\end{align*}
\tag{18a}
\tag{18b}
\]
which is essentially a scaled version of the original dynamics. Similarly, the total time of the manoeuvre ($T$) can be obtained as
\[ T = \int_0^T 1dt = \int_0^\pi \frac{1}{s} ds, \tag{19} \]
where $\pi$ is the (normalized) total length of the track.

#### B. Road Definition and Boundaries

One of the requirements that must be incorporated in the problem formulation is the track constraint, i.e., the vehicle must be, at all times, within the road boundaries. To simplify the analysis, it will be assumed that it is enough to keep only the vehicle CoG within the road boundaries. To formally characterize the road, consider that the coordinates of the road center line are known and defined through the vector $c(s) \in \mathbb{R}^2$, parameterized as a function of $s$. From this, the vectors tangent ($c'(s)$) and parallel ($c_\perp(s)$) to the road center can be readily computed. Next, assuming that the distance between the track center and the road boundaries is $\pm w(s)/2$, the lower and upper bounds of the road can be expressed as
\[
\begin{align*}
c(s) &= c(s) + \frac{w(s)}{2} \frac{c_\perp(s)}{\|c_\perp(s)\|} \tag{20} \\
c(s) &= c(s) - \frac{w(s)}{2} \frac{c_\perp(s)}{\|c_\perp(s)\|}. \tag{21}
\end{align*}
\]
where $\| \cdot \|$ is the Euclidean norm. Following a similar methodology to the one described in [9], the track constraints can now be formulated as a set of linear equalities and inequalities of the form
\[
\begin{align*}
c_\perp(s)^T (P z_1(s) - c(s)) &\leq 0 \tag{22a} \\
-c_\perp(s)^T (P z_1(s) - c(s)) &\leq 0 \tag{22b} \\
c'(s)^T (P z_1(s) - c(s)) &= 0 \tag{22c} \\
P &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{22d}
\end{align*}
\]
The first two inequalities state that, for a given $s$, the position of the vehicle CoG should not exceed the lower and upper bounds of the road, while the last constraint imposes that $z_1$ should lie within the road boundaries. For a fixed $s = s_i$, the intersection of these three constraints produces a straight line, which can be regarded as a "wayline", which the vehicle should go along.

C. Problem Formulation and Solution

It is now possible to formulate the benchmark optimization problem:

$$\min_{\dot{s}, z_1, z_2, s, \delta_1} \int_0^s \frac{1}{s} ds$$

s.t. \hspace{1em} (18), \hspace{1em} $\dot{s} = f_s(z_1, z_2, s), \hspace{1em} \dot{s} > 0, \hspace{1em} s \in [0, \bar{s}]$ \hspace{1em} (22), \hspace{1em} $F = f_F(z_1, z_2, T, T_1, T_2, T_1, T_2, \delta_1)$

$$z_1(0) = p_0, \hspace{1em} z_1(\bar{s}) = p_f, \hspace{1em} z_2(0) = [\dot{X}_0 \hspace{1em} Y_0 \hspace{1em} \psi_0]^{T}$$ \hspace{1em} (23)

where: i) the first set of constraints is related with the vehicle equations of motion: given that we are interested in minimum-time results, it was assumed that the vehicle (normalized) speed should be always positive, which also avoids a division by zero in (18); ii) the second set of constraints is concerned with the road boundaries and the friction force model; the function $f_F$ is a nonlinear algebraic map, that results from aggregating (2)-14; iii) the third set of constraints imposes the initial and final position of the vehicle, as well as its initial speed, while the last inequalities are due to the actuators saturation limits.

Due to the nonlinearities present in (23), mainly due to the friction force model, it is difficult to analytically establish the optimality conditions for the problem. Consequently, to ease the solution of this problem, a direct optimization approach was adopted, also known as the transcription formulation [10]. This means that the dynamic equations in (23) were discretized (in this work through the Euler forward method), yielding a large nonlinear optimization problem that was solved with the help of the ipopt solver [11].

The discretization of the optimization problem also brings in an interesting simplification in the computation of $\dot{s}$. In fact, although one may determine $f_s(.)$ in continuous time, the discrete version is not only simpler, but also more intuitive. To better see this, let us consider that the vehicle CoG is at the position $z_1(s_i)$, with velocity $z_2(s_i)$, and that it reaches, after one integration step, the position $z_1(s_{i+1})$. Assuming forward Euler discretization, one can approximate $\dot{s}[s_{i+1}]$ as

$$\dot{s}[s_{i+1}] \approx \frac{s_{i+1} - s_i}{\Delta t} = \frac{\|Pz_2(s_i)\|}{\|P(z_1(s_{i+1}) - z_1(s_i))\|}$$

Replacing this approximation in the total manoeuvre time one has

$$T = \int_0^\bar{s} \frac{1}{s} ds \approx \sum_{i=0}^{N-1} \frac{\|P(z_1(s_{i+1}) - z_1(s_i))\|}{\|Pz_2(s_i)\|} (s_{i+1} - s_i),$$

which reveals that, to minimize the time $T$, the vehicle speed ($\|Pz_2\| = \sqrt{X^2 + Y^2}$) should be maximized and the travelled distance between successive "waylines" ($\|P(z_1(s_{i+1}) - z_1(s_i))\|$) should be as low as possible. This analysis is in accordance with the traditional approach of racing drivers, which normally try to "cut the corners" (thus minimizing the travelled distance) with the maximum possible speed.

IV. CAUSAL TORQUE ALLOCATION STRATEGY

The methodology described in the previous section allows gaining insight on the optimal torque allocation for the IWMs. However, in practice, this approach is not appropriate for real-time implementation because, in addition to the high computational burden, the vehicle trajectory cannot in general be assumed to be known in advance. This section will thus introduce a causal allocation strategy that aims at emulating the previous baseline optimal solution in a real-time environment with causality constraints. To facilitate the design of the allocator, three normalization factors are introduced ($\gamma_0, \gamma_1, \gamma_2$) which, together with $u_T$, parameterize the torque distribution

$$T_{1l} = u_T \gamma_0 (1 - \gamma_1) \hspace{1em} T_{1r} = u_T \gamma_0 \gamma_1$$ \hspace{1em} (25)

$$T_{2l} = u_T (1 - \gamma_0) (1 - \gamma_2) \hspace{1em} T_{2r} = u_T (1 - \gamma_0) \gamma_2,$$

where: $\gamma_0$ is the front/rear allocation ratio ($\gamma_0 = 1$ means all the torque is applied to the front axle) $\gamma_1$ is the left/right allocation ratio of the front axle ($\gamma_1 = 1$ means all the front axle torque is on the right wheel) $\gamma_2$ is the left/right allocation ratio of the rear axle. Notice that, in conventional vehicles, the ratio $\gamma_0$ is a well known quantity for distributing the braking force among the axles; our idea here is to generalize this concept for the left/right torque split in the front and rear axles.

The torque allocation problem, as stated above, has been previously addressed in the literature, see e.g., [12], [13]. These studies assume that the vehicle is operating close to the quasi-steady-state (QSS) conditions, i.e., constant accelerations and constant yaw-rate, which simplifies the analysis of the nonlinear vehicle model.

To devise a solution that can be applicable in real time, it is worth noting that, while the baseline solution presented in the previous section seeks the minimization of the manoeuvre time, this cost function is not appropriate for a causal allocation strategy, as it asks for a full manoeuvre preview. To overcome this limitation, the proposed suboptimal formulation is inspired to the gg-diagram concept, a tool that generalizes the tire friction circle concept to the vehicle CoG forces/accelerations. It is well known in the racing community that, in order to minimize the lap-time, the vehicle should operate close to the boundaries of the gg-diagram.

To design the torque allocator, it is convenient to consider the CoG forces represented in polar coordinates

$$F_x + jF_y = \rho e^{j\phi},$$

(26)
where $\rho$ is the force magnitude and $\phi$ the force angle in relation to the $x$ axis. It will be assumed that the vehicle is operating under QSS conditions and that the desired force angle $\phi$ is fixed and known (note that the force direction can be straightforwardly inferred from the current value of $a_x$ and $a_y$). The goal is to find the tire forces $F^x$ that maximize the force magnitude $\rho$ in the direction $\phi$. Formally, this problem can be formulated as

$$\min_{\rho, F^x} \rho$$

s.t. $\begin{bmatrix} B_x & B_y & B_{\psi} \end{bmatrix}^T F^x y - \rho \cos(\phi) = 0$

$$B_y^T F^x y - \rho \sin(\phi) = 0$$

$$B_{\psi}^T F^x y = 0$$

$$(C_i^T F^x y)^2 + (D_i^T F^x y)^2 \leq \left( \pi E_i^T \left( F^0_0 + A_x \frac{\rho}{m} \cos(\phi) + A_y \frac{\rho}{m} \sin(\phi) \right) \right)^2$$

$$i = \{1l, 1r, 2l, 2r\},$$

where $B_x, B_y, B_{\psi}$ are the matrices that compose $B$ in (6).

In the above optimization problem, the first two constraints ensure that the force applied to CoG is applied in the desired direction, while the third constraint is related with the constant yaw-rate assumption (resulting from the QSS conditions); the fourth constraint takes into account the restrictions introduced by the friction circle, defined in (12), where $C_i, D_i, E_i$ are the matrices that extract the $x, y$ and $z$ force component for each tire.

Since our ultimate goal is to determine the IWM torques, and in view of the linear relation between torque and longitudinal force expressed by (13), it is desirable to express the forces $F^x y$ in the $LC$ coordinate system, fixed to the wheel. To do this, one can exploit the analytical results given in [12], obtaining

$$F_{Li} = \pi F_{zLi} \cos(\delta_l) \cos(\phi) + \sin(\delta_l) \sin(\phi)$$

$$F_{Ci} = \pi F_{zLi} \sin(\delta_l) \cos(\phi) + \cos(\delta_l) \sin(\phi)$$

Combining the previous equations with (13) and (25), the allocation ratios can be determined as

$$\gamma_0 = \frac{F^r_{Li} + r F^r_{Li} + r F^r_{Li r} + F^r_{Li 2f} + r F^r_{Li 2r}}{r F^r_{Li 1f} + r F^r_{Li 1r} + F^r_{Li 2f} + r F^r_{Li 2r}} \left( \frac{\cos(\phi)}{\cos(\phi) \cos(\delta_l) + \sin(\phi) \sin(\delta_l)} \right) \left( \frac{F_{zLi} + F_{zLi r}}{F_{zLi} + F_{zLi r}} \right)^{-1}$$

$$\approx \left( \frac{a_x}{F_{zLi} + F_{zLi r}} \right)$$

$$\gamma_1 = \frac{F^r_{Li r} + F^r_{Li r}}{r F^r_{Li 1f} + r F^r_{Li r} + F^r_{Li 2f} + r F^r_{Li 2r}}$$

$$\gamma_2 = \frac{F^r_{Li 2f} + r F^r_{Li 2r}}{r F^r_{Li 1f} + r F^r_{Li r} + F^r_{Li 2f} + r F^r_{Li 2r}}$$

Notice that, during the derivation of (29a), the force direction $\phi$ was inferred from the vehicle acceleration: $ma_x \approx \rho \cos(\phi), ma_y \approx \rho \sin(\phi)$. Inspecting the above formulas, it is worth pointing out that $\gamma_1$ and $\gamma_2$ (left/right allocation ratios) are not directly affected by the force direction $\phi$ or by the steering angle. Actually, they only depend on the ratio of the vertical forces, which generates fairly intuitive allocation results: i) during straight line manoeuvres, equal torques are applied to the wheels of the same axle$^1$; ii) during cornering, the outside wheel receives more torque than the inside one (lighter wheel), in accordance with the left-right and front-rear weight shift that affects the vehicle. It is also interesting to note that all ratios are independent of the tire-road friction peak $\mu$.

Since the tires vertical forces are intrinsically related with the longitudinal and lateral accelerations, c.f. (5), the torque ratios can also be expressed in the following equivalent form

$$\gamma_0 = \left( \frac{1}{a_y} + \frac{a_x}{a_y} \cos(\delta_l) \sin(\delta_l) \right) \left( \frac{ag + ha_x}{gb - ha_x} \right)^{-1}$$

$$\gamma_1 = \frac{gb - ha_x}{gb + ha_x}$$

$$\gamma_2 = \frac{gb - ha_x}{gb + ha_x}$$

To investigate the evolution of the front/rear ratio $\gamma_0$ with combined acceleration/cornering, consider the allocation results illustrated in Fig. 2. From these results, one can see that, as $|a_x|$ increases, $\gamma_0$ converges to the situation with $\delta_l = 0$ (straight line allocation). On the other hand, for small longitudinal accelerations (almost pure cornering), there is a discontinuity in $\gamma_0$, being more pronounced at low speeds; since this discontinuity happens mainly when $a_x$ is close to zero (thus the torque applied to the wheels is small), this is not a concern. As for the evolution of $\gamma_1, \gamma_2$, one can observe that these variables always take values larger than 0.5, which implies that the allocator will generate an

$^1$assuming 50/50 weigh distribution in the left/right wheels
“understeer” yaw-moment during braking (and an “oversteer” during acceleration).

V. SIMULATION RESULTS

In this section, a comparative study between the two allocations strategies discussed in the previous two sections will be carried out. In the causal allocation problem, it is important to stress that the optimization framework is only used to emulate the “ideal” driver, who will minimize the manoeuvre time of the EV configured with a fixed torque split strategy. To test the performance of the proposed approach, different cornering maneuvers were considered (180, 130 and 90 deg), designed with different radii. Analyzing the results obtained for the so-called Hairpin corner (180 deg), shown in Fig. 3(b), it can be verified that, as the corner radius increases, the time difference between the causal and baseline allocation policies is reduced from 1% to the 0.40 – 0.55%. This evolution can be explained by the yaw-rate response, depicted in Fig. 3(c): as the radius increases, the yaw-rate gets smoother, making the manoeuvre closer to the QSS conditions considered in the design of the causal allocator. Looking at the remaining cases, it can be noticed that: i) the 130 deg corner yields a similar trend as the hairpin corner and ii) the 90 deg corner produces lower errors when the radius is small (which, again, can be explained by the smoother transient in the yaw-rate).

To compare the allocation strategies in a more realistic setting, the last 900m of the Monaco track were considered, as shown in Figure 4. Similar to the previous tests, one can verify that the causal allocation yields an evolution (driver inputs, vehicle states and allocation ratios) very close the baseline solution. This proximity between solutions is also reflected in the total manoeuvre time: 30.43s for the baseline

and 30.64s (+ 0.70%) for the causal allocation.

VI. CONCLUDING REMARKS

In this work, torque allocation strategies for EVs endowed with 4IWM have been studied, targeting the full exploitation of the available performance. A minimum-time optimal problem was formulated, the solution of which provided a benchmark against which the designed causal strategy was objectively compared, showing a performance loss of less than 1% with respect to the optimal one.

REFERENCES