Optimal stochastic control for parking systems: occupancy-driven parking pricing
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Abstract—Efficient parking management can mitigate both traffic and parking congestion, and reduce the social costs and environmental impact. Parking pricing and information provision relying on parking sensors jointly serve as a dynamic stabilized controller for traffic demand management. Optimal parking pricing is formulated as a stochastic control problem. We obtain measurements of parking occupancy in real time and are used to update the optimal parking prices considering stochasticity on demand and travelers’ parking choice. The stochastic control formulation is solved using dynamic programming. There exists a critical occupancy for each time period, beyond which the parking prices should be set effective (namely, not free of charge). The numerical experiments show that the optimal parking policies based on stochastic control models can deal with different demand levels and generally outperforms the deterministic pricing schemes.

I. INTRODUCTION

Efficient parking management can mitigate both traffic and parking congestion, and reduce social costs and environmental impact of transportation. The objective of this paper is to investigate how sensing can be used to analyze parking behavior in response to pricing, and in turn, how pricing can be set with prior knowledge of parking behavior. Therefore, we propose an optimal parking pricing strategy relying on real-time sensing.

Either pricing or sensing in parking has been discussed in the literature [1], but previous studies usually investigate each of them separately (mostly pricing), and their relationships are not fully understood yet. There are several descriptive and empirical studies on parking policies regarding pricing and other types of management [2], [3], [4], while theoretical studies on parking modeling are few, and most of them focus on the steady-state pricing [5], [6], [7]. Dynamic parking pricing was modeled in various recent papers [8], [9], [10], but non-recurrent traffic was not considered, and therefore the advantage of dynamic pricing cannot be achieved when real-time information is available.

Starting almost two decades ago, parking guidance information (PGI) systems have been deployed in many cities. One of the features in PGI is to provide historical or real-time parking occupancy information. The parking occupancy information is mostly regarding the number of available spaces in certain parking garages. In most cases, occupancy information is provided through variable message signs (VMS), traffic radio or phone applications. Although such occupancy information has the potential of improving parking efficiency and reducing cruising cost [11], empirical studies and simulations show that the benefits of solely relying on this parking availability information may be marginal in congested networks [12]. This is mainly because such information provision does not influence travelers’ decisions on where to park and when to leave. Providing the occupancy information alone is usually not sufficient to improve the transportation system. Sensing should be connected to real-time demand “control”. The real-time occupancy data should be analyzed to reflect the real-time parking demand and therefore a corresponding real-time pricing strategy can be set to efficiently direct travelers to different parking lots. In that sense, traffic demand can be controlled in a more efficient way by sensing and pricing.

The main idea of this paper is to model optimal parking pricing as a stochastic control problem. Relying on parking sensors, parking pricing and information provision jointly serve as a dynamic stabilized controller for the traffic demand management. We obtain the measurement on parking occupancy by sensors in real time. The occupancy is then analyzed for a prediction of future demand and flow diversion (to parking lots), which in turn determines an optimal parking price for the next time period. All the parking information along with parking prices, is then provided to travelers to help them make real-time parking choices. The diversion of traffic flow to parking lots will further be detected by parking sensors in the parking occupancy at the end of the next time period. Overall, parking pricing and sensing alternate and control the traffic jointly.

We consider two types of stochasticity, demand uncertainties and traveler heterogeneity. First, demand varies from day to day. Significant demand change in response to flow disruptions, such as incidents, accidents and events, is expected. The optimal prices should be set to minimize the average system cost from day to day. Second, users may respond to the same parking price differently, as reflected by their respective heterogeneous Value of Time (VOT). This paper proposes a dynamic solution to the optimal parking pricing problem accounting for both types of stochasticity.

This paper is organized as follows. We first propose the basic parking model, notations and set-ups in Section 2. Next we discuss how to use prices to control the flow diversion to each of the parking lots in Section 3. It is then followed in Section 4 by the solution derivation and algorithm. We apply our model to an experimental example in Section 5. We illustrate the optimal pricing strategy in the numerical case, as well as its performance compared to the benchmark.
II. PARKING MODEL

This paper considers a simplified parking network as depicted in Figure 1 to mimic commuters’ parking choices. The parking spaces near the destination are generally divided into two parking clusters (or areas), cluster 1 and cluster 2. Cluster 1 contains parking spaces in the central area, while cluster 2 is generally located in the peripheral area of the destination. Spaces in cluster 1 are usually more convenient and expensive than cluster 2.

\[ \tau_2 = \varphi_t(p_t, \alpha_t) \]  

(6)

If he chooses to park in the closer cluster, but cannot find a spot and finally returns to the farther cluster to park, his generalized travel time is

\[ C^1(t, \alpha_t) = \tau_2 + (2\xi + \xi), \]  

(4)

where \( \xi \) is the searching time spent on cluster 1. \( 2\xi + \xi \) can be seen as the additional travel time for being unfortunate to choose the closer cluster.

III. STOCHASTIC CONTROL FORMULATION

A parking system collects real-time parking occupancy, which conveys the demand intensity and reveals parking choices made by commuters so far. We can then set parking prices to balance the supply and demand of the parking in real time. The impact of pricing is then detected by parking sensors as time progresses. Therefore, pricing and sensing act jointly. The ultimate planner’s goal is to minimize the total system cost, namely the total system travel time (TTT) in this case.

Consider a dynamical parking system as follows. At the beginning of each time period \( t \), the up-to-date parking occupancy of the closer cluster, \( k_{t-1} \), is collected from the sensors. The real-time occupancy, \( k_t \), denotes the state of the system at stage \( t \). Let the set \( Y_t \) for \( t = \{0, 1, \ldots, T\} \) denote all the information that is available to us at the beginning of stage \( t + 1 \) (i.e., in the end of stage \( t \)). Note that \( Y_0 \subseteq Y_1 \subseteq \cdots \subseteq Y_T \).

At the beginning of stage \( m + 1 \) (i.e. the end of stage \( m \)) and \( m = \{0, 1, \ldots, T - 1\} \), information \( Y_m \) becomes available. The up-to-date occupancy \( k_m \) and the traffic demand up to time \( m, \lambda_m \), are revealed as well as part of the information, namely, \( k_m \in Y_m, \lambda_m \in Y_m \). The conditional distribution of the future parking demand, \( \{\lambda_{m+1}, \lambda_{m+2}, \ldots, \lambda_T | Y_m\} \) can then be determined.

Given the information \( Y_m \) at the beginning of stage \( m + 1 \), the parking system makes a decision on the optimal parking fee for the closer cluster, \( p_{m+1} \). The parking occupancy \( k_m \) and the prices \( p_{m+1} \) being executed during time period \( m + 1 \) will be provided to all the commuters at the beginning of stage \( m + 1 \). Thus pricing and sensing are using in alternating fashion until the end of the analysis time horizon.

From now on, we write the future demand \( \lambda_t \) given the information \( Y_m \) as

\[ \lambda_t^m = \lambda_t | Y_m, \forall t > m. \]

The random dynamics of the parking system is

\[ k_t = \min \{k_{t-1} + x_t \lambda_t, K\} \]  

(5)

where the number of spaces used in the closer cluster during time period \( t \) is the number of travelers directed to it, subtracted by those who are unable to find a spot due to limited capacity. The proportion of travelers that are diverted to the closer parking cluster \( x_t \) during time period \( t \) is a function of the parking prices \( p_t \) broadcast at the beginning of that time period, and also dependent on the the distribution of the VOT during that time period \( \alpha_t \).

\[ x_t = \varphi_t(p_t, \alpha_t) \]  

(6)
The dynamic pricing policy \( \phi_t(\cdot) \) at stage \( t \) is dependent on the up-to-date occupancy \( k_{t-1} \) and population attributes \( \text{VOT} \).

\[ p_t = \phi_t(k_{t-1}, \alpha_t) \] (7)

The stage cost of the time period \( t \), represented by \( \ell_t \), is defined as the total generalized travel time during time period \( t \). Given the available information at the end of stage \( t-1 \), the stage cost for a future stage \( t \) is dependent on the decision variable \( p_t \) and the future demand,

\[ \ell_t(p_t, \lambda_t)Y_{t-1} = \tau_1\lambda_{t-1}^{-1} + \tau_2\lambda_{t,2}^{-1} + (\tau_2 - \tau_1 + 2\zeta + \xi + \epsilon)(k_{t-1} + \lambda_{t,1}^{-1} - K) \] (8)

where \( k_{t-1} + \lambda_{t,1}^{-1} - K \) is the number of travelers that are directed to the closer cluster during the time period \( t \) but are unable to find a spot there. They are subject to the round-trip driving time between the two clusters (2\( \zeta \)), cruising time \( \xi \), additional composite travel time \( \tau_2 - \tau_1 \) and an additional penalty to the system \( \epsilon \). The stage cost \( \ell_t \) consists of the planned total travel time and the additional travel time due to limited capacity of the preferred parking cluster. \( \epsilon \geq 0 \) should be assigned by the parking operator as a penalty for directing travelers to a fully occupied parking lot. Given the desire to reduce the total cost over all the stages, a greater \( \epsilon \) decreases the risk of directing commuters to a fully packed parking cluster, but it may also increase the total stage cost as commuters are more likely to be directed to the farther cluster even if the closer one is not fully used.

Let \( \tau_2 - \tau_1 + 2\zeta + \xi + \epsilon = \eta \) and substitution \( \lambda_t \) using Equation (1), we have,

\[ \ell_t(p_t, \lambda_t)Y_{t-1} = \tau_2\lambda_{t-1}^{-1} - \Delta \tau \lambda_{t-1} \tau_t x_t \] (9)

The objective of the parking control system is, at the beginning of any stage \( m \in \{0, 1, \ldots, T-1\} \), to minimize the expected value of the sum of the stage costs up to a horizon \( T \),

\[ \min_{p_m \geq 0} J_m = \sum_{t=m}^{T} \mathbb{E}(\ell_t(p_t, \lambda_t^{m-1})|Y_{m-1}) \] (10)

The expectation is taken over the forecast traffic demand \( \{\lambda_{m-1}^{m-1}, \lambda_{m+1}^{m-1}, \ldots, \lambda_{T-1}^{m-1}\} \) and the VOT distribution \( \{\alpha_m, \alpha_{m+1}, \ldots, \alpha_T\} \).

For any time period \( t \), the perceived travel time of travelers preferring the farther cluster and the closer one is \( \tau_2 \) and \( \tau_1 \), respectively. The parking price of both clusters is \( 0 \) and \( p_t \), respectively. Any travelers whose VOTs satisfy \( \alpha \leq \Delta \tau/p_t \) will prefer the farther cluster (and use it), while those with VOT of \( \alpha \geq \Delta \tau/p_t \) prefer the closer one (but may not be able to find an available spot). Thus,

\[ \mathbb{E}(x_t) = Pr(\alpha_t \geq \frac{\Delta \tau}{p_t}) = \int_{\Delta \tau/p_t}^{\infty} f_{\alpha_t}(y)dy \] (11)

where \( f_{\alpha_t}(\cdot) \) is the probability density function of VOT \( \alpha_t \) during time period \( t \).

Since price \( p_t \) and expectation of diversion ratio \( x_t \) is one-to-one mapping, the objective function (omitting the expectation over \( \alpha_t \)) can be converted with respect to the diversion ratio \( x_t \),

\[ \min_{0 \leq x_t \leq 1} J_m(k_{m-1}) = \sum_{t=m}^{T} \mathbb{E}(\ell_t(x_t, \lambda_t^{m-1})|Y_{m-1}) \] (12)

The parking price \( p_m \) is then represented by a function of \( x_m \) due to Equation (11). We make a decision on the proportion of travelers being directed to the closer cluster at every stage, and this is equivalent to the decision on the parking prices \( p_m \) given that we know the distribution of VOT at stage \( m \). The decision variable \( x_m \) is only dependent on the up-to-date occupancy.

\[ p_m = \phi_m(x_m), x_m = \varphi_m(k_{m-1}) \] (13)

IV. OPTIMAL PARKING PRICING

Without loss of generality, we can solve the stochastic control problem (14) by letting \( m = 1 \). For any stage \( m \geq 1 \), we simply update the distribution of forecasted future demand \( \{\lambda_{m-1}^{m-1}, \lambda_{m+1}^{m-1}, \ldots, \lambda_{T-1}^{m-1}\} \), and use the same solution algorithm to produce the optimal decision variable (or equivalently the optimal parking price) at stage \( m \). In this section, we solve for the optimal policy at the beginning of stage \( 1 \) with information revealed \( Y_0 \).

\[ \min_{0 \leq x_t \leq 1} J_1(k_0) = \sum_{t=1}^{T} \mathbb{E}(\ell_t(x_t, \lambda_t)|Y_0) \] (14)

For simplicity in the solution derivation, we omit the superscript 0 for the forecasted demand \( \{\lambda_1^{0}, \lambda_2^{0}, \ldots, \lambda_T^{0}\} \). But keep in mind that the demand is forecasted based on the information revealed so far, \( Y_0 \).

A. Optimal solution using dynamic programming

The optimal value functions (also known as cost-to-go functions) are defined as,

\[ J_t^*(k_{t-1}) = \min_{0 \leq x_t \leq 1} J_t(k_{t-1}) \]

\[ = \min_{0 \leq x_t \leq 1} \mathbb{E}(\ell_t(x_t, \lambda_t) + J_{t+1}^*(k_t)|Y_{t-1}) \] (15)

The optimal policy and the optimal value functions can be found by a backward iteration involving the Bellman operator. The terminal cost-to-go function is

\[ J_{T+1}(k_T) = 0 \] (16)

Standard dynamic programming yields the following lemma [13].

Lemma 1.1: For every initial parking occupancy in the closer cluster \( k_0 \), the optimal total travel time \( J_t^*(k_0) \) can be calculated iteratively with Equation (15) and Equation (16). Furthermore, if \( x_t^* = \varphi_t^*(k_{t-1}) \) minimizes \( J_t(k_{t-1}) \) in Equation (15) for each \( t \) and \( k_{t-1} \), then the policy \( x_0^* = \varphi_0^*(k_0) \) is the optimal solution to Problem (14).
We let \( f_{\lambda_t}(\cdot) \) denote the probability density function of demand \( \lambda_t \) during time period \( t \), and define that
\[
G_t(y) = \int y z f_{\lambda_t}(z) dz \quad (17)
\]
\[
\bar{G}_t(y) = \int 0 y z f_{\lambda_t}(z) dz = \mathbb{E}_{\lambda_t}(\lambda_t) - G_t(y) \quad (18)
\]
\[
F_t(y) = \int y f_{\lambda_t}(z) dz \quad (19)
\]
Now we state the optimal control solution in the following propositions, and they are proven in [14].

**Proposition 1.1: Optimal solution for the last time period** \( T \). The optimal solution is,
\[
x_T^* = \varphi_T^*(k_{T-1}) = \begin{cases} 1 & \text{if } 0 \leq k_{T-1} < K - C_T \\ \frac{K - k_{T-1}}{C_T} & \text{if } K - C_T \leq k_{T-1} \leq K \end{cases}
\]
where \( C_T \) is a positive real number dependent solely on the distribution of the demand during stage \( T \), and satisfies,
\[
G_T(C_T) = \mathbb{E}_{\lambda_T}(\lambda_T) \frac{\Delta T}{\eta} \quad (20)
\]
**Proposition 1.2: Approximate optimal solution for any time period** \( t < T \). The optimal solution can be approximated by
\[
x_t^* = \varphi_t^*(k_{t-1}) = \begin{cases} 1 & \text{if } 0 \leq k_{t-1} < K - C_t \\ \frac{K - k_{t-1}}{C_t} & \text{if } K - C_t \leq k_{t-1} \leq K \end{cases}
\]
where \( C_t \) is a positive real number dependent solely on the distribution of the demands during stages \( t, t+1, \ldots, T \), and satisfies,
\[
G_t(C_t) + V_{t+1} \bar{G}_t(C_t) = \mathbb{E}_{\lambda_t}(\lambda_t) \frac{\Delta T}{\eta} \quad (21)
\]
\[
V_t = F_t(C_t) + V_{t+1}(1 - F_t(C_t)), \forall t, \text{ and } V_{T+1} = 0 \quad (22)
\]
, if \( F_t(C_t - C_{t+1}) \approx 1 \) for any \( t \).

Proposition 1.2 gives the approximate optimal solutions that preserve the convexity of the minimization problem. The approximation error is very small when certain conditions (described in Proposition 1.2) holds.

**B. Properties of the solution**

The solution described in Proposition 1.2 has strong policy indications. \( C_t \) is identified as the **critical occupancy** for the closer parking lot. Depending on our forecast of future demand and population attributes, the critical occupancy changes over time with respect to the occupancy. We see that when the pricing control is effective (e.g. not free), the relation between the proportion of travelers directed to the preferred cluster and the number of remaining vacant spots is positively proportional. Note that the relation between the prices and the number of vacant spots are then not linear, since the price is a nonlinear function of \( x_t \) dependent on the prediction of the demand distributions. This policy is approximately optimal in general for any time stage, which can serve as the pricing strategy in principle for any parking clusters. Such pricing policies can be rather efficient if we have sufficient data to support a good estimation/prediction of the demand and VOT distributions.

**V. Numerical Experiments**

In this section, we quantitatively characterize stochastic parking pricing policies in an experimental setting, and measure its effectiveness in reducing the total travel time compared to the deterministic optimal parking pricing.

Suppose we have a parking lot with total capacity of 1000 spaces. This lot serve the commuting demand which is greater than the space capacity. The synthetic data is as follows. We assume 15-min demand (between 7am to 10am) follows a normal distribution for each 15-min period. In this numerical experiment, we simply assume the forecast future demand distribution at each stage stays the same. The demand mean increases in the beginning of the peak hour and reduces afterwards. The value of time \( \alpha \) is assumed to follow a normal distribution where the mean increases linearly over time while the variance remains the same. The average value of time is set to 35 US dollars per hour (approximately the average hourly pay rate on campus). The mean and variance of the 15-min demand and VOT (demand is rounded to the nearest integers) are shown in Table I. Other parameters are set to: \( \tau_1 = 5\text{min}, \tau_2 = 20\text{min}, \zeta = 5\text{min}, \xi = 10\text{min}, \epsilon = 10\text{min} \).

**TABLE I**

<table>
<thead>
<tr>
<th>Time period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand mean (veh)</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>130</td>
<td>150</td>
</tr>
<tr>
<td>Demand S.D. (veh)</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>VOT mean ($/15 min)</td>
<td>6.9</td>
<td>7.1</td>
<td>7.3</td>
<td>7.5</td>
<td>7.7</td>
<td>7.9</td>
</tr>
<tr>
<td>VOT sd ($/15 min)</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

A. **Optimal parking policies**

We first use Propositions 1.1 and 1.2 to compute the optimal parking policies that are determined by the up-to-date occupancy for each of the 15 min time periods. The critical occupancy and the optimal policies are shown in Figure 2.

The critical occupancy of the parking lot ranges approximately from 78% to 94%. In general, for each time period, the more demand we predict, the smaller critical occupancy we would expect. For instance, in the last stage, the demand is significantly lower than other stages, for merely 50 vehicles on average. Thus the critical occupancy is highest, at 94%. If we have more than 60 parking spaces available in the closest lot at time 9:45am, and given our knowledge about the incoming demand (on average 50 vehicles with variance 15 vehicles), we should set the parking price identical to the farther one so that all travelers prefer the closer one. Moreover, we predict a high demand in time period 6 with mean 150 vehicles and standard deviation 25 vehicles, and
the critical occupancy for that period is low, at 78%. The less vacancy we have, the more diversion we would need for the purpose of system optimum.

The optimal parking pricing policies are such that the diversion portion to the closer cluster is linearly decreasing with respect to the remaining vacancy once the observed occupancy is above the critical occupancy.

B. Comparisons to deterministic pricing schemes

We now investigate the effectiveness of the optimal stochastic parking pricing by comparing it to those deterministic pricing schemes. We will randomly sample the traffic demand for consecutive 1000 days, and compare the true total travel time among those pricing schemes over the entire analysis horizon.

We sample the demand randomly from the normal distribution for 1000 trials, and compute the total travel time resulting from stochastic pricing policy and the two deterministic pricing schemes. The results are shown in Table II. Over the entire course of 1000 day, the time-varying parking policies produce merely minor congestion (1 hours total) compared to the benchmark case, and the closer parking lot is always fully used (terminal occupancy 1). However, if we use the deterministic pricing scheme, the resulted additional travel time (consisting of parking congestion and additional walking time) could be large, evaluated at 2507 hours if real-time occupancy information is provided, or 2829 hours if no occupancy information is provided. We find that the closer parking lot is sometimes under-used, and its overall terminal occupancy is 94% for both deterministic cases.

| Total travel time for various pricing schemes (1000 days) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| Scheme                          | Benchmark       | Stochastic      | D1              | D2              |
| Total travel time (hr)          | 165,425         | 165,426         | 167,933         | 168,255         |
| Terminal occupancy (%)          | 100%            | 100%            | 94%             | 94%             |
| Additional travel time (hr)     | -               | 1               | 2,507           | 2,829           |

Note: “D1” is the deterministic pricing scheme that provides occupancy information to travelers, while “D2” is the pricing scheme without such provision.

To sum up, we see that the optimal parking policies based on stochastic control models can deal with random demand levels and approach the best possible system performance. Adopting the occupancy-driven parking policies is significantly more efficient than using the deterministic pricing schemes. Occupancy-driven parking policies can “learn” the real-time supply/demand and set the parking prices accordingly as the feedback to the demand, so that the incoming demand will approximately match the remaining supply in the preferred parking cluster. This is very close to the case as if we know exactly the parking demand in advance. The occupancy-driven parking pricing policies are efficient in managing the traffic and reducing the parking congestion.

For the deterministic pricing scheme, providing real-time information alone (without setting the occupancy-driving parking prices) seems useful, but not compelling, to reduce the cruising and parking congestion. The numerical experiments show that the improvement to the total travel time.
is around 12%. This is essentially because the deterministic pricing scheme works the best for the demand consistent with the historically average demand. If the demand is greater than the historical average, the parking congestion in the closer lot usually occurs in the very end of the commuting period. Therefore, providing the real-time occupancy information alone does not reduce as much the parking congestion until the end. In addition, if only the occupancy is provided without setting appropriate parking prices, a fraction of travelers are still willing to search for a convenient parking space as long as there are several vacant spots. Providing real-time information alone does not necessarily improve the system performance as much as with coordination with the dynamic parking policies.

VI. CONCLUSIONS

This paper proposes management of parking demand using sensing and dynamic pricing. We model the optimal parking pricing as a stochastic control problem. The measurement of parking occupancy is obtained from parking sensors in real time. The occupancy is then used to predict the future system cost, which in turn determines an optimal parking prices in real time. Parking information along with parking prices are provided for travelers to make real-time parking choices. As time progresses parking prices are adjusted based on the occupancy. We take into account two types of randomness to produce the optimal policy: demand uncertainties and user heterogeneity in Value of Time.

The final optimal parking policy is such that for each time period, there exists a critical occupancy. The parking prices should be set effective when the up-to-date occupancy is above the critical occupancy. The diversion portion to the closer cluster is linearly decreasing with respect to the number of remaining available spaces. The theory is easy to implement in real cases and thus is practically useful. Numerical experiments show that the optimal parking pricing considering uncertainties generally outperforms the deterministic pricing schemes. It can approach the minimum possible total travel time in most of the cases as if we know the true traffic demand in advance of the commuting time. Occupancy-driven parking policies are able to “learn” the real-time supply/demand over time, set the prices adaptive to the real-time usage.

In future research, we will extend our research to accommodate multiple parking clusters. Commuters may follow a particular sequence when searching for preferred parking lots among those clusters. The parking prices of each cluster are not only dependent on its own occupancy, but also the coordinations between those lots to achieve the system optimum. Last but not least, multiple objectives for parking management other than the total travel time should be discussed in the future research, such as total travel cost, revenue of the parking operators and so forth.

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