Optimal Control of Manifold Filling During VDE Mode Transitions

Rohit Gupta¹, Jennifer S. Hudson², Anthony M. Bloch³ and Ilya V. Kolmanovsky⁴

Abstract—During mode transitions in advanced engines and powertrains, actuators (electronic throttle, fuel, spark timing, variable valve timing, etc.) need to be optimally coordinated to obtain the correct state for the next mode. Specific examples of mode changes include cylinder deactivation/reactivation transitions in variable displacement engines, combustion mode transitions in HCCI and DISI engines and clutch transitions between locked and unlocked states in dual clutch transmissions. Mode transitions lead naturally to optimal control problems that can be analyzed using the maximum principle and solved numerically. From this perspective, the paper considers in detail one of the simplest problems of its kind (which to the authors’ knowledge has not been treated), the optimal control of intake manifold filling using the electronic throttle. This problem is relevant to variable displacement engines and to the fuel economy that these engines can achieve. The analysis of minimum time and Nonlinear Quadratic (NLQ) optimal control problems is presented for the nonlinear manifold filling dynamics based on the maximum principle and simulated optimal trajectories are reported. Then a recently proposed Iterative Model and Trajectory Refinement (IMTR) strategy for solving trajectory optimization problems is validated using the intake manifold pressure filling as a case study. Specifically, we demonstrate the convergence of IMTR to trajectories that are close enough to the trajectories obtained from the maximum principle.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_e )</td>
<td>Engine Speed (rad/sec)</td>
</tr>
<tr>
<td>( a )</td>
<td>Throttle Time Constant</td>
</tr>
<tr>
<td>( f_{th, g_{th}} )</td>
<td>Nonlinear Functions</td>
</tr>
<tr>
<td>( k_{0,1} )</td>
<td>Constants</td>
</tr>
<tr>
<td>( p )</td>
<td>Manifold Pressure (kPa)</td>
</tr>
<tr>
<td>( p_{des} )</td>
<td>Desired Manifold Pressure (kPa)</td>
</tr>
<tr>
<td>( R )</td>
<td>Gas Constant (kJ/(kgK))</td>
</tr>
<tr>
<td>( T_{amb} )</td>
<td>Ambient Air Temperature (K)</td>
</tr>
<tr>
<td>( \theta_{th} )</td>
<td>Throttle Angle (deg)</td>
</tr>
<tr>
<td>( \theta_{th,c} )</td>
<td>Commanded Throttle Angle (deg)</td>
</tr>
<tr>
<td>( \theta_{th,c} )</td>
<td>Maximum Commanded Throttle Angle (deg)</td>
</tr>
<tr>
<td>( \theta_{th,des} )</td>
<td>Desired Throttle Angle (deg)</td>
</tr>
<tr>
<td>( V_m )</td>
<td>Intake Manifold Volume (m³)</td>
</tr>
<tr>
<td>( W_{cyl} )</td>
<td>Cylinder Flow (kg/sec)</td>
</tr>
<tr>
<td>( W_{th} )</td>
<td>Throttle Flow (kg/sec)</td>
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</table>

I. INTRODUCTION

Advanced engines and powertrains undergo many mode changes, such as cylinder deactivation/reactivation transitions in variable displacement engines, combustion mode transitions in HCCI and DISI engines and clutch transitions between locked and unlocked states in dual clutch transmissions. During these mode transitions, actuators (electronic throttle, fuel, spark timing, variable valve timing, etc.) need to be optimally coordinated to obtain the correct state for the next mode (see [1], [2], [8], [9]). These optimal control problems can be analyzed using Pontryagin’s maximum principle and solved numerically.

This paper considers optimal control of intake manifold pressure filling in variable displacement engines with electronic throttle control. When variable displacement engines transition from a large number of cylinders to a smaller number, the intake manifold pressure must increase. An optimal control – one that minimizes transition time while achieving the correct final state – improves the vehicle fuel economy by enabling the engine to spend more time in a fuel efficient mode.

In this paper, the optimal manifold filling control problem is addressed using three different approaches. The first is a minimum time optimal control approach, which uses Pontryagin’s maximum principle and numerical solution of a two point boundary value problem. The second approach uses a Nonlinear Quadratic (NLQ) control method, again with Pontryagin’s maximum principle, to minimize a quadratic cost function of the manifold pressure and the throttle position. The third approach uses Iterative Model and Trajectory Refinement (IMTR) to optimize the same quadratic cost function through optimization of a low fidelity model.

IMTR is a recently proposed algorithm for optimizing systems represented by high complexity models. The algorithm uses intertwined steps of trajectory optimization on a low fidelity model and disturbance estimation to adjust the low fidelity model to mimic a high fidelity model. In [5] and [6], this approach was successfully applied to spacecraft orbital maneuvering problems and automotive powertrain launch optimization problems, respectively.

The paper is organized as follows. In Section II, the manifold pressure control optimization problem is described in more detail. In Section III, the minimum time optimal control problem is solved using Pontryagin’s maximum principle. In Section IV, the same problem is solved using NLQ optimal control. Section V discusses the IMTR method and presents the results of its application to the manifold pressure control problem. Section VI compares the results obtained from the NLQ optimal control approach using Pontryagin’s maximum principle and the IMTR approach. Finally, concluding remarks are made in Section VII.
II. PROBLEM DESCRIPTION

A variable displacement, naturally aspirated engine is considered during transition from 8 to 4 cylinders. Fast control of manifold pressure and cylinder flow is important to enable rapid mode changes and fuel economy improvements (see [3], [7]). The intake manifold pressure filling of the engine is described by

\[ p = \frac{RT_{amb}}{V_m} (W_{th} - W_{cyl}). \]  

(1)

The throttle and cylinder flows are characterized by

\[ W_{th} = f_{th}(p)g_{th}(u_{th}), \]

\[ W_{cyl} = k_0 + k_1 \omega_e p, \]

(2)

(3)

where \( \omega_e \) is the engine speed, and \( k_0, k_1 \) can also depend on \( \omega_e \). A detailed description of the nonlinear functions \( f_{th}, g_{th} \) can be found in [4]. The throttle dynamics are given by

\[ u_{th} = -au_{th} + a u_{th,c}, \]

(4)

where \( u_{th,c} \) is the commanded throttle position, prescribed on a time interval \([0, T]\). The symbols in Equations (1)-(4) are defined in the Nomenclature Table above.

During the transition, the manifold pressure must approximately double, so we consider the terminal constraint,

\[ p_{des} = p(T) = 2p(0). \]

(5)

The optimal control objective is to minimize a transition metric discussed further below.

To demonstrate the three optimal control approaches, a test problem is defined and solved using each approach. Table I shows the parameters of the test problem.

### TABLE I

**TEST PROBLEM TO BE SOLVED BY NLQ OPTIMAL CONTROL APPROACH AND THE IMTR APPROACH**

<table>
<thead>
<tr>
<th></th>
<th>p(0)</th>
<th>( u_{th}(0) )</th>
<th>( p_{des} )</th>
<th>( u_{th,des} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40 kPa</td>
<td>7.951 deg</td>
<td>80 kPa</td>
<td>15.5299 deg</td>
</tr>
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</table>

III. MINIMUM TIME OPTIMAL CONTROL

The optimal control problem for the intake manifold is first solved using a minimum time approach. The two point boundary value problem is formulated as follows.

The objective is to minimize the final time \( T \) subject to bounds on the commanded throttle position

\[ 0 \leq u_{th,c} \leq u_{th,c}, \]

(6)

and terminal constraints

\[ p(T) = p_{des}, \]

\[ W_{th}(T) = W_{cyl}(T). \]

(7)

(8)

The necessary conditions for optimality are defined and the problem is solved using Pontryagin’s maximum principle. The Hamiltonian for the problem is

\[ H = \Psi_0 + \Psi_1 \frac{RT_{amb}}{V_m} [f_{th}(p)g_{th}(u_{th}) - k_0 - k_1 \omega_e p] \]

\[ + \Psi_2 (-au_{th} + a u_{th,c}). \]

Pointwise minimization of the Hamiltonian leads to

\[ u_{th,c} = \begin{cases} 0 & \text{if } \Psi_2 > 0 \\ \overline{u}_{th,c} & \text{if } \Psi_2 < 0 \end{cases}. \]

The case \( \Psi_2 = 0 \) represents a singular arc and can be ruled out with further analysis.

The adjoint equations are

\[ \dot{\Psi}_1 = -\Psi_1 \frac{RT_{amb}}{V_m} \left[ \frac{\partial f_{th}(p)}{\partial p} g_{th}(u_{th}) - k_1 \omega_e \right], \]

\[ \dot{\Psi}_2 = -\Psi_1 \frac{RT_{amb}}{V_m} f_{th}(p) \frac{\partial g_{th}(u_{th})}{\partial u_{th}} + \Psi_2 a. \]

The transversality conditions are

\[ H(T) = 0. \]

Note that \( \Psi(T) \) is not constrained. The initial state vector

\[ x_0 = \begin{bmatrix} p(0) \\ u_{th}(0) \end{bmatrix}, \]

(14)

is fixed. The system equations and adjoint equations are solved numerically. The values \( \Psi_1(0), \Psi_2(0) \) and \( T \) are optimized such that Equations (7) and (8) hold.
Fig. 3. Throttle and cylinder flow from the minimum time optimal control approach.

Fig. 4. Adjoint variables \( \Psi_1 \) and \( \Psi_2 \) from the minimum time optimal control approach.

Fig. 5. Optimal switching time from the minimum time optimal control approach.

Fig. 6. Actual vs. estimated switching time from the minimum time optimal control approach.

Figures 1-4 show the results of the minimum time optimal control approach for the test problem defined in Table I. Figure 1 shows the optimal manifold pressure trajectory. Figure 2 shows the optimal throttle angle trajectory. Figure 3 shows the optimal throttle and cylinder flow trajectories. Figure 4 shows the trajectories for the adjoint variables. Next, the problem was solved for different values of \( p(0) \) and \( p_{des} \). Figure 5 shows the optimal switching time as a function of initial and desired manifold pressure. The surface shown in Figure 5 is constructed by using a finite number of values for \( p_{des} \) and \( p(0) \) and then interpolating for other values of \( p_{des} \) and \( p(0) \). Figure 6 shows the actual optimal switching time compared to the switching time obtained from the surface shown in Figure 5.

IV. NONLINEAR QUADRATIC OPTIMAL CONTROL

The manifold filling optimal control problem is next solved using NLQ control with the finite horizon cost defined as

\[
J = \frac{1}{2} \int_{0}^{T} \left[ q_1 [p(t) - p_{des}]^2 + q_2 [u_{th}(t) - u_{th,des}]^2 + \rho [u_{th,c} - u_{th,des}]^2 \right] dt + \frac{1}{2} f_1 [p(T) - p_{des}]^2 + \frac{1}{2} f_2 [u_{th}(T) - u_{th,des}]^2,
\]

where \( q_1, q_2, \rho, f_1, f_2 \) are positive weights and \( T \) is the fixed terminal time. To minimize the cost, Pontryagin’s maximum principle is used again. The Hamiltonian for the NLQ minimization problem is

\[
H = \frac{1}{2} \Psi_0 [q_1 [p(t) - p_{des}]^2 + q_2 [u_{th}(t) - u_{th,des}]^2 + \rho [u_{th,c} - u_{th,des}]^2]
\]

\[
+ \Psi_1 \frac{RT_{amb}}{V_m} \left[ f_{th}(p) g_{th}(u_{th}) - k_0 - k_1 \omega_c p \right]
\]

\[
+ \Psi_2 [-a u_{th,c} + a u_{th,c}].
\]  \hspace{1cm} (16)

Pointwise minimization of the Hamiltonian leads to

\[
u_{th,c} = u_{th,des} - \frac{\Psi_0}{\rho}.
\]  \hspace{1cm} (17)

Assuming \( \Psi_0 = 1 \), the adjoint equations are

\[
\dot{\Psi}_1 = -\Psi_1 \frac{RT_{amb}}{V_m} \left[ \frac{\partial f_{th}(p)}{\partial p} g_{th}(u_{th}) - k_1 \omega_c \right]
\]

\[
- q_1 [p(t) - p_{des}],
\]  \hspace{1cm} (18)

\[
\dot{\Psi}_2 = -\Psi_1 \frac{RT_{amb}}{V_m} f_{th}(p) \frac{\partial g_{th}(u_{th})}{\partial u_{th}} + \Psi_2 a
\]

\[
- q_2 [u_{th} - u_{th,des}].
\]  \hspace{1cm} (19)

The transversality conditions are

\[
H(T) = 0,
\]  \hspace{1cm} (20)

\[
\Psi(T) = \left[ \begin{array}{c} f_1 [p(T) - p_{des}] \\ f_2 [u_{th}(T) - u_{th,des}] \end{array} \right].
\]  \hspace{1cm} (21)

Figures 7-10 show the results of the NLQ optimal control approach for the test problem defined in Table I. The values \( q_1 = 1, q_2 = 1, \rho = 1, f_1 = 0, f_2 = 0 \) and \( T = 0.3 \) sec were used in the cost function defined in Equation (15).
Figure 7 shows the optimal manifold pressure trajectory. Figure 8 shows the optimal throttle angle trajectory. Figure 9 shows the commanded throttle angle trajectory. Figure 10 shows the optimal throttle and cylinder flow trajectories.

V. ITERATIVE MODEL AND TRAJECTORY REFINEMENT

IMTR is a method of trajectory optimization for a high complexity system represented by both high fidelity and low fidelity models. The high fidelity model accurately represents the system response in simulations, but is not amenable to trajectory optimization, due to long simulation times, nonlinearities, different time scales of the dynamics and/or being of “black-box” type (no access to states or subsystems). The low fidelity model approximates the system response with a larger error than the high fidelity model, but is more amenable to optimization (e.g. adjoint equations can be easily derived and implemented, etc.). The IMTR approach involves iterating between steps of disturbance estimation and trajectory optimization of the low fidelity model to efficiently solve the optimization problem without requiring numerical optimization of the high fidelity model.

The IMTR algorithm is shown in Table II. Superscripts indicate iteration number.

The nonlinear, high fidelity model given by Equations (1) and (4) has the form

$$\dot{x}_h^n = f(x_h^n, u_{th,c}^n),$$

where

$$x_h = \begin{bmatrix} p \\ u_{th} \end{bmatrix}.$$  

The low fidelity model can be approximated by the linearization of the high fidelity model at $p_{des} = 80 \text{ kPa}$ and $u_{th,des} = 15.5299 \text{ deg}$. The low fidelity model has the form given below

$$\dot{x}_l^n = Ax_l^n + Bu_{th,c}^n + d^n, \quad (24)$$
$$y_l^n = Cx_l^n, \quad (25)$$

where $x_l^n(0) = x_h^n(0)$ and $d^n$ is a time varying disturbance. In the numerical experiments, $d^n$ is represented by a piece-wise linear function of time.

The finite horizon LQ cost is given by Equation (15). The LQ optimization problem for the low fidelity model over the time interval $[0, T]$ is solved with the Riccati differential equations given below

$$-\dot{P}^n = A^T P^n + P^n A - P^n BR^{-1} B^T P^n + C^T Q C, \quad (26)$$
$$-\dot{r}^n = (A - BR^{-1} B^T P^n)^T r^n - P^n d^n - P^n B u_{th,des} + C^T Q \begin{bmatrix} p_{des} \\ u_{th,des} \end{bmatrix}, \quad (27)$$

where $u_{th,c} = u_{th,des} - R^{-1} B^T P^n x_l^n + R^{-1} B^T r^n, \quad (28)$

In Step 4 of the IMTR algorithm, the disturbance is updated by adding a quantity proportional to the difference in the time rate of change of the states of the high fidelity model.
and the low fidelity model, and is given by the equation below
\[ d^{n+1} = d^n + k(\dot{x}^n_k - \dot{x}_n^0), \quad k \in (0,1]. \]  

Figures 11-13 show the results of the IMTR approach for the test problem defined in Table I with the same values of \( q_1, q_2, \rho, f_1, f_2 \) and \( T \) as used in the previous section. Figure 11 shows the initial and final manifold pressure trajectories for the low fidelity and high fidelity models. Figure 12 shows the initial and final throttle angle trajectories for the low and high fidelity models. Figure 13 shows the initial and final commanded throttle angle trajectory. Note in Figures 11-12, the initial trajectories are quite far away from the desired trajectories, but over iterations of the IMTR algorithm, the final trajectories approach the desired trajectories (some of the trajectories are not visible in Figures 11-12 due to an overlap).

VI. COMPARISON OF RESULTS OBTAINED FROM THE MAXIMUM PRINCIPLE AND IMTR

The results from Section IV and Section V, obtained by the maximum principle and the IMTR approach, respectively, are now compared. Figure 14 compares the manifold trajectory obtained from the maximum principle to the trajectory obtained from the IMTR approach. Figure 15 compares the throttle angle trajectory obtained from the maximum principle with the one obtained from the IMTR approach. Figure 16 compares the commanded throttle angle trajectory obtained from the maximum principle with the one obtained from the IMTR approach. Note that in Figures 14-16, the IMTR approach converges to trajectories close to the trajectories obtained from the maximum principle.

Figure 17 shows how the cost for the high fidelity model decreases with the iterations of the IMTR algorithm. Figure 17 also demonstrates that the final cost obtained after IMTR algorithm convergence is close to the optimal cost obtained using the maximum principle. Figure 18 compares the cost for the high fidelity model obtained using the maximum principle and the IMTR approach for different values of \( p(0) \) and \( p_{des} \). Figure 18 shows that for each case, the final cost obtained from the IMTR algorithm is close to the optimal cost obtained using the maximum principle. Note that for each case in Figure 18, \( u_{th}(0) \) is chosen such that \( W_{th}(0) = W_{cap}(0) \).

The different case studies indicate that the trajectories obtained from the IMTR approach and the maximum principle are close enough, with the IMTR approach being easier to implement as compared to the maximum principle.
VII. CONCLUSIONS

The problem of optimal control of automotive engine intake manifold filling using an electronic throttle was approached with three methods: minimum time optimal control, NLQ optimal control and IMTR. The analysis of minimum time and NLQ optimal control problems was based on the maximum principle and simulated optimal trajectories were reported. The IMTR algorithm for solving trajectory optimization problems was validated using intake manifold pressure filling as a case study. Convergence of IMTR to trajectories that are close to the trajectories obtained from the maximum principle was demonstrated.

Our results in this paper, along with previous spacecraft control and automotive examples, increase confidence in applying IMTR strategy to more complex systems.

VIII. ACKNOWLEDGMENTS

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REFERENCES