Building Temperature Control with Adaptive Feedforward

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Abstract—A common approach to the modeling of temperature evolution in a multi-zone building is to use thermal resistance and capacitance to model zone and wall dynamics. The resulting thermal network may be represented as an undirected graph. The thermal capacitances are the nodes in the graph, connected by thermal resistances as links. The temperature measurements and temperature control elements (heating and cooling) in this lumped model are colocated. As a result, the input/output system is strictly passive and any passive output feedback controller may be used to improve the transient and steady state performance without affecting the closed loop stability. The storage functions associated with passive systems may be used to construct a Lyapunov function, to demonstrate closed loop stability and motivate the construction of an adaptive feedforward control to compensate for the variation of the ambient temperature and zone heat loads (due to changing occupancy). The approach lends itself naturally to an inner-outer loop control architecture where the inner loop is designed for stability, while the outer loop balances between temperature specification and power consumption. Energy efficiency consideration may be added by adjusting the target zone temperature based on user preference and energy usage. The initial analysis uses zone heating/cooling as input, but the approach may be extended to more general model where the zonal mass flow rate is the control variable. A four-room example with realistic ambient temperature variation is included to illustrate the performance of the proposed passivity based control strategy.

INTRODUCTION

Heating, ventilation, and air conditioning (HVAC) system is a major energy consumer in buildings. For the analysis of building temperature evolution under HVAC control, a common approach is to model it as an interconnected network of lumped thermal capacitors and resistors. Thermal resistance models the heat flow due to temperature difference: \( Q = \Delta T/R \), where \( Q \) (in W) is the rate of heat transfer across the resistance, \( \Delta T \) is the temperature difference (in K), and \( R \) is the thermal resistance (K/W). Thermal capacitance (or thermal mass) models the ability of a space (or wall) to store heat: 

\[
C \frac{d\Delta T}{dt} = Q, \text{where } C \text{ has the unit J/K.}
\]

We model a single zone as a single thermal capacitor and use the standard \( 3R2C \) model [1] for the wall (i.e., the wall is characterized by three thermal resistors in series shunted by two thermal capacitors at the nodes). As shown in [2], the temperature dynamics of a thermal RC network modeled as a graph consisting of \( n \) nodes (capacitors) and \( \ell \) links (resistors) is given by

\[
CT = -DR^{-1}DT + B_0T_\infty + Bu + Bw \tag{1}
\]

where \( C \in \mathbb{R}^{n \times n} \) is a diagonal, positive definite matrix consisting of the thermal capacitances, \( R \in \mathbb{R}^{\ell \times \ell} \) is a diagonal, positive definite matrix consisting of the link thermal resistances, \( D \in \mathbb{R}^{\ell \times n} \) is the incidence matrix of the graph, \( B_0 = -DR^{-1}d_0 \in \mathbb{R}^{n} \) is a column vector with non-zero elements as the thermal conductance of nodes connected to the ambient, \( T_\infty \) is the ambient temperature, \( u \in \mathbb{R}^m \) is the controlled heat input and \( w \in \mathbb{R}^m \) is the environmental heat input into each zone, and \( B \in \mathbb{R}^{n \times m} \) is the corresponding input matrix. Note that \( w \) and \( u \) both enter the dynamics through the same input matrix since we assume no heat generation within the walls.

We assume a fully connected graph, so \( D \) is full row rank, and \( DR^{-1}DT \) is positive definite. Each zone contains a heater, implying that \( B \) has full column rank. We address temperature regulation of the zones that are directly affected by active heating/cooling devices. Therefore, the output of interest is

\[
y = B^T T. \tag{2}
\]

Since \( DR^{-1}DT \) is positive definite, the open loop system (with \( u = 0 \)) is exponentially stable. If \( T_\infty \) and \( w \) are constants, then the steady state temperatures are given by

\[
T_{ss} = (DR^{-1}DT)^{-1}(B_0T_\infty + Bw). \tag{3}
\]

II. PASSIVITY BASED CONTROL

A. Feedforward Control: Stable Plant Inversion

The purpose of the feedforward control is to shift the system equilibrium to the desired operating point. First consider the setpoint control case where the ambient temperature, \( T_\infty \), desired zone temperature, \( y_{des} \), and disturbance heat input \( w \), are constant vectors. Given a desired temperature set point \( y_{des} \) and known \( T_\infty \) and \( w \), we can solve for the equilibrium temperature, \( T^* \), and feedforward control, \( u^* \), from (1) (with \( T^0 = 0 \)):

\[
\begin{bmatrix}
DR^{-1}DT \\
B^T \\
0 \\
\end{bmatrix}
\begin{bmatrix}
T^* \\
u^* \\
\end{bmatrix}
= \begin{bmatrix}
B_0T_\infty + Bw \\
y_{des} \\
\end{bmatrix}. \tag{4}
\]

The following shows that a unique solution may always be found.

Proposition 1: For constant \((T_\infty, y_{des}, w)\), the solution \((T^*, u^*)\) that satisfies (4) is given by

\[
T^* = \begin{bmatrix}
I - (B^T A^{-1}B)^{-1}B^T y_{des} \\
B^T (B^T A^{-1}B)^{-1}B_0 T_\infty \\
\end{bmatrix}
\]

\[
u^* = -w + (B^T A^{-1})^{-1} \\
(y_{des} - B^T A^{-1}B_0 T_\infty) \tag{6}
\]
where \( A := DR^{-1}D^T \), \( B^+ \in \mathbb{R}^{m \times n} \) is the Moore-Penrose pseudo-inverse and \( B^\perp \in \mathbb{R}^{n-m \times n} \) the annihilator of \( B \):

\[
B^+ = (B^TB)^{-1}B^T, \quad B^\perp = I_{n-m} - B^+B^T
\]

If \((T_{\infty}, y_{des}, w)\) are time varying, \((T^*, u^*)\) that solves (1) is given by a stable dynamical system as depicted in Figure 1 and stated in the proposition below.

**Proposition 2:** For time varying \((T_{\infty}, y_{des}, w)\), the solution \((T^*, u^*)\) that satisfies (1) (with \((T, y)\) replaced by \((T^*, y_{des})\)) is given by the output of the following stable dynamical system:

\[
\dot{\xi} = -(B^+CB^\perp)^{-1}B^+(-AB^\perp\xi + B_0T_{\infty} - AB^+\dot{y}_{des} - CB^+\dot{y}_{des})
\]

\[
T^* = B^+\dot{y}_{des} + B^\perp\dot{\xi}
\]

\[
u^* = -w + (B^T C^{-1}B)^{-1}(B^T AB^\perp\xi + B^T C^{-1}AB^+\dot{y}_{des} + \dot{y}_{des} - B^T C^{-1}B_0T_{\infty})
\]

**Passivity Property of Building Thermal Systems**

A system with state \( x \), input \( u \), and output \( y \) is passive if there exists a continuously differentiable storage function \( V(x) \geq 0 \) such that

\[
\dot{V} \leq -W(x) + u^Ty
\]

for some function \( W(x) \geq 0 \). If \( W(x) \) is positive definite, then the system is strictly passive [3]. The notion of passivity is motivated by physical systems that conserve or dissipate energy, such as passive circuits and mechanical structures, where \( V(x) \) corresponds to an energy function. Passivity is a useful tool for nonlinear stability analysis and control design, particularly for large scale interconnected systems as in network flow control [4] and formation control [5]. Indeed, the celebrated Passivity Theorem states that, if two passive systems \( H_1 \) and \( H_2 \) with positive definite and radially unbounded storage functions \( V_1(x) \) and \( V_2(x) \) respectively, are interconnected as in a negative feedback interconnection, then the equilibrium of the interconnection is stable in the sense of Lyapunov [3].

A physical system that conserves or dissipates energy is passive if appropriate, dual input/output pairs are chosen – so that the product between the input and output vectors is the power delivered to the system. Examples include multi-port RLC circuits with port voltages as inputs and port currents into the circuit as outputs (or vice versa), and mass-spring-damper networks with force (or torque) as input and collocated velocity (or angular velocity) as the output. This is sometimes known as the sensor/actuator collocation condition. Similarly, the thermal system modeled as a thermal RC network with collocated heat input and zone temperature output is strictly passive (due to the thermal resistances) without \( T_{\infty} \) and \( w \), which is easily shown using the storage function \( V = CT^T\).

We consider a standard feedback/feedforward control architecture as shown in Figure 2. Decompose the control input as

\[
u = u_{fb} + u^*
\]

where we will design \( u_{fb} \) as a passive feedback and \( u^* \) will be the feedforward from Section II-A.

**Controller Architecture**

1) **Model Based Feedforward:** Consider the output tracking problem (with regulation as a special case): Given time varying \((T_{\infty}, y_{des}, w)\), find \( u \) based on feedback of \( T \) to drive \( y \) to \( y_{des} \). The desired zone temperature, \( y_{des} \), is assumed to be known, but we will consider both cases when \((T_{\infty}, w)\) may or may not be measured. First form the error system based on \((T^*, u^*)\) from (8):

\[
C\delta T = -DR^{-1}D^T\delta T + B\delta u, \quad \delta y = B^T\delta T
\]

where \( \delta T := T - T^* \), \( \delta u := u - u^* \), \( \delta y := y - y_{des} \). The stability of the first order system and collocation of the input and output immediately suggests inherent passivity of the system. Thus, a passivity-based stabilizing control law can be designed as stated in the following theorem:

**Theorem 1:** Given

\[
u = u^* - K(y - y_{des})
\]

where \( K \) is a passive (possibly dynamic) system and \( u^* \) satisfies (4), the equilibrium \( T^* \), in (4), is a globally exponentially stable equilibrium, and \( y \to y_{des} \) exponentially.

**Proof:** Consider the Lyapunov function candidate for (10):

\[
V(\delta T) = \frac{1}{2}\delta T^T \Sigma C\delta T.
\]

The derivative along the solution is

\[
\dot{V} = -\delta T^T DR^{-1}D^T\delta T + \delta TB\delta u.
\]

Substituting in the controller (11), we get

\[
\dot{V} = -\delta T^T DR^{-1}D^T\delta T - \delta T^TBK(B^T\delta T).
\]

Using the passivity of \( K \) and the fact that \( DR^{-1}D^T > 0 \), it follows that \( V \to 0 \) exponentially.

This result implies that we have a large class of stabilizing controllers to draw from in building control, with virtually no model information necessary. (Although we do need model information to compute the feedforward, \( u^* \), however,
error in \( u^* \), while influencing the steady state, does not affect stability.) Any available model information may be used to design \( K \) towards an optimization objective, e.g., energy efficiency, while preserving the passivity structure. For example, the \( H_2 \) optimization problem subject to positive realness constraint may be posed as a convex optimization and solved using linear matrix inequality (LMI) approach [6], [7].

The controller \( K \) may contain saturation and still preserves passivity. In fact, the entire \( u \) may be constrained, as long as the saturation level is larger than \( u^* \). This simply means that the effective gain is reduced when \( u \) is in the saturation region. Furthermore, the controller gain may be time varying, as long as passivity is preserved.

For heating only, \( u \) is restricted to be non-negative. In this case as well, as long as \( T_\infty < y_{des} \), asymptotic stability of the closed loop system is preserved. This class of passivity based controllers possesses a high level of robustness, i.e., the inherent passivity in the system implies robust stability even when the operating condition changes. For example, if windows or doors are open and changing the thermal resistance, the closed system would remain stable.

D. Adaptive Feedforward Control

The passivity property of the building system also allows direct extension to feedforward adaptive control when \( u^* \) is unknown or uncertain. The key observation is since the inverted plant is stable that the feedforward \( u^* \) becomes a linear combination of \( (T_\infty, y_{des}, w) \) after the transient in \( \xi \) in (8) dies out:

\[
u^* = F_0 y_{des} + F_1 T_\infty - w.	ag{13}\]

If \( T_\infty \) can be measured, the following theorem states that \( F_0, F_1, \) and \( w \) may be adaptively updated and ensure that \( y \to y_{des} \).

**Theorem 2:** Consider the passive controller with adaptive feedforward in the following form:

\[
\begin{align*}
u &= \hat{u}^* - K (y - y_{des}) \\
\dot{\hat{u}} &= F_0 y_{des} + F_1 T_\infty - \hat{w} \\
\dot{F}_0 &= -\Gamma_0 (y - y_{des}) y_{des} T_\infty \\
\dot{\hat{F}}_1 &= -\Gamma_1 (y - y_{des}) T_\infty \\
\dot{\hat{w}} &= -\Gamma_2 (y - y_{des})
\end{align*}
\tag{14)-(18}
\]

where \( K \) is passive and \( \Gamma_i > 0, i = 0, 1, 2 \). Then \( y \to y_{des} \) asymptotically as \( t \to \infty \).

**Proof:** Consider the Lyapunov function candidate:

\[
V(\delta T) = \frac{1}{2} \delta T^T C \delta T + \frac{1}{2} \text{tr} (\delta F_0^T \Gamma_0^{-1} \delta F_0) + \frac{1}{2} \text{tr} (\delta F_1^T \Gamma_1^{-1} \delta F_1) + \frac{1}{2} \delta w^T \Gamma_2^{-1} \delta w.	ag{19}\]

Substituting in the controller (14)-(18), we get

\[
\dot{V} = - \delta T^T D R^{-1} D^T \delta T - \delta T^T B K (B^T \delta T).
\]

Integrating both sides and using Barbalat's Lemma [3], we have \( \delta T \to 0 \) asymptotically. However, the convergence is no longer exponential. \( \blacksquare \)

**Remark:** If \( T_\infty \) is unknown but may be expressed as a linear combination of known basis functions:

\[
T_\infty = \sum_{i=1}^{k} \alpha_i \phi_i (t).	ag{20}\]

then, an adaptation law on \( \alpha_i \) may be used.

**Remark:** The adaptive feedforward for \( w \) is simply the integral control. If additional time varying characteristics of \( u^* \) is known, it is straightforward to incorporate into the adaptive controller.

**Remark:** In addition to robust stability, the adaptive feedforward strategy proposed can also take into account changes in the building behavior. For example a window opening or closing will affect \( F_1 \), which will be counteracted by the adaptive feedforward correction of \( \hat{F}_1 \) to drive \( y \to y_{des} \). Thus, we can assure robust performance of the controller in the presence of typical uncertainties in the building's HVAC System Model

We have considered the heat into each zone as directly controllable, but in practice, heating is provided through the building heating-ventilation-air-conditioning (HVAC) system. There are numerous architectures and design choices for HVAC systems. For example, a model suggested in [8]-[10] consists of a central heating/cooling unit, zone heating/cooling coils, zone dampers, and fan as shown in figure 3. In this paper, we only consider heating, though generalization to heat and cooling is straightforward. In this model, the heat input into each zone is given by

\[
u_i = c_p \dot{m}_{s_i} (T_s - y_i)	ag{21}\]

where \( c_p \) is the specific heat of air (we approximate it as a constant for dry air at 1.0J/g K), \( \dot{m}_{s_i} \) is the air mass flow rate into zone \( i \) and \( T_s \) is the supply air temperature given by

\[
T_s = \rho T_r + (1 - \rho) T_\infty + \Delta T_h
\]

where \( \rho \) is the return-air/outside-air ratio, \( \Delta T_h \) is the temperature increase through the central heater. The return air temperature is assumed to be a weighted average of the zone temperature with weights determined by the mass flow ratio (return air mass flow rate is assumed to be the same as the supply air mass flow rate for each zone, i.e., no accumulation in a room):

\[
T_r = \frac{\sum_i \dot{m}_{s_i} y_i}{\sum_i \dot{m}_{s_i}}	ag{23}\]

Instead of \( u_i \), the control variable is now \( \dot{m}_{s_i} \), consisting of the air mass flow rate into each zone, \( \dot{m}_{s_i} \). However, the same control approach may be applied. We first ensure \( T_s > \max_i y_i \) by adjusting \( T_h \) (in practice, this would be the setpoint for the central heater or boiler control system). A
simple strategy could be the following hysteresis controller (to avoid chattering), as shown in Figure 4:

\[ \Delta \dot{T}_h = \begin{cases} \alpha & \text{if } T_s \leq \max_i y_i + M_1 \text{ and } \dot{T}_s > 0 \\ -\beta & \text{if } T_s > \max_i y_i + M_2 \text{ and } \dot{T}_s < 0 \end{cases} \] (24)

where \( M_2 < M_1 \). This would result in \( T_s - \max y_i \) hovering in the range \([ M_2, M_1 ]\), and reduces unnecessary additional heating.

Once we have \( T_s - T_i > 0 \), we may apply the same passivity controller as before.

**Theorem 3:** Consider the following controller

\[ \dot{\hat{y}} = -K(y - y_{\text{des}}) + \dot{m}_{s_i}^a \] (25)

where \( K \) is memoryless and passive, and the \( i \)th component of \( \dot{m}_{s_i}^a \) is given by

\[ \dot{m}_{s_i}^a = \frac{\hat{u}_i^*}{c_p(T_s - y_i)} \] (26)

with \( \hat{u}_i^* \) given by either the feedforward \( u^* \) in (8) or adaptive feedforward \( \dot{u}_i^* \) in (15). Then the closed loop system is stable and \( y \to y_{\text{des}} \) asymptotically.

**Proof:** Substitute (25) into (21), we have the same controller as in (11) or (14) except that the controller \( K \) is now a memoryless time varying gain, \( K = \text{diag}(T_s - y_i) \). The stability result then follows from Theorem 1 or Theorem 2. Note that upper and lower bounds on \( \dot{m}_{s_i} \) may be imposed without affecting stability. When \( \dot{m}_{s_i} \) is in saturation, the effective gain is reduced.

For example, consider a quadratic objective function

\[ J = \frac{1}{2}(y_{\text{des}} - y^*)^T \Lambda(y_{\text{des}} - y^*) + R_1^T \dot{m}_s + \frac{1}{2} \dot{m}_s^T R_2 \dot{m}_s \] (27)

where \( y^* \) is the ideal zone temperature, \( \dot{m} \) is given by the control law (25), \( \Lambda \) and \( R_2 \) are positive definite weight matrices, and \( R_1 \) is a vector of positive entries. For simplicity, we only focus on the \( y_{\text{des}} \) dependence in the feedback portion of (25). The update of \( y_{\text{des}} \) along the gradient descent direction of \( J \) is then given by

\[ y_{\text{des}} = -a_i(\Lambda_i(y_{\text{des}} - y^*) + K_i R_1^i + K_i R_2 \dot{m}_s) \] (28)

where \( a_i \) is the update gain. A lower bound of \( y_{\text{des}} \) should also be specified for the minimum acceptable temperature level. The effect of \( R_1 \) is to cause \( y_{\text{des}} \) to fall below the target value \( y^* \) and \( R_2 \) regulates this drop based on the flow rate (lower \( y_{\text{des}} \) would in turn reduce the flow rate). Note that when there is no power (energy) consumption cost, i.e., \( R_1 = R_2 = 0 \), then we recover the output tracking case with \( y_{\text{des}} = y^* \).

**III. SIMULATION EXAMPLE**

To illustrate results in the paper, consider a four-room temperature control example as shown in Figure 5. This example is taken from [11], with the added heat transfer to the ambient for all the rooms. We have also used this example in our previous work [2]. For the 4 rooms and 8 walls, the number of capacitive elements is \( N = 4 + 2 \times 8 = 20 \). There are 27 thermal resistance elements, so \( L = 27 \). Hence, the dimension of the incidence matrix \( D \) (without the ambient node) is \( 20 \times 27 \). For the purpose of numerical simulation we assume that the dimensions of two larger rooms are \( 3m \times 4m \), and the two smaller ones are \( 3m \times 3m \). The passages between the rooms have a width of 1m, the rooms are all 2.5m high and the walls are assumed to be 15cm thick. Further, we assume that the insulation (thermal resistance) of the material used for walls of room 2 is poorer than the other rooms, to simulate for example, a glass paneled room. Using values of volumetric heat capacities from [12] and values of thermal resistances from [13], we have a model of the form as in (1–2).

![Fig. 3. HVAC architecture of system](image)

![Fig. 4. Hysteresis control of supply air](image)

**F. Optimization for Energy Efficiency**

So far, we have considered the output tracking control problem with specified \( y_{\text{des}} \). However, \( y_{\text{des}} \) may be elastic depending on the cost of energy. We may choose \( y_{\text{des}} \) to balance between user comfort and energy usage by choosing \( y_{\text{des}} \) to minimize the instantaneous comfort and power cost

\[ J_{\text{comfort}}(y_{\text{des}}) + J_{\text{power}}, \text{ where } J_{\text{power}} \text{ depends either on } u \text{ (for direct room heating control) or } \dot{m}_s \text{ (for air flow rate control)}, \text{ which in turn depend on } y_{\text{des}} \text{ through the control law.} \]
zone temperatures set at $y^* = 20\,^\circ C$ and the feedback gain $K = 0.05$. The maximum mass flow rate is set at 2Kg/s.

The adaptation gains are chosen to be $\Gamma_1 = 5$ and $\Gamma_2 = 0.1$. First set $y_{des}$ to be just $y^*$. In this case, we may combine the $F_0$ term with $\hat{w}$, i.e., $\Gamma_0 = 0$. The heater, $\Delta T_h$, is controlled by (24) with $\alpha = 0.1$, $\beta = 0.05$, $M_1 = 5$, $M_2 = 1$.

When $y_{des} = y^*$, the adaptive control almost completely compensated for the ambient temperature variation, as shown in Figure 7. The mass flow rate is at maximum initially to warm up the room and then reduces to a steady state level to maintain the room temperature, as shown in Figure 8. At around $t = 35$ hr, the outside temperature becomes higher, and the flow rate is reduced, as expected. Note that the mass flow rate for room 2 is the highest due to its poor insulation. The supply air and heater temperatures track the ambient temperature variation to maintain steady room temperature level as shown in Figure 9. The oscillation in the flow rate is due to the hysteresis supply air temperature control.

When the power consumption terms are included, with $R_2$ set at 100, the $y_{des}$ is moved away below $y^*$ as shown in Figure 10. As Room 2 is poorly insulated, lowering its desired temperature would most significantly affect the energy usage. Again at around $t = 35$ hr, the desired room temperatures increase as the outside air warms up. The temperature controller tracks the desired room temperature profile closely as shown in Figure 11. As a result of increasing the energy penalty, the energy consumption over 2 days is reduced by 16%.

The RC lumped model is only a coarse approximation of the actual room temperature distribution and evolution. To verify the applicability of the analysis to a higher fidelity distributed model, we use the computational fluid dynamics (CFD) package Fluent [14] to simulate the temperature control in a 3D model of the four-room example. The ambient temperature and the initial room temperatures are...
set to 12°C. Proportional feedback is used to adjust the mass flow rate into each room (restricted to be between 0 and 0.1 Kg/sec). The temperature sensor is assumed to be a point sensor located at the center of the room. The temperature distribution after one minute, five minutes, and one hour are shown in Figure 12. The air inlets are assumed at the corners of the room, showing heated air entering the rooms. After one hour, the room distribution mostly reaches the specified temperature set point. As seen in the five-minute snapshot, the poorly insulated room 2 takes longer to heat up to the desired temperature. The averaged vs. center temperature evolution in each room is shown in Figure 13. It is clear that the proportional feedback to the mass flow rate is effective to steer the overall room temperature to the set point.

**Fig. 12.** Room temperature distribution after one minute, five minutes, and one hour, using Fluent CFD simulation

**Fig. 13.** Averaged and centroid room temperature evolution, using Fluent CFD simulation

**IV. CONCLUSION**

This paper presents a passivity based adaptive control strategy for building thermal control. By including all thermal capacitances as nodes, we show that the building thermal control problem is inherently passive. This allows a large class of stabilizing controller to be constructed, as well as adaptation for the feedforward compensation for ambient temperature and room occupancy variations. Additional criteria such as energy minimization may also be included. A four-room simulation example under realistic ambient conditions is illustrated.

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