Adaptive Mirror Control for an Optical Resonator Cavity

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Abstract—An adaptive mirror control strategy for tuning an optical resonator is described. The control approach is model-free and uses a small-amplitude perturbation signal to enable real-time estimation of the cost function gradient. The strategy has similarities with the approaches of ‘stochastic approximation’ and ‘extremum seeking control’, which are well-known methods in the control community. Particular fine-tuning of the algorithm, such as perturbation filtering, cost function shaping and disturbance feedforward are discussed. The control strategy has been validated on a (quasi-optical) millimetre wave resonator system. A significant disturbance acting on this set-up is the non-stationary frequency of the wave source (gyrotron). In various experiments it has been demonstrated that the control approach is well capable of keeping the optical cavity in resonance, to track wavelength variations and to reject disturbances in the low and mid-frequency range. The only performance limitation of the approach occurs in cases which require fast disturbance tracking. This is due to the bandwidth restrictions in the underlying mirror motion system.

I. INTRODUCTION

Optical resonators are widely applicable for laser systems, interferometry and optical wave filtering. Generally, optical resonators have a high Q-factor, such that the spectral transmission properties are characterised by a narrow passband and a wide stopband. For that reason accurate tuning of the resonator towards the desired passband frequency (or set of passband frequencies) is essential. This particularly holds in the presence of external disturbances.

A basic form of an optical resonator is the plane-parallel Fabry-Perot cavity, of which a sketch is shown in Fig. 1. An incoming wave is resonating between the two mirrors and is partially reflected and partially transmitted by the resonator cavity. The distribution of reflected power and transmitted power depends - among other factors - on how well the wavelength fits within the optical pathlength of the cavity.

![Diagram of a basic plane-parallel Fabry-Perot resonator (with equal mirrors)](image)

Fig. 1. Diagram of a basic plane-parallel Fabry-Perot resonator (with equal mirrors) and a wide stopband. For that reason accurate tuning of the resonator towards the desired passband frequency (or set of passband frequencies) is essential. This particularly holds in the presence of external disturbances.

A. Transmission Properties

For the plane-parallel Fabry-Perot resonator of Fig. 1 the transmitted and reflected power transfer functions can be written as:

\[
H_T(kL) = \frac{(1 - R_m)^2 \sqrt{R_{ca}}}{1 + R_m^2 R_{ca} - 2 R_m \sqrt{R_{ca}} \cos(kL)}
\]

(1)

\[
H_R(kL) = \frac{R_m}{1 + R_m^2 R_{ca} - 2 R_m \sqrt{R_{ca}} \cos(kL)}
\]

(2)

in which \(H_T\) and \(H_R\) are the power transmission and reflectance functions respectively, as a function of \(kL\), in which \(k = 2\pi/\lambda\) is the wave number and \(L\) the optical resonator pathlength. The coefficient \(R_m\) represents the intensity reflectivity of the cavity mirrors and \(R_{ca}\) the transmission factor per round trip; see also Chapter 4 of [1]. Note that the transfer expressions only hold for the operational range of the resonator, in particular the operational wavelength. An example graph of the power transmission and reflectance curves is shown in Fig. 2. This shows the common transfer behaviour of a resonator with a high Q-factor, having sharp transmission peaks and a wide reflection range. Here \(\Delta(kL)\) is the detuning of the resonator from its nominal, resonant state; \(\Delta(kL) = kL - (kL)_{\text{res}}\).

B. Disturbances

For most applications the resonator needs to be tuned to be ‘in resonance’, which means a maximum \(H_T\) and a minimum \(H_R\). This holds when the resonator optical pathlength \(L\) is an exact integer multiple of the wavelength \(\lambda\). In practice,
disturbances may drive the resonator out of resonance. These are for instance:

1) Structural vibrations acting on the resonator cavity.
2) Expansion of the cavity due to a temperature gradient.
3) Non-stationarity in the frequency of the wave field source.

In order to reject the disturbances described above, the cavity pathlength can be adapted such that the ideal condition of $\Delta(kL) = 0$ is maintained. This could be achieved for instance by making one the resonator mirrors movable. Denoting the mirror position by $x$ and the mirror movement by $\Delta x$, the relation between the change in cavity geometric pathlength and the mirror position is:

$$\Delta L = 2 \frac{\Delta x}{\cos \theta}$$ (3)

in which $\theta$ is the angle of incidence of the wave field on the movable mirror. To maintain the resonator’s functionality, $\Delta x$ should be small such that the resultant deviation from the nominal incident angle at the mirrors is negligible.

II. ADAPTIVE CONTROL OF THE MIRROR POSITION

The main objective is to control the mirror position $x$ such that cavity resonance is achieved, throughout its operation and in the presence of external disturbances. Denoting the resonator transmitted power $P_T$ as $P_T = H_T P_{in}$ - where $P_{in}$ is the total input wave field power - and the resonator reflected power $P_R$ as $P_R = H_R P_{in}$, a straightforward and suitable cost function $J$ would be the normalised resonator reflected power:

$$J = \frac{P_R}{P_T + P_R}$$ (4)

Note that for a (near) loss-free cavity the sum of $P_T$ and $P_R$ is (nearly) constant and independent of $\Delta(kL)$.

Referring to (1), (2) and (3) the map $J = f(x)$ is non-linear and static. Since in practical cases: $0 < R_m < 1$ and $0 \leq \theta < \pi/2$, the function $f(\cdot)$ is smooth and at least 2 times differentiable. In the range $[-\pi < \Delta(kL) \leq \pi]$ $f(\Delta(kL))$ has a unique global minimum $J^* = f(x^*)$ for which it holds:

$$\frac{\partial J(x)}{\partial x} = 0 \quad \text{for} \quad x = x^*$$
$$\frac{\partial^2 J(x)}{\partial x^2} > 0 \quad \text{for} \quad x = x^*$$

Based on complete and exact information of $f$ and the physical parameters of the resonator cavity, the optimal mirror position $x^*$ could be determined directly to yield the minimum $J^*$. In practice, however, we have to assume that $f(\cdot)$ and the cavity parameters are unknown or contain model uncertainties. To find the minimum of $J$ the method of gradient descent could be selected as optimisation algorithm. In its basic form it reads:

$$x(n + 1) = x(n) - \alpha(n) \hat{g}(x(n))$$ (5)

where $\hat{g}(x(n)) = \frac{\partial J(x)}{\partial x}|_{x=x(n)}$, in which the real-valued step-size parameter $\alpha(n) > 0$ and the integer $n$ is the sampling index. This algorithm requires accurate knowledge of the cost function gradient $g(x)$ and under the assumptions stated above (smoothness of $f$ and no local minima) it will converge to the optimal $x^*$ under reasonably wide conditions. In practice however, exact information of the cost function’s gradient is unavailable and neither is a direct measurement of $g(x)$. The mere information available is a measurement of $J(x)$, possibly corrupted by noise and disturbances. To minimize $J$ we will follow a recursive gradient approach from the family of stochastic approximation algorithms, in which an approximate gradient is used in the update equation; see for instance [3].

$$x(n + 1) = x(n) - \alpha(n) \hat{g}(x(n))$$ (6)

Here $\hat{g}(x(n))$ is an estimate of the true gradient based on measurements of the cost function $J$. In order to acquire an estimate of the gradient, at each iteration a small-amplitude perturbation signal $\delta x$ is added to the mirror position $x$. This perturbation signal $\delta x$ can be written as the product of a small amplitude $\delta A > 0$ and a quadratically normalised, zero mean perturbation signal $x_p$, such that $\delta x(n) = \delta A x_p(n)$, where $\text{AV} \{ x_p(n) \} = 0$ and $\text{AV} \{ x_p^2(n) \} = 1$. The averaging operator $\text{AV} \{ \cdot \}$ is defined as [4]:

$$\text{AV} \{ g(n) \} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N} g(n)$$ (7)

with $m$ a finite integer. The injection of a perturbation on $x(n)$ yields a cost function value $J(x(n) + \delta x(n))$. Evaluating $J(x(n) + \delta x(n))$ around $x(n)$ in a Taylor series approximation gives:

$$J(x + \delta x) = J(x(n)) + \delta x(n) \frac{\partial J}{\partial x}|_{x=x(n)}$$ (8)

$$+ \frac{1}{2} \delta x^2(n) \frac{\partial^2 J}{\partial x^2}|_{x=x(n)} + O(\delta x^3(n))$$

Multiplying by $x_p(n)$ and applying the averaging operator
This gives:
\[
A V \left\{ x_p J(x + \delta x) \right\} \approx A V \left\{ x_p(n) J(x(n)) \right\} (9)
\]
\[
+ A V \left\{ \delta A x_p(n) \frac{\partial J}{\partial x} \bigg|_{x=x(n)} \right\}
\]
\[
+ A V \left\{ \frac{1}{2} \delta A^2 x_p(n) \frac{\partial^2 J}{\partial x^2} \bigg|_{x=x(n)} \right\}
\]
\[
+ O(\delta A^3)
\]  

For a perturbation signal \( x_p(n) \) with zero mean and unit energy this can be simplified to:
\[
A V \left\{ x_p J(x + \delta x) \right\} \approx \delta A \left\{ \frac{\partial J}{\partial x} \bigg|_{x=x(n)} \right\} + O(\delta A^3) \quad (10)
\]

From the above expression it follows that for sufficiently small \( \delta A \), the gradient estimate \( \hat{g}(x(n)) \) can be written as:
\[
\hat{g}(x(n)) \delta A = A V \left\{ x_p(n) J(x(n) + \delta x(n)) \right\} \quad (11)
\]

This result is a basic building block in the approaches of model-free gradient type optimisation, such as Stochastic Approximation [3] and also of the closely related technique of Extremum Seeking Control [4]. By correlating cost functions values with known perturbation signal values the gradient search direction towards the optimum can be retrieved without prior knowledge of the map \( f(\cdot) \).

For a sufficiently small \( \delta A \), the instantaneous product of cost function \( J(n) \) and perturbation \( x_p(n) \) can be inserted in the update rule:
\[
x(n + 1) = x(n) - \mu(n) \left\{ x_p(n) J(x(n) + \delta x(n)) \right\} \quad (12)
\]
in which \( \mu(n) \) is again a step-size parameter that scales with \( \delta A \). The convergence properties of stochastic approximation and extremum seeking algorithms in discrete-time have been studied in detail by many researchers. The conditions for convergence are relatively weak, both for deterministic-type and stochastic-type perturbation signals; see for instance the references [3], [4], [5], [6]. These references however hardly address disturbance tracking issues.

The update rule of (12) represents the basic recursive algorithm for controlling the optical resonator towards resonance. In the next Sections further details of the algorithm are discussed together with the specific conditions of controlling a resonator for high-power millimetre waves in a quasi-optical setting.

III. APPLICA TION TO A MILLIMETRE-WAVE RESONATOR

A. Description of the Resonator

Based on the principle of the Fabry-Perot resonator, the FADIS (FAst DIrectional Switch) resonant diplexer has been developed for high-power millimetre waves; see [2]. It is a quasi-optical system consisting of 4 mirrors which are mounted in a rigid aluminum support structure. Two of the mirrors have gratings to serve as input and output couplers. A schematic diagram of this four-port resonator is shown in Fig. 3.

This type of resonant diplexers has a large field of potential applications, for instance in Electron Cyclotron Resonance Heating (ECRH) systems for nuclear fusion and in plasma diagnostics. The functional applications include (a) the tunable distribution of mm-wave power over the output ports, (b) mode purification, (c) a narrow band-pass filter to discriminate between low power Electron Cyclotron Emission (ECE) signals and high power ECRH in a line-of-sight ECE sensing configuration and (d) the combination of gyrotron power from two input ports into a single transmission line. Further details of the versatile application field can be found in [2], [7], [8].

The transmission properties of the FADIS resonator diplexer are similar to those of the Fabry-Perot resonator (1) and (2), in which - in the terminology of Section I and Fig. 3 - the reflected wave travels to output port 1 and the transmitted wave to output port 2. The nominal values of operation for the FADIS resonator considered here are: \( \lambda = 2.14 \text{ mm} \) and \( L = 2.1 \text{ m} \). Furthermore, the grating scattering coefficients are such that the resonance width (FWHM power) equals 12 MHz. The range for which \( H_R < 0.01 \) amounts to 1.1 MHz in terms of frequency band or 12 \( \mu \text{m} \) in terms of mirror movement.

B. Mirror Position Control

In order to tune the resonator to its desired functionality and to reject external disturbances of the type mentioned in Section I, one of the cavity mirrors has been made movable. A dedicated, mirror drive mechanism allows movement of the mirror in the desired direction. It consists of a voice coil actuator and an elastically deformable parallel leaf spring mechanism to guide the mirror. Note that this type of guiding is friction-free. The leaf spring mechanism has a high stiffness in five degrees of freedom of the mirror, whereas it is compliant for motion in the desired direction. To enable control of the position of the mirror relative to its casing a position sensor has been added. This optical encoder measures the displacement of the mirror base relative to the frame with a resolution of 0.1 \( \mu \text{m} \). More details of the mirror drive mechanism can be found in [9].

Given the mechanical design, the mirror motion system would have a limited bandwidth of \( \approx 8 \text{ Hz} \). This is due to the large weight of the diplexer mirror (around 17 kg) and the stiffness of the parallel leaf spring construction. Using the
encoder signal from the position sensor as input, a feedback control loop has been designed to enhance the mirror motion system bandwidth. The position controller has been tuned on the basis of loop shaping techniques. This low order position controller has led to a closed-loop bandwidth of approximately 100 Hz; see Fig. 4.

Fig. 4. TOP: Open-loop amplitude response of the mirror drive mechanism. BOTTOM: Closed-loop amplitude response of the mirror drive mechanism using low order feedback of the mirror position signal.

C. Resonator Output Feedback Control

The mirror position feedback controller will be used in an inner control loop. For the main resonator performance control, measurements of the output beam powers $P_T$ and $P_R$ are used as inputs to the resonator controller, since these variables directly characterise the resonator behaviour. Moreover, output power feedback will render the required absolute positioning accuracy to the mirror motion system. The cost function $J$ as introduced earlier in (4) serves as a suitable performance metric to recursively drive the resonator to its optimum along the principles of the gradient-based stochastic approximation algorithm (see Section II). The structure of the power feedback set-up is shown in the block diagram of Fig. 5. Following the principles of the

actual mirror position $x$. The perturbation signal must not be correlated to any other signal picked up by the power sensors. The underlying adaptive algorithm does not require any a priori information of the shape of the cost function, although knowledge of the characteristics of the cost function may help to improve the convergence properties (as shown in the next Section). Note that at each sampling instant the outcome of the algorithm (6) is used as a position command to the mirror system.

D. Characterisation of Disturbances

In the introductory description of optical resonators 3 types of external disturbances have been mentioned. For the specific case of the high-power millimetre wave resonator in an experimental nuclear fusion environment the following can be stated.

**Frequency of the mm-wave**: The non-stationarity of the frequency of the high-power mm-wave source (gyrotron) forms the dominating disturbance source. Fig. 6 shows a characteristic time recording of the temporal frequency behaviour of a gyrotron at Wendelstein 7-X. Its main characteristics are the initial frequency drop, which is mainly caused by thermal expansion of the gyrotron cavity and the irregular and fast frequency jumps, possibly caused by mm-wave reflections back to the gyrotron cavity.

![Fig. 6. LEFT: temporal frequency behaviour of a W7-X gyrotron. RIGHT: zoomed detail of the frequency behaviour.](image)

**Thermal deformation**: A straightforward calculation shows that the thermal elongation of the round-trip length $L$ for the FADIS resonator amounts to $5.10^{-5}$ mK$^{-1}$. So, already a step of 1°C in ambient temperature will effectively move the resonator far out of resonance; 50 µm of $\Delta L$ at 140 GHz yields $H_R > 0.5$.

**Structural vibrations**: Regarding structural vibrations acting on the cavity, measurements have shown that these may result in variations of the round-trip length in the order of several µm’s.

IV. FINE-TUNING OF THE ADAPTIVE CONTROL ALGORITHM

A. Selection of Perturbation Signal

The parameters of the perturbation signal play an essential role in the behaviour of the adaptive stochastic approximation algorithm. In this particular case the bandwidth of the (closed-loop) mirror motion system is significantly lower than the sample rate of the resonator power control loop...
(100 Hz vs. 10 kHz). From the candidate signal types, such as sinusoidal, triangular, square wave and stochastic variants, a sine wave has been selected after an extensive set of experiments. The frequency of the sine wave has been maximized to 120 Hz in order to speed up the convergence of the gradient algorithm. The amplitude of the sinusoidal perturbation has been set to a value sufficiently low (order 1 μm) such that it allows for a near-optimal resonator performance (note that $H_R < 0.01$ over a position range of 12 μm).

**B. Filtering**

A basic building block in the family of extremum seeking algorithms is the high-pass filter after the cost function evaluation. This would eliminate the 0-th order term in the Taylor expansion (9); i.e. $\mathbf{A}V\{x(n)J(x(n))\}$. For that reason high-pass filtering would be beneficial for the accuracy of the gradient estimation. However, in this particular case where fast step-wise frequency variations occur, the high-pass filter would also attenuate these disturbances from the cost function evaluation. This is an undesirable side-effect and for that reason the high-pass filter has not been applied in the adaptive control set-up.

The underlying principle of the stochastic, approximated gradient algorithm is to correlate cost function values with perturbations on the control variable. Therefore, it is essential that both variables are temporally aligned. Since the perturbation on the mirror position is a command to the mirror motion system, the closed-loop mirror position transfer function effectively filters the perturbation signal before the static map $f(\cdot)$ from mirror position to cost function $J$ takes place.

To account for this filtering effect, in the estimation of the gradient a filtered version of the mirror perturbation has to be applied:

$$\hat{g}(x(n)) = \mathbf{A}V\{x^p(n)J(x(n)) + \delta_p(n)\} \quad (13)$$

where $x^p(n) = H_m(z)x_p(n)$ and $H_m(z)$ is the closed-loop mirror system transfer function with $z$ the forward shift operator.

**C. Cost Function Shaping**

In minimising cost function (4) towards resonance the derivative of the nonlinear map $J$ to $x$ is effectively a gain in the recursive update loop of $x(n)$. As can be observed in the graph (Fig. 7) of the original $J$-curve this derivative is strongly varying with $x$. This effect slows down the gradient search convergence speed, since the step-size parameter needs to be limited to cope with the largest value of the $J$ derivative. In order to have a more equalised slope of the cost function, for this particular case $J$ can be modified in the following manner:

$$J_2(x) = \frac{P_R}{P_T + P_R} + \gamma \frac{P_R}{P_T} \quad (14)$$

in which the scaling factor $\gamma$ is specifically tuned to normalise $P_R/P_T$ and to equalise the gradient of the cost function. Figure 7 shows the result for $\gamma = 1.3$ after normalisation of $P_R/P_T$.

**D. Simulation Experiment**

The adaptive mirror control algorithm based on the approximate gradient descent algorithm (12), together with the sine wave perturbation signal, the specifically tuned cost function (14) and the filtered perturbation signal in the gradient estimation (13), has been validated in a simulation experiment. Based on a resonator model of the form (1) and (2), an artificial gyrotron wave frequency signal (with an initial exponential decay and 3 fast, step-wise jumps) and a closed-loop mirror motion model with 50 Hz bandwidth, the real millimetre wave experimental conditions could be reasonably well approximated. In Fig. 8 the results are shown. In general the adaptive mirror controller is capable of tracking the frequency curve and keeping the resonator cavity in resonance; i.e. low reflected power and high transmitted power. Only at start-up the controller needs time ($\approx 0.5s$) to converge. This convergence is complicated by the fast initial decay of the wave frequency. Secondly, the controller is not capable of tracking the very fast frequency jumps (both positive and negative). The adaptive algorithm however shows quick recovery to its original performance. Furthermore, the relatively slow mirror response certainly has a retarding effect here as well.

**E. Frequency Signal Feedforward**

The adaptive feedback control method is well capable of keeping the system in resonance. A drawback of the approach is the potential of slow convergence. The online and recursive gradient estimation algorithm requires sufficient measurement data to converge to the correct value. This may hamper the performance, especially in the case of fast gyrotron frequency variations. To enhance the tracking performance, a measurement of the gyrotron frequency signal itself can be used as an additional feedforward input to the mirror control system. Inspection of the transmission expressions (1) and (2) reveals that with known values of the frequency and therefore the wave number $k$, the right mirror position can be calculated to minimise $H_R(kL)$. This would give an instantaneous update of the mirror position, which is likely much faster than the adaptive feedback algorithm can
produce. It should be stressed however, that the feedforward link requires highly accurate knowledge of both the gyrotron wave frequency and the round-trip resonator length $L$. For instance, to get the resonator system into resonance based on the feedforward calculation rule only, $L$ must be known with an accuracy better than 10 $\mu$m. Moreover, this resonator length will vary during operation of the diplexer due to thermal effects. Combined with the adaptive feedback control loop however, the frequency feedforward will improve the tracking speed of the mirror system, whereas the feedback loop will compensate for the potential errors induced by the feedforward link; see Fig. 5 with the feedforward part shown between dashed lines.

V. EXPERIMENTAL RESULTS

The adaptive mirror control system has been integrated with the mirror drive system and the FADIS resonator (version MkIIa [2]). The overall system has been extensively tested and validated in various nuclear fusion experiments with high-power millimetre waves; i.e. with the Wendelstein 7-X (W7-X) ECRH system at IPP Greifswald and at the ASDEX Upgrade tokamak (AUG) at IPP Garching. A selection of these experiments has been reported in previous publications [7], [8], [9], [10], in which the emphasis was put on the impact a controlled FADIS resonator may have in ECRH applications. Here we report on two fundamental high-power experiments with the adaptively controlled mirror, both of which cases deal with controlling the resonator to resonance.

The first test shown took place at W7-X; see Fig. 9. The frequency of the high-power gyrotron source (upper graph) shows a clear non-stationary behaviour with a large initial frequency chirp, followed by a number of fast step-wise frequency jumps. This erratic behaviour is difficult to track for a mirror motion system with a bandwidth of 100 Hz. The performance of the basic adaptive feedback controller - see Fig. 5 - is shown in the lower graphs of Fig. 9. The adaptive control algorithm needs about 0.5s to converge. After this initial phase, the mirror is well capable of tracking the gyrotron wave frequency, which results in a near-maximum power transmission curve over time. Only at the very instants of the wave frequency jumps the controller temporally loses performance.

In a second test the adaptive control system was extended with a frequency feedforward link, as shown in Fig. 5. With a properly tuned feedforward controller, the system should be able to have a faster initial convergence. This test took place at AUG, of which the gyrotron wave frequency shows a less erratic behaviour with time; see the upper graph of Fig. 10. In this experiment the adaptive mirror controller indeed has fast initial convergence and keeps the transmitted power curve at its near-maximum value for the complete time interval. The mirror position accurately tracks the wave frequency curve. Comparing this to the previous experiment it can be concluded that a frequency feedforward link has a beneficial effect on the convergence properties of the adaptive feedback controller.

For both experiments it holds that the added perturbation signal on the mirror position $\delta x$ is not visible in the output power curves.

VI. CONCLUSIONS

An adaptive mirror control strategy has been developed to tune the behaviour of optical resonators, based on power feedback of the resonator output waves. The intrinsic optimisation algorithm is of gradient-descent type, in which an on-line estimator of the cost function gradient is used. For this purpose a small-amplitude perturbation signal is added to the mirror position. This approach shows strong similarities with the techniques from 'stochastic approximation' and 'extremum seeking control’. Further fine-tuning of the adaptive approach has been achieved by perturbation signal filtering, specific cost function shaping and an additional wave frequency feedforward link. Both in simulation tests and in various experiments in a high-power millimetre wave setting with a quasi-optical resonator, the adaptive mirror control approach has proven to perform well. It is capable of compensating for temporal variations in the gyrotron wave frequency, the effects of thermal expansion of the resonator.
cavity and of structural vibrations. The mere performance limitation of the approach occurs in cases which require fast disturbance tracking. An accurately tuned feedforward link could be beneficial in these cases as well as a higher bandwidth of the mirror motion system.

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REFERENCES