A Novel Mid-Ranging Approach for Idle Speed Control of a Hybrid Electric Powertrain

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Abstract—In this paper the problem of idle speed control for the powertrain of a hybrid electric vehicle is investigated. On the one hand, the proposed control structure ensures control of the powertrain’s idle speed, by using the electric motor as a secondary actuator besides the combustion engine. On the other hand, it enables the use of the electric motor as a generator by appropriate load level shifting. The coordination of the two actuators is provided by a novel control structure, which is based on the idea of mid-ranging control. Based on a nonlinear system description, a suitable design model is derived, capturing the fundamental properties of the powertrain. The nonlinear dynamic behavior of the intake manifold pressure is handled via an inversion-based linearization. The controller design itself is based upon a frequency shaped LQG design, which allows for explicit consideration of the powertrain’s oscillatory behavior, which is due to a dual mass flywheel. Time delays, which occur in both the plant input and output channels, are encountered by extending the LQG Kalman filter with a predictor. Finally, measurements from a passenger car are presented to validate the performance of the new controller structure.

I. MOTIVATION AND INTRODUCTION

Rigorous emission regulations result in a continuous increase of requirements regarding engine control performance. To meet these regulations, the combination of an internal combustion engine (ICE) with an electric motor (EM) to a hybrid electric vehicle (HEV) is a recent trend in powertrain development. During the powertrain design, a smaller ICE with fewer cylinders and/or smaller displacement volume can be considered to reduce pollution emission. To meet requirements regarding the vehicle’s longitudinal dynamics, an EM supports the ICE during acceleration, while it enables recuperation of brake energy during deceleration.

The introduction of hybrid powertrains offers a wide range of opportunities to increase fuel economy and improve emission behavior. On the other hand, these additional degrees of freedom result in increased system complexity and lead to new challenges regarding the design of control loops within the HEV powertrain.

The HEV which is investigated in this paper is a parallel type (PHEV), consisting of a spark ignited engine and a permanently excited synchronous machine. The actuators are simultaneously able to transmit torque to the driven axle, cf. Fig. 1. Instead of directly coupling the two actuators, they are typically connected via a dual mass flywheel (DMF), in order to reduce the drivers perception of noise, vibration and harshness. This powertrain setup enables a new control strategy during engine idle operation which has, to the authors’ best knowledge, not been investigated heretofore.

The task of controlling the idle speed of a spark ignited engine, which has been subject to research for more than three decades [1], [2], is consisting of a variety of features. On the one hand, the reference idle speed should be selected as low as possible, improving fuel consumption and thus emission behavior. On the other hand, constraints have to be taken into account. These are imposed by comfort issues as robustness against load torque disturbances and actuator limits. Therefore, the choice of the reference speed can be seen as a trade-off between these requirements. To enable a low idle speed in view of the above constraints, a so-called torque reserve is typically introduced [3]. This actuator reserve is achieved by adjusting the ignition angle to late values, which affects the combustion’s efficiency regarding torque production. This reserve can be demanded almost instantaneously, within one combustion cycle. Unfortunately, the torque reserve in turn leads to an increase in fuel consumption. Further difficulties in the idle speed control (ISC) arise from the so-called induction to power stroke delay (IPS), which is immanent to all ICES and is due to the discrete torque production behavior. Lowering the number of engine cylinders naturally increases this delay, as does a reduction of the engine idle speed.

Regarding the ISC problem, the PHEV offers the possibility of employing the EM as a high-bandwidth actuator.
besides the ICE. Fluctuation in the engine speed can be counteracted quickly through appropriate control action via the torque of the EM. Consequently, the introduction of a torque reserve is not required anymore and the associated additional fuel consumption can be avoided. The ICE is thus operated at the optimal ignition angle, which enables lowest possible emissions with respect to its operating point.

Besides the challenges in traditional ISC, such as the nonlinear gas dynamic in the ICE’s inlet manifold, the PHEV powertrain introduces additional issues. Since the EM also serves as a generator to charge the hybrid battery, a suitable control strategy must be able to exploit the EM’s high-bandwidth capabilities during transients, while the EM torque must be returned to a desired generator torque during steady state. The lightly damped DMF introduces a backlash behavior between the crankshaft and the armature of the EM. Consequently, the ISC must avoid strong excitation of corresponding modes.

The EM itself constitutes a redundant actuator to the ICE. To deal with redundant actuators, the topic of control allocation (CA) has emerged mainly from aerospace and marine engineering. Worth mentioning are: 1) daisy chaining, where redundant actuators are used in a serial manner; if one actuator saturates, the remaining control signal is diverted to another actuator. [4]. 2) optimization-based approaches considering inequality constraints. [5]. For a survey see [6] and the references therein. Most of the existing approaches consider strong input redundancy, as defined in [7], where arbitrarily many input vectors can result in the same dynamic system behavior. The control problem on hand features only weak input redundancy [7], where loosely speaking different input vectors can only result in equal steady state outputs, but not in the same dynamic behavior. Furthermore, it is the author’s intention to present a control strategy which can be implemented on state of the art electronic control units (ECU), featuring fixed-point arithmetic microcontrollers. Therefore, the solution of an optimization problem using implicit methods, within sampling times common for an ISC, is not possible.

Recently, [8] investigated the dynamic torque allocation for a PHEV in the context of drivability, using the results from [7]. However, due to a different powertrain layout, with the EM mounted downstream to the gearbox, the authors did not consider the ISC problem. In contrast to this paper, the authors of [8] used a simplified mechanical model, neglecting gearbox lash and twist of the drive train, which leads to a setup featuring strong input redundancy. Finally, the method in [7] is not able to return one of the inputs strictly to a desired steady state value, without influencing the allocation dynamic itself and it is more suitable to problems exhibiting strong input redundancy.

To cope with the extra actuator input given by the EM, an alternative technique to well known CA methods is presented, which is based on the idea of mid-ranging (MR) control [9]. This approach immediately solves the problem of returning the torque of the EM to a desired value in steady state operation. To encounter the oscillatory behavior between the crankshaft and the EM and to consider the different capabilities of the actuators, the use of linear quadratic Gaussian control with frequency shaped weightings (FLQG) [10], [11] is proposed. Therefore, the nonlinear dynamic of the inlet manifold is linearized by an approach recently presented in [12], resulting in a linear design model regarding the ICE.

We note that an \( H_\infty \) loop shaping controller [13] could also solve the ISC problem. Unfortunately, the resulting controller would be of high order. Since today’s engine ECUs are based on fixed-point arithmetic microcontrollers, solid bounds and appropriate fixed-point resolutions have to be determined for the \( H_\infty \) controller states, which are difficult, if not impossible, to find.

The paper is organized as follows. In Section II the powertrain model is presented. In Section III a new control strategy solving the ISC problem is proposed, followed by experimental results in Section IV. Finally, concluding remarks are given in Section V.

II. MODEL OF A PHEV POWERTRAIN

After a presentation of the fundamental assumptions on the plant, this section introduces a nonlinear torque model of the ICE, which is reasonably simplified and linearized. Afterwards, modelling of the EM and the DMF as the coupling element is addressed.

Most of the following assumptions are specific to an ISC oriented model and are reasonable since engine speed is low and assumed to vary only in a small interval:

- a mean value description is used for the ICE, neglecting effects such as the segmentation of input variables, which arise from its event-triggered character [14]
- the IPS delay is constant
- the dynamic of the throttle valve is neglected due to its high bandwidth, compared to the gas dynamic in the inlet manifold
- hysteresis effects of the DMF are neglected, since they especially appear for engine speeds above idling [15].

A. Torque Model of the ICE

The ICE model is based on a nonlinear mean value description which can be found for example in [14]. It holds for spark ignited, 4-stroke direct injection engines. Based on such a description, a torque-based system structure is embedded in modern engine ECU software [16]. It includes an interface that realizes certain propulsive torque which can be demanded by software functions such as the ISC. The torque demand is realized via feed-forward of the throttle plate actuator, the injection system and the ignition timing, based on a static nonlinear expression for the indicated torque \( T_{\text{ind}} \):

\[
T_{\text{ind}} = f(m_{\text{air,cyl}}, \omega_{cs}, \ldots).
\]  

The ICE on hand is assumed to be operated with homogeneous air to fuel ratio. Therefore, \( T_{\text{ind}} \) mainly depends on the air charge which is available in the combustion chamber, since the injected fuel quantity is fixed by the
stoichiometric ratio. To achieve optimal emission behavior as well as fuel consumption, manipulation of the ignition angle is not considered, as discussed above. It is assumed, that for each 4-stroke cycle, an optimal triple of ignition angle, injection timing and fuel quantity is selected, according to the operating point. The above discussion results in (1), depending mainly on the cylinder air charge \( m_{\text{air,cyl}} \) and the crankshaft angular velocity \( \omega_{\text{cs}} \), since the relevant remaining arguments are determined by the torque structure.

In the ISC operating range, it can be observed that (1) shows nearly linear behavior with respect to \( m_{\text{air,cyl}} \) and no dependency on \( \omega_{\text{cs}} \), resulting in

\[
T_{\text{ind}}(t) = K_T m_{\text{air,cyl}}(t - \delta_{\text{IPS}})
\]

where \( K_T \) relates torque with air charge and \( \delta_{\text{IPS}} = 4\pi/N_{\text{cyl}}\omega_{\text{cs}} \) denotes the IPS delay time, with \( N_{\text{cyl}} \) the number of cylinders. The input delay in (2) is due to intake and compression phases which are prior to the combustion stroke.

The total air mass within the combustion chamber during intake consists of a fresh gas and a residual gas part: \( m_{\text{cyl}} = m_{\text{air,cyl}} + m_{\text{res,cyl}} \), where \( m_{\text{res,cyl}} \) is assumed to be constant, since valve overlap is not changed during idle operation. The ideal gas law for the combustion chamber during intake valve closure gives

\[
m_{\text{air,cyl}} + m_{\text{res,cyl}} = \frac{V_{\text{cyl}}}{R_L \vartheta_{\text{cyl}}} (p_{\text{air,cyl}} + p_{\text{res,cyl}})
\]

assuming that the gas constant and temperature of the residual gas equal the gas constant \( R_L \) and temperature \( \vartheta_{\text{cyl}} \) of the fresh air. Furthermore, \( V_{\text{cyl}}, p_{\text{air,cyl}} \) and \( p_{\text{res,cyl}} \) are the cylinder volume and partial pressures of fresh air and residual gas, respectively. Using (3) and assuming a complete pressure balance between intake manifold and combustion chamber, \( p_{\text{air,cyl}} + p_{\text{res,cyl}} = p_{\text{im}} \), eq. (2) can be reformulated to

\[
T_{\text{ind}}(t) = K_T \frac{V_{\text{cyl}}}{R_L \vartheta_{\text{cyl}}} (p_{\text{im}}(t - \delta_{\text{IPS}}) - p_{\text{res,cyl}})
\]

delivering an expression for the indicated torque in dependence of the manifold pressure \( p_{\text{im}} \). The manifold pressure is described by again applying the ideal gas law: \( p_{\text{im}} V_{\text{in}} = m_{\text{im}} R_L \vartheta_{\text{im}} \). Using the continuity equation \( \dot{m}_{\text{im}} = \dot{m}_{\text{th}} - \dot{m}_{\text{air,cyl}} \), with \( \dot{m}_{\text{th}} \) the mass flow through the intake throttle valve, the differential equation describing the inlet manifold pressure is obtained:

\[
\dot{p}_{\text{im}} = \frac{R_L \vartheta_{\text{im}}}{V_{\text{in}}} (\dot{m}_{\text{th}} - \dot{m}_{\text{air,cyl}})
\]

where \( \dot{\vartheta}_{\text{im}} \) has been neglected due to its small rate of change compared to the other variables involved. Using (5) and averaging the air charge over one time segment, \( \dot{m}_{\text{air,cyl}} = m_{\text{air,cyl}}/\delta_{\text{IPS}} \), the time derivative of (4) gives

\[
\dot{T}_{\text{ind}}(t) = k_1 \left( K_T \dot{m}_{\text{th}}(t - \delta_{\text{IPS}}) - \frac{1}{\delta_{\text{IPS}}} T_{\text{ind}}(t) \right)
\]

with \( k_1 = V_{\text{cyl}} \dot{\vartheta}_{\text{im}}/\vartheta_{\text{cyl}} V_{\text{in}} \). In (6), \( \dot{m}_{\text{th}} \) represents the manipulated variable, since it can be influenced via the throttle plate actuator. Given a desired \( \dot{m}_{\text{th}} \), the required throttle angle is determined via feed-forward by the torque structure. Choosing \( \dot{m}_{\text{th}} = T_{\text{ind,ss}}/K_T \delta_{\text{IPS}} \) determines the desired throttle plate mass flow and introduces an new input variable \( T_{\text{ind,ss}} \), representing the desired steady state torque, which can be accessed by the ISC.

To enable a linear controller design, the following feed-forward law is introduced to linearize eq. (6):

\[
T_{\text{ind,ss}} = \frac{1}{k_2 \omega_{\text{cs}}} \left( T_{\text{ind,lin}} + k_3 \omega_{\text{cs}} T_{\text{ind,lin}} \right)
\]

where the desired trajectory \( T_{\text{ind,lin}} \) and its first derivative \( T_{\text{ind,lin}} \) are determined by a linear first order state variable filter, as it has been proposed by [12]:

\[
t_{\text{ICE}} \dot{T}_{\text{ind,lin}} = -T_{\text{ind,lin}} + T_{\text{ind,des}}.
\]

Here \( t_{\text{ICE}} \) is the desired time constant of the linearized dynamic and \( T_{\text{ind,des}} \) is a new virtual input. The parameters in (7) are given by \( k_2 = (k_1 N_{\text{cyl}})/(4\pi) \) and \( k_3 = (R_L \vartheta_{\text{im}} N_{\text{cyl}})/(V_{\text{in}} K_T 4\pi) \). The resulting error differential equation caused by (7) is stable if \( k_2 > 0 \) and \( k_3 > 0 \), which is true for the reasonable operation regime of the ICE. Under the assumption that the error \( e_{\text{IPS}}(t) = T_{\text{ind}}(t) - T_{\text{ind,des}}(t - \delta_{\text{IPS}}) \) converges, the indicated torque is finally given by

\[
t_{\text{ICE}} \dot{T}_{\text{ind}}(t) = -T_{\text{ind}}(t) + T_{\text{ind,des}}(t - \delta_{\text{IPS}}).
\]

To demonstrate the usefulness of the presented ICE model, Fig. 2 shows an excerpt of a comparison between the measured and modeled manifold pressure. It originated from a test sequence, manipulating the desired indicated torque, which is realized by the torque structure. Fig. 2 shows the response of the manifold pressure to step-wise changes of the desired indicated torque. The modelled pressure \( p_{\text{im,mod}} \) is calculated from (6), with \( T_{\text{ind}} \) replaced by \( p_{\text{im}} \) using (4). It can be observed that the dynamic as well as the stationary behavior are captured satisfactorily by the presented model. The parameter \( K_T \) is obtained from test bench experiments.

Fig. 2. Comparison of measured and modelled manifold pressure.

It should be noted, that the feed-forward (7) offers the possibility of making adjustments to varying operation conditions regarding \( \vartheta_{\text{im}} \) and \( \vartheta_{\text{cyl}} \). Since the rate of change of the two temperatures is by orders of magnitude smaller compared to the rate of change of the remaining variables, this can be done without possible destabilization of the ISC loop [17].
B. Electric Motor

The EM of the considered powertrain is supported by a separate control unit, which is connected via the vehicle’s communication bus with the ECU of the ICE. It delivers a torque interface which can be accessed by the ISC. The dynamic behavior of propulsive torque generated by the permanently excited synchronous machine $T_{EM}$ is realized by a field oriented subordinate control and can be described by a first order lag:

$$\tau_{EM} \dot{T}_{EM}(t) = -T_{EM}(t) + T_{EM,des}(t - \delta_{EM})$$ (10)

where $T_{EM,des}$ represents the interface variable and $\tau_{EM} \approx 30ms$ is experimentally determined. The effective input delay is due to the bus communication of the ECUs and approximately $\delta_{EM} \approx 30ms$.

C. Dual Mass Flywheel and Coupling of the Actuators

The DMF is originally purposed to dampen the event discrete torque production behavior of the ICE. Modern passenger cars feature a DMF, located between ICE and gearbox, to increase powertrain comfort. In view of the ISC problem, the DMF serves as a coupling element between the ICE’s crankshaft and the armature of the EM, see Fig. 3 for the mechanical model. It therefore consists of two inertias, the primary flywheel $I_{DMF1}$ and secondary flywheel $I_{DMF2}$. These are connected via toroidally mounted springs. Conclusively, a displacement angle $\varphi_{DMF}$ between the two inertias results in a torque transmission from one inertia to the other: $T_{DMF} = -c_{DMF} \varphi_{DMF,eff}$, with $c_{DMF}$ the combined stiffness of the toroidal springs and $\varphi_{DMF,eff}$ the effective displacement angle. The latter originates from mechanical lash, resulting in zero torque transmission for small displacement angles. Similar to [18] the effective displacement angle is therefore described by:

$$\varphi_{DMF,eff} = \varphi_{DMF} - sat_{\varphi_0}(\varphi_{DMF})$$ (11)

with $sat_{\varphi_0}(\cdot)$ the saturation function that saturates at $\varphi_0$, the displacement angle where the lash ends. Thereby, a nonlinear behavior is introduced in the DMF model. Additionally, weak damping which is proportional to $\dot{\varphi}_{DMF}$ by the factor $d_{DMF}$ is introduced due to mechanical friction. Together with the rotational dynamics of crankshaft and armature, the mechanical model is stated by:

$$\begin{align*}
I_1 \dot{\omega}_{cs} &= T_{ind} - d_{DMF} \dot{\varphi}_{DMF} - T_{FICE} + T_{DMF} \\
I_2 \dot{\omega}_{EM} &= T_{EM} + d_{DMF} \dot{\varphi}_{DMF} - d_{EM} \omega_{EM} - T_{DMF} \\
\dot{\varphi}_{DMF} &= \omega_{cs} - \omega_{EM}
\end{align*}$$ (12)

where $I_1 = I_{ICE} + I_{DMF1}$, $I_2 = I_{DMF2} + I_{EM}$ are the lumped inertias of the ICE and primary flywheel as well as the secondary flywheel and the EM. $T_{FICE}$ denotes the brake torque, introduced by the friction and pumping losses of the ICE, which is assumed to be constant. The rotational speed of the EM is denoted $\omega_{EM}$ and $d_{EM}$ is its damping factor, respectively.

D. Analysis of the Model

To apply a linear controller design, the time delays and the lash are neglected in the first step of the design and the constant friction of the ICE, $T_{FICE}$ will be compensated by a disturbance observer. The following analysis is intended to justify the choice of the controller design, which is described in the succeeding section. The basis for the analysis is given by the linearized torque model consisting of (9), (10), (12). The corresponding state, input and output vectors are defined as follows:

$$\begin{align*}
x(t) &= [\omega_{cs}(t) \quad T_{ind}(t) \quad \omega_{EM}(t) \quad \varphi_{DMF}(t) \quad T_{EM}(t)]^T \\
u(t) &= [T_{ind,des}(t - \delta_{IPS}) \quad T_{EM,des}(t - \delta_{EM})]^T \\
y(t) &= [\omega_{cs}(t - \delta_{IPS}/2) \quad \omega_{EM}(t - \delta_{EM})]^T.
\end{align*}$$ (13)

Measurements of $\omega_{EM}$ are again delayed by the bus communication between the ECUs. To overcome the effect of the discrete event character of the ICE’s torque production, leading to strong fluctuation of raw crankshaft velocity measurements, $\omega_{cs}$ is evaluated by a moving average over one IPS interval. It is therefore delayed by $\delta_{IPS}/2$, approximately.

Remark 1: For the purpose of idle speed control, only $\omega_{cs}$ is used as a controlled variable. Deviation from its reference speed is perceptible to the driver through acoustical and vibrational behavior, which is not the case for changes in $\omega_{EM}$. Thus, $\omega_{EM}$ is only used as an additional measurement to improve state prediction.

The underlying parameters are obtained from physical powertrain data of a 1.4l, 4-cylinder gasoline passenger car engine. Via the DMF, a pair of conjugate complex roots is introduced in the transfer function matrix of the linearized model, with natural frequency $\omega_0 \approx 62 \text{rad s}^{-1}$ and damping ratio $\zeta = 0.018$. Thus the two lumped inertias in (12) tend to a weakly damped oscillation once they are appropriately excited. It is therefore desirable to either avoid excitation of these modes or introduce additional damping through the EM. During normal operation, the ICE does not excite these modes, since the frequency of torque production during idle at $\omega_{cs,ref} = 89 \text{rad s}^{-1}$ is $\omega = 178 \text{rad s}^{-1}$.

In order to achieve good emission behavior for the PHEV powertrain, it is desired to use the ICE as a low-bandwidth actuator. Fast transients, where knowledge of the cylinder air charge is uncertain, can lead to injected fuel quantities off the desired stoichiometric quantity. The EM is therefore used to meet requirements regarding closed loop performance. With the latter being predominantly limited by the delay connected to the input channel of the EM and the minimum delay of the measurement signals. The combined actuator and sensor
delay is therefore \( \delta_{min} = \delta_{EM} + \delta_{IPS}/2 \approx 47 \text{ms} \), if a reference idle speed of \( n_{cs,ref} = \omega_{cs,ref} \pi/30 = 850 \text{rpm} \) is assumed. According to [13] an upper bound of the bandwidth in terms of the loop transfer function is given by \( \omega_{c}^{\text{max}} < 1/\delta_{min} \approx 21 \text{rad s}^{-1} \). It is thus vital to design the controller such that the spectrum of \( T_{EM} \) is limited to frequencies below \( \omega_{c}^{\text{max}} \), with a reasonable roll off, to avoid excitation of the DMF oscillation. Since \( \omega_{c}^{\text{max}} < \omega_{0} \), actively damping the DMF induced oscillation is not possible anyway.

### III. Idle Speed Controller

In this section a frequency shaped LQG like controller is used to solve the ISC problem. The LQG controller structure is therefore modified, such that redundant actuators are only used during transients and are returned to specified desired values in steady state. This modification is inspired by the so-called MR control structure [9], cf. Fig 4. It is a decentralized structure, solving over-actuated control problems, which is particularly useful if actuators of different capabilities are available, e.g. in terms of bandwidth and operation costs. Typically, the controller \( K_2 \) is responsible for fast transient behavior or required disturbance rejection and corresponds to the high-bandwidth actuator. Controller \( K_1 \) ensures steady state operation and ”midranges” the input \( u_2 \) via the error \( e_{u2} \) to its desired steady state value \( u_{2,ss} \).

The challenge in MR control is the appropriate design of the two controllers. On the one hand, performance requirements regarding tracking or disturbance attenuation have to be met via \( K_2 \). On the other hand, \( K_1 \) might introduce counter acting control action within certain frequencies, or even stability issues. To resolve these problems, some strategies have been proposed in the past. In [9] an appropriate MPC formulation is proposed, to achieve MR like behavior. Since this solution handles the midranges of the redundant actuators via inequality constraints, which requires an online optimization, it is not a suitable solution for the ISC problem presented in this paper. Another technique has been presented in [19], where a master-slave method is employed to determine \( K_1 \) and \( K_2 \). Unfortunately, it involves a two-step design procedure which does not exhibit the clear structure and tunability of an LQG controller. These challenges are resolved by the approach presented in the following subsection.

![Fig. 4. Mid-ranging control structure.](image)

#### A. LQG with Mid-Ranging Properties

The design model used in this paper is based on the state space description

\[
\dot{x} = Ax + Bu, \quad y = Cx \tag{14}
\]

where \( x \in \mathbb{R}^n, u \in \mathbb{R}^p \) and \( y \in \mathbb{R}^m \), with \( p > m \), represent state, input and output vectors. It is assumed that \( (A, B) \) is controllable and \( (C, A) \) is observable. Furthermore, the input matrix can be partitioned by \( B = [B_1 \ B_2] \), with \( B_1 \in \mathbb{R}^{m} \), where the columns of \( B_2 \) are associated to inputs that are redundant to the ones corresponding to \( B_1 \), i.e.

\[
\text{rank}
\begin{bmatrix}
A & B_1 \\
C & 0
\end{bmatrix}
= n + m \tag{15}
\]

which is required to achieve tracking of the controlled output to any constant reference. The input vector can be written as \( u^T = [u_1^T \ u_2^T] \) accordingly, where \( u_2 \) contains inputs that are required to settle to a desired value during steady state.

The LQG controller, solving this problem, is designed to minimize the cost functional

\[
J = \int_0^\infty (\hat{x}(t)^T Q_C \hat{x}(t) + \hat{u}(t)^T R_C \hat{u}(t)) \, dt \tag{16}
\]

where \( \hat{x} = \hat{x} - x_{ss}, \hat{u} = u - u_{ss} \) and \( Q_C = Q_C^T \geq 0, R_C = R_C^T > 0 \) are constant weighting matrices. The state estimate \( \hat{x} \) is obtained from the Kalman filter, incorporated in the LQG controller:

\[
\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{17}
\]

To achieve the MR property, the steady state values \( x_{ss} \) and \( u_{ss} \) are determined such that \( u_2 = u_{2,ss} \) in steady state, by

\[
\begin{bmatrix}
x_{ss} \\
u_{1,ss}
\end{bmatrix}
= \begin{bmatrix}
-A & -B_1 \\
C & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
B_2 u_{2,ss} \\
y_{ref}
\end{bmatrix} \tag{18}
\]

The resulting control law

\[
u = -K_{LQG}(\hat{x} - x_{ss}) + u_{ss} \tag{19}\]

where \( u_{ss} = [u_{1,ss}^T \ u_{2,ss}^T] \), minimizes (16). The gains \(-K_{LQG}\) and \( L \) are determined solving the corresponding control algebraic Riccati equations (ARE) [13], using the model (14), \( Q_C, R_C \) and the process and sensor noise covariance matrices \( Q_{KF}, R_{KF} \). Using (17), (18) and (19) a controller is obtained, featuring the properties of the MR structure, while avoiding the difficulties connected to a two-step MR controller design.

It is important to note that handling control problems with more than two redundant actuators, using the above procedure, is drastically simplified, compared to the traditional MR structure.

In order to achieve an intuitive tuning of the LQG controller and respect actuator capabilities in terms of frequency domain characteristics, it is proposed to extend the LQG controller by a frequency shaped cost functional [10]. This allows to consider the requirements derived in Subsection II-D. Following the separation principle, the cost functional associated with the controller (16) is modified independently of the Kalman filter.

To introduce frequency weights into the quadratic cost, (16) is transformed into the frequency domain using Parseval’s theorem:

\[
J = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\hat{x}(\omega)^T Q_C \hat{x}(\omega) + \hat{u}(\omega)^T R_C \hat{u}(\omega)) \, d\omega \tag{20}
\]
where \( R_C \) is replaced by \( \bar{R}(\omega) = F(\omega)^T R_C F(\omega) \). It is assumed that \( F(\omega) \) is a biproper filter [20] that can be used to emphasize certain frequencies not to appear in the control signal \( u \). A state space representation of \( \bar{F}(\omega) \) is supposed to be given by

\[
\dot{x}_F = A_F x_F + B_F u, \quad u_F = C_F x_F + D_F u. \tag{21}
\]

The order of the filter is determined by the required roll-off rate of the complementary sensitivity function. Using \( x_{aug}^T = [x^T \ x_F^T] \), eqs. (14) and (21) are combined to an augmented controller design model:

\[
\dot{x}_{aug} = \begin{bmatrix} A & 0 \\ 0 & A_F \end{bmatrix} x_{aug} + \begin{bmatrix} B \\ B_F \end{bmatrix} u \tag{22}
\]

and the cost functional (20) can be expressed in the time domain as

\[
J = \int_0^\infty \left( x_{aug}^T(t) \bar{Q} x_{aug}(t) + u^T(t) \bar{R} u(t) \right) dt. \tag{23}
\]

The optimal controller gain matrix, minimizing (23) with respect to (22), can be determined by the solution of the associated ARE with weighting matrices

\[
\bar{Q} = \begin{bmatrix} Q_C & 0 \\ 0 & C_F R_C C_F \end{bmatrix}, \quad \bar{R} = \begin{bmatrix} C_F R_C D_F \\ D_F^T R D_F \end{bmatrix},
\]

Example 1: This example considers a randomly generated plant, lend from [7], with state space representation:

\[
A = \begin{bmatrix} -0.157 & -0.094 \\ -0.416 & -0.45 \end{bmatrix}, \quad B = \begin{bmatrix} 0.87 & 0.253 & 0.743 \\ 0.39 & 0.354 & 0.65 \end{bmatrix},
\]


\[
C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

A stabilizing LQG controller as presented above is designed, using identity matrices for \( Q_C, R_C, Q_{KF}, R_{KF} \). It is assumed that inputs \( u_2, u_3 \) correspond to high-bandwidth actuators, which have to be reset to a desired steady state value after transients. Therefore, \( u_1 \) is used to operate the system in steady state and is assumed to be a coarse/low-bandwidth actuator. The desired property of \( u_1 \) is considered by appropriate selection of \( F_1(j\omega) = (j\omega + 0.3162)/(0.1j\omega + 0.1) \) and \( F_2 = 1, F_3 = 1 \), cf. (20). Fig. 5 shows output and input signals to a step on the reference at \( t = 1s \), followed by a step of the desired steady state values from \( u_{2,ss} = u_{3,ss} = 0 \) to \( u_{2,ss} = 1, u_{3,ss} = -1 \) at \( t = 25s \). It can be observed, that the controller employs \( u_2, u_3 \) to drive the system to the reference value and returns the two inputs to their desired steady state values after the transient.

B. Offset-free Tracking

To enable offset-free tracking and desirable disturbance rejection, the design model for the Kalman filter (17) is augmented by an integrating disturbance state [21], capturing constant load torque disturbances and plant-model mismatch:

\[
\begin{bmatrix} \dot{x} \\ d \end{bmatrix} = \begin{bmatrix} A & B_{dis} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{24}
\]

where \( B_{dis} \) is designed according to knowledge about the origin of the disturbance. The corresponding process noise covariance matrix is extended to \( Q_{KF} = diag(Q_{KF}, \rho) \), where \( \rho \) is a tuning parameter. The disturbance estimate \( \hat{d} \) is incorporated into the steady state values \( x_{ss} \) and \( u_{ss} \) by extending (18) to

\[
\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} = \begin{bmatrix} -A & -B_1 \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} B_2 u_{2,ss} + B_{dis} \hat{d} \\ \ref{y} \end{bmatrix}. \tag{25}
\]

The main source of load disturbances is due to the accessories, like the A/C compressor or the electric power steering, which act directly on the crankshaft. Furthermore, the friction and pumping losses \( T_{F\&P} \), cf. (12), which are assumed to be constant, have to be estimated by the disturbance state in (24).

C. Time-delay Compensation

The LQG like controller that has been developed to this point does not consider any time delay present in the model, cf. (13). To achieve required performance in terms of sensitivity bandwidth and transient behavior, a two step time-delay compensation is used. As the considered plant is a multiple-input multiple-output system with different delays in the specific input and output signals, estimation of the non-delayed state and prediction for the input calculation has to be carried out separately. Therefore, a non-delayed state estimate is obtained, followed by an input individual state prediction.

Since delay compensation can only be carried out for time-delays which are multiples of the ECU sampling time \( \tau_s = 10\text{ms} \), the delay compensator is added to the controller after discretization.

To obtain a non-delayed state estimate, an output predictor, based on the past input sequence, is used [22]. Assuming a discretized plant model, where the input delay is ignored in the first step:

\[
x(k+1) = A_d x(k) + [B_{d,1} \ B_{d,2}] \begin{bmatrix} u_1(k - \delta_1) \\ u_2(k - \delta_2) \end{bmatrix} \tag{26}
\]

\[
[ y_1(k)] \quad [ y_2(k)] = \begin{bmatrix} C_{d,1} x(k - \delta_1) \\ C_{d,2} x(k - \delta_2) \end{bmatrix} \tag{27}
\]
with $\hat{\delta}_i, i = \{1, 2\}$, the rounding of $\delta_j/\tau_s, j = \{IPS, EM\}$, to an integer, an undelayed output $\bar{y}_i$ can be computed based on the measurement information at time step $k$, using

$$\bar{y}_i(k + \hat{\delta}_i | k) = y_i(k) + \phi_{\delta_i} U_{\delta}(k)$$  \hspace{0.5cm} (28)

where

$$\psi_{\delta_i} = C_d A_d^{-\hat{\delta}_i} \left[ A_d^{\hat{\delta}_i-1} B_d \ldots B_d \right]$$  \hspace{0.5cm} (29)

$$U_{\delta_i}(k) = \left[ u^T(k - \hat{\delta}_i + 1) \ldots u^T(k) \right]$$  \hspace{0.5cm} (30)

and the z-transform of the output predictor, $\Psi_i = \mathcal{Z}\{\psi_{\delta_i} U_{\delta}(k)\}$, is defined as

$$\Psi_i(z) = \psi_{\delta_i} \left[ z^{-(\hat{\delta}_i-1)} z^{-(\hat{\delta}_i-2)} \ldots 1 \right].$$  \hspace{0.5cm} (31)

$\bar{y}^T = [\bar{y}_1 \bar{y}_2]^T$ is used as measured input to the Kalman filter, to get an estimate of the non-delayed state, under the assumption of an undelayed input signal.

In the second step, to compensate for the input delays in (26), a $\hat{\delta}_i$-step ahead prediction $\bar{x}(k)$ is used, based on the state estimate $\hat{x}(k)$:

$$\bar{x}(k) = \hat{x}(k + \hat{\delta}_i | k) = A_d^{\hat{\delta}_i} \hat{x}(k) + \phi_{\delta_i} U_{\delta}(k)$$  \hspace{0.5cm} (32)

$$\phi_{\delta_i} = \left[ A_d^{\hat{\delta}_i-1} B_d \ldots B_d \right].$$  \hspace{0.5cm} (33)

The predictor structure is depicted in Fig. 6, where $FLQR$ contains the steady state value calculation (18) and the state feedback part (19), $\Psi = [\Psi_1 \Psi_2]$ and $\Phi(z)$ is defined analog to (31) and $A = \begin{bmatrix} A_d^{\hat{\delta}_i} & 0 \\ 0 & A_d^{\hat{\delta}_i EM} \end{bmatrix}$. Assuming perfect knowledge about the delays, no plant-model mismatch and only constant disturbances, the FLQG controller can be designed for the delay free plant, as presented in the previous subsection.

![Controller structure including the state prediction.](image.png)

**IV. EXPERIMENTAL TEST RESULTS**

In this section the effectiveness of the proposed controller structure is shown through experimental validation tests. The controller is therefore discretized according to an ECU sampling time of $\tau_s = 10 \text{ ms}$. It has been embedded into the torque structure of the ECU and is thereby executed on series production hardware. Experimental data is based on a 1.4l, 4-cylinder passenger car engine. Initial parametrization of the controller has been found using a high-dimensional nonlinear powertrain model, courtesy of IAV company, resulting in the following setting: $Q_C = \text{diag}(1, 0, 1, 0, 1)$, $R_C = \text{diag}(3, 1, 5)$, $Q_{KF} = 0.01 \cdot I$, $\rho = 1$, $R_{KF} = \text{diag}(1, 1)$. The filter weights $F_i(s) = \frac{s}{(s^2 + \omega_d \cdot 2 \pi s + \omega_d^2)^2}$, $i = \{IPS, EM\}$ haven been chosen to be of second order, with $\omega_{IPS} = 1 \text{ rad/s}^{-1}$ and $\omega_{EM} = 10 \text{ rad/s}^{-1}$, such that the EM has control authority over input signals up to frequencies of an order of magnitude higher compared to the ICE, while penalizing excitation frequencies above $\omega_{EM}^{\max}$ with a magnitude of $30 \text{ dB}$. The validation data features step response results, where steps are applied to the reference idle speed. Fig. 7 depicts an excerpt of an experiment, where the reference idle speed is changed from $n_{ss,ref} = 850 \text{ rpm}$ to $n_{ss,ref} = 1050 \text{ rpm}$ at time $t = 11 \text{ s}$ and back to its initial value at time $t = 32 \text{ s}$. The desired steady state value for the generator torque of the EM is fixed at $T_{EM,ss} = -10 \text{ Nm}$. It can be observed that the controller features excellent tracking behavior, realizing smooth and aperiodically damped transitions. The EM torque $T_{EM,des}$ is returned to its desired steady state value, as it is required for appropriately charging the hybrid battery. Note that despite of the potential oscillatory behavior of the mechanical setup, the magnitude of the oscillation between crankshaft and EM is below the resolution of the plot.

![Crankshaft speed and input variables for EM and ICE during reference steps.](image.png)

Fig. 7. Crankshaft speed and input variables for EM and ICE during reference steps.

Fig. 8 shows the response to steps on the desired steady state torque of the EM. At $t \approx 70 \text{ s}$, the generator torque is changed from $T_{EM,ss} = -45 \text{ Nm}$ to $T_{EM,ss} = +5 \text{ Nm}$ and returned to its initial value at $t \approx 82 \text{ s}$. Despite the large load level change, with the DMF traversing the backlash, the influence on the measured crankshaft speed is below a level which can be recognized by the driver. The EM torque is quickly transferred to its desired values, showing no overshoot, while the load delivered by the ICE is appropriately adjusted.

To indicate the robustness as well as the disturbance rejection capability of the controller, Fig. 9 shows the crankshaft speed for a drive-off maneuver. The passenger car considered is equipped with an automated gearbox, that facilitates drive-off and creeping by releasing the brake pedal. Thereby, the idle reference speed is increased by the gearbox at the
moment of brake pedal release, $t \approx 89.5$ s. This is followed by closure of the clutch, causing a large load torque. As it can be observed, the proposed ISC is able to avoid engine stall and the car is set in motion, quickly. As intended by the calibration, it therefore makes primarily use of the EM. Furthermore, note that the reference engine speed is drastically increased after closing the clutch.

**V. CONCLUSION**

In this paper we have investigated the ISC problem for a PHEV powertrain. The limited capacity of the battery, being the energy source for the EM, necessitates for a strategy to use the EM as a generator, following a desired steady state load level. Thereby, it has to use the EM to compensate for load disturbances, to avoid the use of a torque reserve for the ICE. To deal with the EM as a redundant actuator an LQG controller exhibiting MR properties has been presented, resulting in a unified design methodology for idle speed and battery charging control. This controller simplifies the design task for redundant control systems, compared to a design using the classical MR structure. Validation of the resulting ISC has been carried out using experiments from a passenger car, demonstrating the performance of the proposed ISC. Intuitive calibration is enabled by offering frequency dependant weighting of the control effort.

**REFERENCES**


