Extended LMI Approach to Coherent Quantum LQG Control Design

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Abstract—A coherent quantum controller is itself a quantum system that is required to be physically realizable. Thus, additional non-linear and linear constraints must be imposed on the coefficients of a physically realizable quantum controller, which differs the quantum Linear Quadratic Gaussian (LQG) design from the standard LQG problem. The purpose of this paper is to propose one numerical procedure based on extended linear matrix inequality (LMI) approach and new physical realizability conditions proposed in [14] to design a coherent quantum controller. The extended synthesis linear matrix inequalities are, in addition to new analysis tools, less conservative in comparison to the conventional counterparts since the optimization variables related to the system parameters in extended LMIs are independent of the symmetric Lyapunov matrix. These features may be useful in the optimal design of quantum optical networks. For comparison, we apply our numerical procedure to the same example given in [9].

I. INTRODUCTION

Quantum feedback control can be simply thought of as an interconnection of a quantum plant and a controller in such a way that the closed loop system satisfies desired performance requirements, where the controller may be a quantum or classical controller [1], [2], [3]. If the controller is a classical system implemented by standard analog or digital electronics and measurement is involved in the feedback loop, this is known as measurement-based feedback control [4]. However, the state of quantum systems is easily affected by interaction with measurement devices, which causes the loss of quantum information. This thus motivates the replacement of the classical controller in measurement-based feedback control loop by a coherent quantum controller, which is directly interconnected with a quantum plant without any interfaces (eg. homodyne detectors, modulators) involved [5], [6], [7]. The notion of physical realizability proposed in [8] imposes some linear and nonlinear constraints on the system matrices of a physically realizable quantum controller, which complicates the problem of quantum controller design. Classical control theory and control methods continue to play an important role in the creation of new quantum technologies [8], [9]. The problem of applying Linear Quadratic Gaussian (LQG) techniques to quantum systems has been addressed in [9], where a numerical procedure based on standard LMI approach [10] was proposed for finding a quantum controller to achieve desired performance specifications with a given upper bound on the LQG cost, which is computed as the squared $H_2$-norm of the plant-controller system.

In previous works [11], [12], [13], extended LMIs techniques have been applied to designs of classical controllers, which characterize stability and performance specifications. Recalling some knowledge about extended LMIs approach: A linear system with system matrices $A, B, C$ (assume that $D = 0$) is Hurwitz stable and the squared $H_2$-norm of its transfer function $T$ satisfies $\|T\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Tr}(T(i\omega)^T T(i\omega)) < \gamma$ if and only if there exists a general matrix $F$, symmetric matrices $P > 0$ and $Q$ such that

$$
\begin{bmatrix}
FA + A^TF^T & P - F + A^TF^T & FB \\
F - FT + FA & -F - FT & FB \\
B^T F^T & B^T F^T & -I
\end{bmatrix} < 0, \quad (1)
$$

It can be seen from (1) that the extra instrumental variable $F$ introduced in extended LMIs gives a suitable structure in which the system matrices are completely independent from the Lyapunov matrix and provides a more positive impact on the design of quantum controllers compared with standard LMI conditions used in [9], [10]. In a significant way, the problem of minimizing the norm on one channel, subject to some moderate $H_\infty$ performance requirement on another channel can be addressed by employing different Lyapunov matrices to test all the objectives, which gives us less conservative solutions [11], [12], [13].

Therefore, the purpose of this paper is to propose a new numerical procedure based on extended LMI approach and new physical realizability conditions presented in [14] to design quantum controllers. We may optimize over extra parameters in extended LMIs and new physical realizability conditions to improve the LQG control performance of a closed-loop plant-controller system.

This paper is organized as follows. Section II introduces some notations and gives a brief overview of quantum systems. Section III formulates the set-up of a closed-loop quantum system with a physically realizable quantum controller, and then we present a quantum LQG problem to be solved in this section. Section IV proposes a numerical procedure based on extended LMIs approach to solve the quantum LQG problem. Section V applies our numerical procedure proposed in Section IV to the same example given in [9] for comparison. Finally, Section VI gives the conclusion of this paper.
II. PRELIMINARIES

A. Notation

The notations used in this paper are as follows: \( i = \sqrt{-1} \); the commutator is defined by \( [A, B] = AB - BA \). If \( X = [x_{jk}] \) is a matrix of linear operators or complex numbers, then \( X^\# = [x^*_jk] \) denotes the operation of taking the adjoint of each element of \( X \), and \( X^\dagger = [x^*_jk]^T \). We also define \( \Re(A) = (X + X^\#)/2 \) and \( \Im(A) = (X - X^\#)/2i \). \( J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \), and \( \text{diag}_n(M) \) denotes a block diagonal matrix with a square matrix \( M \) appearing \( n \) times on the diagonal block. The symbol \( I_n \) denotes the \( n \times n \) identity matrix and \( \delta_{jk} \) is the Kronecker delta function. The symbol \( \langle \cdot \rangle = \mathbb{P}(\cdot) \) indicates quantum expectation with respect to a given quantum state \( \mathbb{P} \).

B. Quantum Linear Stochastic Systems and Physical Realizability

Consider an open quantum harmonic oscillator (\cite[Theorem 3.4]{8}) consisting of \( n \) one degree of freedom open quantum harmonic oscillators coupled to boson fields (e.g., optical beams), \cite{6}, \cite{4}. Each oscillator may be represented by position \( q_j \) and momentum \( p_j \) operators \( (j = 1, \ldots, n) \), while each field channel is described by analogous field operators \( w_{q_j}(t), w_{p_j}(t), (k = 1, \ldots, m) \). The oscillator variables are canonical if they satisfy the canonical commutation relations \( [q_j, p_k] = 2i\delta_{jk} \), while the field operators (corresponding to the vacuum state) are canonical if they satisfy \( [w_{q_j}(t), w_{p_k}(t)] = i\delta_{jk} dt, [w_{q_j}(t), w_{q_k}(t)] = \delta_{jk} dt = [w_{p_j}(t), w_{q_k}(t)] \). In vector form, we write \( x = [q_1, p_1, q_2, p_2, \ldots, q_n, p_n]^T \), and the commutation relations become

\[
xx^T - (xx^T)^T = 2i\Theta_n,
\]

where, in the canonical case, \( \Theta_{nq} = \text{diag}_n(J) \). Similarly, the Ito products for the fields \( w = [w_{q_1}, w_{p_1}, w_{q_2}, w_{p_2}, \ldots, w_{q_m}, w_{p_m}]^T \) may be written as \( dw(t)dw(t)^T = F_w dt \), where in the canonical case \( F_w = I_{2m} + i\text{diag}_m(J) \). Commutation relations for the noise components of \( w \) can be defined as:

\[
[dw(t), dw(t)^T] = F_w - F_w^T = 2i\Theta_w dt.
\]

The dynamical evolution of an open system is unitary (in the Hilbert space consisting of the system and fields), and in the Heisenberg picture the system variables \( x \) and output field \( w \) operators evolve according to equations of the form

\[
\begin{align*}
    dx(t) &= Ax(t) dt + B dw(t); \\
    dy(t) &= Cx(t) dt + D dw(t),
\end{align*}
\]

where \( A, B, C \) and \( D \) are real constant matrices of suitable dimension. However, for arbitrary matrices \( A, B, C \) and \( D \) equations (5) need not correspond to a canonical open oscillator. The system (5) is said to be physically realizable if and only if matrices \( A, B, C, \) and \( D \) of the system (5) satisfy [14]

\[
\begin{align*}
    A\Theta_n + \Theta_n A^T + B\Theta_w B^T &= 0; \\
    B\Theta_w D^T &= -\Theta_n C^T; \\
    D\Theta_w D^T &= \Theta_y.
\end{align*}
\]

Definition 1: A fully quantum linear stochastic system of the form (5) is said to be standard if the skew-symmetric matrix \( \Theta_n \) and the nonnegative definite Hermitian matrix \( \Theta_w \) are both canonical.

III. PROBLEM FORMULATION

Consider a quantum plant described by non-commutative stochastic models of the following form:

\[
\begin{align*}
    dx_p(t) &= A_p x_p(t) dt + B_{pw} dw_p(t) + B_{pu} du(t); \\
    dy_p(t) &= C_p x_p(t) dt + D_{pw} dw_p(t); \\
    z(t) &= C_{pz} x_p(t) dt + D_{pz} \beta_u(t).
\end{align*}
\]

where \( A_p \in \mathbb{R}^{n \times n}, B_{pw} \in \mathbb{R}^{n \times n_{wp}}, B_{pu} \in \mathbb{R}^{n_{wp} \times n}, C_p \in \mathbb{R}^{n_{wp} \times n}, D_{pw} \in \mathbb{R}^{n_{wp} \times n_{wp}} \) \( (n, n_{wp} \text{ are even}) \). \( x_p \) represents a vector of plant variables and \( w_p \) is a quantum noise. \( u \) is a control input and \( \beta_u(t) = C_{\text{c},x_c}(t) \) is the signal part of \( u(t) \). \( z(t) \) is the performance output. Let initial values \( x_p(0) = x_{p0} \) satisfy the commutation relations: \( \left( x_{p0}, x_{p0}^T \right) = 2i\Theta_{wp} \). We assume that \( \Theta_{wp} = \frac{F_{wp} - F_{wp}^T}{2i} \) with \( dw_p(t)dw_p(t)^T = F_{wp} dt \).

Construct a quantum controller given by:

\[
\begin{align*}
    dx_c(t) &= A_c x_c(t) dt + B_{cw} dw_c(t) + B_{ck} dy_p(t), \\
    du(t) &= C_c x_c(t) dt + D_c dw_c(t),
\end{align*}
\]

where \( A_c \in \mathbb{R}^{n \times n}, B_{cw} \in n \times n_{cw}, B_{ck} \in \mathbb{R}^{n_{cw} \times n}, C_c \in \mathbb{R}^{n_{cw} \times n}, D_c \in \mathbb{R}^{n_{cw} \times n_{cw}} \) \( (n_{cw} \text{ is even, } n_{cw} = n, n_{cw} = n_{wp} \text{ are even}) \). \( x_c \) represents a vector of controller variables of the same order as \( x_p(t) \). The commutation relation for \( x_c(t) \) satisfies

\[
\left( x_c, x_c^T \right) = 2i\Theta_{nc},
\]

where \( \Theta_{nc} \) is an arbitrary anti-symmetric matrix. The quantum Wiener disturbance vectors \( w_{c1}, w_{c2}, w_{pc} \) are independent of each other and satisfy the following relations

\[
\begin{align*}
    [dw_{c1}(t), dw_{c1}(t)^T] &= (F_{wc1} - F_{wc1}^T) dt = 2i\Theta_{wc1} dt, \\
    [dw_{c2}(t), dw_{c2}(t)^T] &= (F_{wc2} - F_{wc2}^T) dt = 2i\Theta_{wc2} dt, \\
    [dw_{c3}(t), dw_{c3}(t)^T] &= (F_{wc3} - F_{wc3}^T) dt = 2i\Theta_{wc3} dt,
\end{align*}
\]

where \( F_{wc1}, F_{wc2}, F_{wc3} \) are nonnegative definite Hermitian matrices and their corresponding \( \Theta_{wc1}, \Theta_{wc2}, \Theta_{wc3} \) are skew-symmetric matrices. A physically realizable quantum controller (10) should require its system matrices \( A_c, B_{cw}, B_{ck}, C_c, D_c \) to satisfy the following conditions:

\[
\begin{align*}
    A_c\Theta_{nc} + \Theta_n A_c^T + B_{cw} \Theta_{wc1}B_{c1}^T + B_{cw} \Theta_{wc2}B_{c2}^T \\
    + B_{cw} \Theta_{wc3}B_{c3}^T &= 0, \\
    B_{cw} \Theta_{wc1} D_{c1}^T &= -\Theta_{nc} C_c^T, \\
    D_c \Theta_{wc1} D_{c1}^T &= \Theta_{wc1}.
\end{align*}
\]
Interconnecting systems (9) and (10) gives:
\[ dx(t) = Ax(t)dt + Bdw(t) \]
\[ z(t) = Cx(t) \quad (14) \]
where \( x = [x^T \bar{x}^T x_c^T] \), \( w = [w_p \ w_c \ w_c^T]^T \), \( A = \begin{bmatrix} A_p & B_p \bar{C}_p \ A_c \\ B_{pw} & B_{pw} \bar{D}_p \ 0 \\ B_{pc} & B_{pc} \bar{D}_p \ B_c \end{bmatrix} \), \( B = \begin{bmatrix} B_{pw} \ 0 \\ B_{pc} \ 0 \end{bmatrix} \), \( C = [C_{pc} \ D_{pc} C_c] \). Along the line of [9], the infinite-horizon LQG cost can be defined as:
\[ J_\infty = \lim_{t \to +\infty} \sup_t \frac{1}{t} \int_0^t (z(s)^T z(s))ds \]
\[ = \lim_{t \to +\infty} \sup_t \frac{1}{t} \Tr(C^TCS(t))ds \]
\[ = \Tr(C^TCS) \quad (15) \]
where the symmetric matrix \( S \) solves the following Lyapunov equation and the solution is unique.
\[ AS + SA^T + BB^T = 0, \quad (16) \]
where \( S < P^{-1} \) and the symmetric matrix \( P \) is shown in (1)-2).

In the next section, we will focus our attention to solve the following problem:

**Problem 1:** Given a cost bound parameter \( \gamma > 0 \), and design a quantum controller of the form (10) for the closed system (14) to satisfy the following statements:

1. There exist symmetric matrices \( P > 0 \) and \( Z \) as well as a general matrix \( F \) satisfying (1)-(3).
2. \( J_\infty < \gamma \).
3. The conditions (11)-(13) should be satisfied.

**IV. MAIN RESULTS**

In this section, we will propose a numerical procedure to design a coherent quantum controller, which can solve Problem 1.

**A. Controller Parametrization**

In order to fit Problem 1 into extended LMI frames, let us redefine our plant as follows without changing the structure of the closed system (14):
\[ dx(t) = A_p x_p(t)dt + B_{pw} \beta_p(t) + \bar{B}_{pw}d\bar{w}_p(t); \]
\[ d\bar{y}_p(t) = \bar{C}_p x_p(t)dt + \bar{D}_{pw}d\bar{w}_p(t); \]
\[ z(t) = C_p x_p(t)dt + D_{pw}\beta_p(t). \quad (17) \]

where \( \bar{w}_p = w, \bar{B}_{pw} = [B_{pw} \ 0] \), \( \bar{C}_p = [0 \ C_p^T]^T, \) and \( \bar{D}_{pw} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \).

Let us redefine our controller as:
\[ dx_c(t) = A_c x_c(t)dt + B_c d\bar{y}_p(t), \]
\[ \beta_c(t) = C_c x_c(t), \quad (18) \]
where \( B_c = [B_{c1} \ B_{c2} \ B_{c3}] \).

To remove the nonlinear terms in (1) and (2), we now follow the ideas of [11], [12] by introducing \( n \times n \) generalized matrices \( X, Y, U, V \). Let \( K = \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}, \) \( F = \begin{bmatrix} X & * \\ U & * \end{bmatrix}, \) and \( F^{-1} = \begin{bmatrix} Y^T & * \\ V^T & * \end{bmatrix}, \)
\[ F \begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} X \\ U \end{bmatrix} \]
can be inferred from \( F^{-1} = I \).

Then, we have \( FT_1 = T_2, \) where \( T_1 = \begin{bmatrix} I \ 0 \\ Y^T \ V^T \end{bmatrix} \) and \( T_2 = \begin{bmatrix} X & I \\ U & 0 \end{bmatrix} \). Define the following nonlinear transformation
\[ \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix} = \begin{bmatrix} V & YB_{pu} \\ 0 & I \end{bmatrix} \begin{bmatrix} U & 0 \\ C_p & I \end{bmatrix} \]
\[ = \begin{bmatrix} \Theta_n & \Theta_n \Theta_n \\ \Theta_n & \Theta_n \Theta_n \end{bmatrix} \]
\[ = \begin{bmatrix} N & J \\ \Theta_n^T & H \end{bmatrix} = T_1^T P T_1, \quad (20) \]
\[ S = YX + VU, \quad (21) \]
where \( A_c = YA_pX + BC_c + YB_{pu}C_c + U + VA_cU, B_c = VB_c \), \( C_c = C_cU, \) \( N, H \) are symmetric matrices and \( J \) is a general \( n \times n \) matrix. Performing congruence transformations on inequalities (1) and (2) with \( \text{diag}(T_1, T_1, I_n) \) and \( \text{diag}(T_1, I_n) \), respectively, we obtain new inequalities (22)-(24).

Multiplying both sides of the left hand side of (11) with \( V \) and \( V^T \) produces new variables \( \Theta_n \), \( \Theta_n \Theta_n \), \( \Theta_n \Theta_n \Theta_n \) satisfying conditions (19)-(23).

If there exists a upper bound \( \gamma \) and matrices \( A_c, B_c, C_c, D_c, X, Y, S, N, J, H, V, U, Q \) satisfying conditions (19)-(27), then matrices \( A_c, B_c \) and \( C_c \) of a quantum controller of the form (10) can be obtained as follows
\[ C_c = C_cU^{-1}, \quad (28) \]
\[ B_c = V^{-1}B_c, \quad (29) \]
\[ A_c = V^{-1}(A_c - YA_pX - BC_cX)U^{-1} - V^{-1}YB_{pu}C_c \quad (30) \]

**B. Numerical Optimization Procedure**

Our Problem 1 can be formulated as minimization of the LQG cost subject to constraints including LMIs and additional nonlinear constraints which are related to rank conditions [9], [21].

In the following we present numerical algorithms based on extended LMI approach. The nonlinear constraints (11)-(13) make proposing numerical algorithm for solving Problem
and a rank constraint
\[
\text{rank}(Z) \leq n. 
\] (32)

Conditions (25)-(27) for physical realizability can be expressed as
\[
-Z_{v8} + Z_{v9} + Z_{v9} + Z_{v10} + Z_{v10} + \\
Z_{v12} + Z_{v14} + Z_{v16} = 0; \\
Z_{v19} + Z_{v20} = 0; \\
Z_{v18} - \text{diag} \, \nu_{nc}(J) = 0. 
\] (33) (34) (35)

If we can employ a semidefinite programming to solve the feasibility problem with constraints (31)-(38) in which decision variables are \( Z, N, J, S, H \) and \( P \), Problem 1 is solvable. Then controller matrices can be built as (28)-(30).

Our numerical procedure can be solved based on Yalmip [16], SeDuMi [17], and LMIRank [20], [21]. The LMIRank solver can only solve feasibility problems and uses a local approach to address the non-convex rank constraints, hence it is essential to find proper starting points for our algorithms [20], [21].

In our procedure, \( \Theta_{nc} \) is allowed to be arbitrary antisymmetric matrices. However, as pointed out in [14], [15], [19], we cannot build a linear quantum stochastic controller as a suitable network of basic quantum devices if it is not in a standard form in the sense of Definition 1 proposed in Subsection II-B. Thus, we need to transform the quantum controller into a standard form once it does not satisfy conditions in Definition 1.

**Theorem 1:** Given an arbitrary real skew-symmetric matrix \( \Theta_{nc} \), there exists a real nonsingular matrices \( S_{nc} \) such that
\[
\Theta_{nc} = S_{nc} \text{diag} \, \nu_{nc}(J) S_{nc}^T. 
\] (39)

Then we have
\[
\begin{align*}
\tilde{A}_c &= S_{nc}^{-1}A_c S_{nc}, \\
\tilde{B}_c &= S_{nc}^{-1}B_c, \\
\tilde{C}_c &= C_c S_{nc}, \\
\tilde{D}_c &= D_c.
\end{align*} 
\] (40) (41) (42) (43)

Furthermore, if the original closed system (14) is asymptotically stable, the closed system of the form (14) with a
new quantum controller built by $\hat{A}_c$, $\hat{B}_c$, $\hat{C}_c$, $\hat{D}_c$ is still asymptotically stable and its LQG cost is the same as original one.

Proof: The similar proof of relation (39) can be found in [18] and hence is omitted here. Substituting (39)-(42) into conditions (11)-(12) with some algebraic manipulations gives

$$\hat{A}_c \text{diag}_{\frac{m}{2}} (J) + \text{diag}_{\frac{m}{2}} (J) \hat{A}_c^T + \hat{B}_c \text{diag}_{\frac{m}{2}} (J) \hat{B}_c^T + \hat{B}_c \text{diag}_{\frac{m}{2}} (J) \hat{B}_c^T = -\text{diag}_{\frac{m}{2}} (J) \hat{C}_c^T.$$

(45)

By applying similarity transformation $\Gamma = \text{diag}(I, S_{n_c}^{-1})$ to the Hurwitz matrix $A$, $B$ and $C$, we have

$$\hat{A} = \Gamma \begin{bmatrix} A_p & B_p C_p & \text{diag}_{\frac{m}{2}} (J) \hat{A}_c^T \end{bmatrix} \Gamma^{-1} \begin{bmatrix} A_p & B_p C_p & \text{diag}_{\frac{m}{2}} (J) \hat{A}_c^T \end{bmatrix},$$

(46)

$$\hat{B} = \Gamma \begin{bmatrix} B_p C_p & B_p D_c & \text{diag}_{\frac{m}{2}} (J) \hat{B}_c^T \end{bmatrix},$$

(47)

$$\hat{C} = \begin{bmatrix} C_{pz} & D_{pz} C_c \end{bmatrix} \Gamma^{-1} = \begin{bmatrix} C_{pz} & D_{pz} C_c \end{bmatrix}.$$

(48)

From (46), we can see that the new closed system of the form (14) with $A$, $B$, $C$, $x$ and $z$ replaced by $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{x} = \Gamma x$ and $\hat{z} = \Gamma z$ is asymptotically stable. Multiplying the left and right hand sides of each term in (16) with $\Gamma$ and $\Gamma^T$ gives

$$\Gamma A \Gamma^{-1} \Gamma S^T + \Gamma C \Gamma^{-1} \Gamma T - \Gamma T - \Gamma A^T = 0,$$

$$\hat{A} \hat{S} + \hat{S} \hat{A}^T + \hat{B} \hat{B}^T = 0,$$

where $\hat{S} = \Gamma S \Gamma^T$.

The LQG cost for the new closed system of the form (14) is given by

$$\dot{J}_\infty = \lim_{t \to +\infty} \sup_{\frac{1}{t}} \int_{0}^{t} (\hat{z}(s))^T \hat{z}(s) ds = \lim_{t \to +\infty} \sup_{\frac{1}{t}} \int_{0}^{t} \text{Tr}(\hat{C}^T \hat{C} \hat{S}(t)) ds = \text{Tr}(\hat{C}^T \hat{C} \hat{S}) = J_\infty$$

(49)

This completes the proof.

V. AN EXAMPLE

The following linear quantum plant is studied in [9, Section 8]:

$$dx(t) = \begin{bmatrix} 0 & 0.1 \\ -0.1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \end{bmatrix} u(t)$$

$$dy(t) = \begin{bmatrix} 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} w(t)$$

For comparison with results in [9, Section 8], we work in Matlab using the same Yalmip prototyping environment and the same semi-definite program solver. Then applying our numerical algorithm with $\gamma = 5.4$ proposed in the Section IV, we get the following solutions:

$$A_c = \begin{bmatrix} -0.0265 & -0.2471 \\ 0.0665 & -0.1558 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.0835 & -0.5259 \\ 0.1740 & -0.0578 \end{bmatrix},$$

$$B_{c2} = 10^{-12} \begin{bmatrix} -0.1212 & -0.0865 \\ -0.0785 & -0.0100 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 0.0578 & -0.5259 \\ 0.1740 & -0.0835 \end{bmatrix},$$

$$D_c = I_2.$$
respectively, which indicate the quantum controller is physically realizable. The eigenvalues of the closed system are $-0.0281 + 0.1030i$, $-0.0281 - 0.1030i$, $-0.0631 + 0.0901i$, $-0.0631 - 0.0901i$, so the plant-controller system is Hurwitz stable. The resulting LQG performance is 4.1601, which is a little better than the LQG cost 4.1793 in [9].

VI. CONCLUSION

In this paper, we propose a new numerical procedure based on extended LMIs approach and new physical realizability conditions, which can provide more parameters for the design of a physically realizable quantum controller of the standard form and give less conservative solutions to quantum LQG problem. For comparison, we reinvestigate the example given in [9]. It turns out that our optimization procedure proposed in this paper may be useful in the optimal design of quantum optical networks.

REFERENCES