An Analytic Price-driven Coordination Scheme For Distributed Model Predictive Control Systems

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Abstract—In this work, we propose an analytic approach to coordinate distributed model predictive control (MPC) systems that are subject to equality constraints. The main idea behind the coordinated distributed model predictive (CDMPC) is to improve the performance of an existing decentralized control system with a minor modification applied to the existing distributed control system. As such, the enhancement can be interpreted as a bilevel optimization problem, which consists of modified local controllers and a coordination level that ensures optimal centralized behavior of the plant. The modification to the decentralized MPC controllers would be equivalent to relaxing local versions of the overall interaction constraint using a price vector to penalize violations. In the proposed price-driven CDMPC scheme, local variables are parameterized in terms of the price vector. As a result, an analytic closed-loop solution to the optimal price vector can be developed. The effectiveness of the proposed CDMPC is illustrated using a chemical process example.

I. INTRODUCTION

Traditionally, centralized and decentralized control are the two primary frameworks for the control of large-scale systems. While the decentralized control is easy to implement, it may lead to degradation of plant-wide performance or even loss of closed-loop stability since in decentralized control the interactions between subsystems are in general neglected. On the other hand, centralized control is expected to give the best performance; however, it may become too complicated to implement as the size of the control problem grows. Motivated by the above, in recent years, significant efforts have been devoted to the development of distributed model predictive control (DMPC) for the control of large-scale systems.

Among different DMPC schemes, the type of DMPC pertaining to the present work is the so-called coordinated DMPC (CDMPC). In a CDMPC scheme, distributed MPCs communicate with coordinator to achieve improved performance. The main idea behind CDMPC is to relate hierarchical dual decomposition of a hypothetical centralized controller with performance enhancement of an existing network of decentralized controllers. This problem can be interpreted as a bilevel optimization problem which consists of modified local controllers and a coordinator to ensure optimal centralized plant behavior is achieved. Different algorithms have been developed for CDMPC including the price-driven approach (i.e., [1]–[4]), the primal decomposition approach (i.e., [5], [6]), the prediction-driven approach (i.e., [3], [7]), the chancd-constraint coordination method [3], and the pseudo-model coordination approach [3]. Basically, the main differences between these coordination methods appear in ways that the interactions between subsystems are addressed [3]. Besides CDMPC, other important recent work includes cooperative DMPC of linear systems (i.e., [8], [9]), and Lyapunov-based sequential and iterative DMPC of nonlinear systems (i.e., [10], [11]).

In this work, we propose an analytic scheme to coordinate the distributed controllers using price-driven strategy. Conventionally, iterative numerical methods are used to find the optimal value of the price vector with finite number of coordination cycles. In the analytic scheme, local variables are parameterized in terms of the price vector. Accordingly, an analytic closed-loop solution to the optimal price vector is provided inside the coordinator. The performance of the proposed CDMPC versus the equivalent iterative procedure [7] is illustrated via the application to a forced-circulation evaporation process.

II. THE PROPOSED CDMPC ALGORITHM

In this section, we present an analytic CDMPC algorithm for linear systems. A schematic of the proposed CDMPC is shown in Fig. 1. In the proposed formulation, a local MPC is formulated for each subsystem and a coordinator coordinates the actions of the subsystem MPCs. Subsystem MPCs and the coordinator communicate and exchange information iteratively every sampling time. The coordinator calculates prices for the subsystem MPCs to coordinate their actions for closed-loop stability and improved performance. Each subsystem MPC calculates control inputs minimizing a local cost function based on subsystem state measurements and the price received from the coordinator.

Fig. 1: Architecture and information flow of the proposed CDMPC.
A. Structure of local MPC Controllers

Local MPCs are formulated based on the discretized model of the plant and the calculated control actions are applied to the continuous subsystems. Specifically, for subsystem $i$, $i = 1, \ldots, m$, the prediction model used in the formulation of the subsystem MPC at time instant $k$ takes the following form:

\[
\hat{x}_i(k + lk|k) = \Psi_{ii,i} \hat{x}_i(k + lk) + \Gamma_{ii,i} \hat{u}_i(k + lk) + \hat{v}_i(k + lk) + x_i(k) \tag{1a}
\]

for $l = 0, \ldots, H_p - 1$ with $H_p$ being the prediction horizon. In addition, $\Psi_{ii,i}$ and $\Gamma_{ii,i}$ are matrices corresponding to sub-system $i$, and $\hat{v}_i$ is defined as the interacting or linking variable that contains unknown interaction information among different subsystems. Note that $\hat{v}_i$ characterizes the interaction of subsystem $i$ with other subsystems. To proceed with the presentation, we define $e_i$ as follows:

\[
e_i(k + lk|k) = \hat{v}_i(k + lk) - \sum_{j \neq i}^m (\beta \Psi_{ij} \hat{x}_j(k + lk) + \Gamma_{ij} \hat{u}_j(k + lk)) \tag{2a}
\]

with

\[
\beta = \begin{bmatrix}
0 & l = 0 \\
1 & l = 1, \ldots, H_p - 1
\end{bmatrix} \tag{2b}
\]

The vector $e_i(k + lk|k)$ denotes the error between the interaction of subsystem $i$ with others captured by the plant-wide model and the one characterized by $\hat{v}_i$. A specific objective of the coordinator is to find a price for subsystem $i$ such that the interaction term $\hat{v}_i$ determined by the price renders $e_i(k + lk|k) = 0$. This will ensure that the CDMPC approaches the performance of the corresponding centralized MPC. The overall interaction error over the prediction horizon can be described as follows:

\[
E(k|k) = \begin{bmatrix}
E_1(k|k) \\
\vdots \\
E_m(k|k)
\end{bmatrix} \tag{3a}
\]

where

\[
E_i = \begin{bmatrix}
e_i(k|k) \\
e_i(k + lk) \\
\vdots \\
e_i(k + H_p - 1k)
\end{bmatrix} \tag{3b}
\]

According to (2a), it can be verified that $E(k|k)$ can be written as an additive separable form:

\[
E(k|k) = \sum_{i=1}^m \Theta_i(k) \begin{bmatrix}
X_i(k) \\
U_i(k) \\
V_i(k)
\end{bmatrix} \tag{4}
\]

where $X_i(k) \triangleq [\hat{x}_i(k + 1k|k)^T, \ldots, \hat{x}_i(k + H_p|k)^T]^T$ is the predicted state trajectory of subsystem $i$, $U_i(k) \triangleq$
and $Q_{ii}$ and $R_{ii}$ are positive definite weighting matrices. Optimization problem (7) provides the basics for the formulation of the subsystem MPCs and the coordinator. Note that the cost function and constraints are separable with respect to the subsystems. The idea is to relax constraint (7c), which characterizes the interactions, via a price vector so that it is also separable in terms of subsystems. Specifically, the equivalent overall problem is formulated as follows:

$$
\min_{p;Z} J_D = \sum_{i=1}^{m} J_{D_i}
$$

subject to

$$
\dot{x}_i(k + l + 1 | k) = \Psi_{ii} \dot{x}_i(k + l | k) + \Gamma_{ii} \hat{u}_i(k + l | k)
$$

for $l = 0, \ldots, H_p - 1$

$$
\hat{u}_i(k + l | k) = \hat{u}_i(U_i | k)
$$

for $H_u \leq l \leq H_p - 1$

where

$$
J_{D_i} = \frac{1}{2} \left( (X_i(k) - X_{i, set})^T Q_{ii} (X_i(k) - X_{i, set}) + U_i(k)^T R_{ii} U_i(k) \right) + p^T \Theta_i \left[ \begin{array}{c} X_i(k) \\ U_i(k) \\ V_i(k) \end{array} \right]
$$

In the separable optimization problem (8), $p$ is a price vector calculated by the coordinator to provide the plant-wide solution for the distributed system.

### B. Formulation of the Analytic Price-Driven Coordinator

According to (8), the price vector $p$ can be considered as the Lagrange multiplier associated with interaction equality constraints. This vector is determined by the coordinator, based on the information provided by the local MPC controllers. In order to find a plant-wide solution to the distributed system the bilevel optimization problem (9) is defined:

$$
\min_p - J_D(p, \bar{Z}^*)
$$

$$
\bar{Z}^* = \arg \min_{\bar{Z}} \{ J_D = \sum_{i=1}^{m} J_{D_i} \}
$$

subject to

$$
A_i \bar{Z}_i(k) = b_i
$$

where

$$
\bar{Z}_i = \left[ \begin{array}{c} X_i(k) \\ U_i(k) \\ V_i(k) \end{array} \right]
$$

$$
A_i = [A_{X_i}, A_{U_i}, A_{V_i}]
$$

$$
A_{X_i} = \left[ \begin{array}{ccc} I_{n_{x_i}} & 0 & \cdots & 0 \\ -\Psi_{ii} & I_{n_{x_i}} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_{n_{x_i}} \end{array} \right],
$$

$$
A_{U_i} = \left[ \begin{array}{ccc} -\Gamma_{ii} & 0 & \cdots & 0 \\ 0 & -\Gamma_{ii} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -\Gamma_{ii} \end{array} \right],
$$

$$
A_{V_i} = I_{H_p n_{x_i}} \times I_{n_{x_i}}
$$

$$
\begin{bmatrix} \Psi_{ij} \dot{x}_j(k) + \Gamma_{ij} u_j(k - 1) \\ \Gamma_{ij} u_j(k - 1) \\ \vdots \\ \Gamma_{ij} u_j(k - 1) \end{bmatrix}
$$

In the bilevel optimization problem (9), the lower level optimization problem represents the distributed network of MPC controllers and the upper level unconstrained optimization problem is handled by the coordinator. The Lagrange function of the lower problem can be written as:

$$
\mathcal{L} = \frac{1}{2} \begin{bmatrix} X^T \\ U \end{bmatrix}^T \begin{bmatrix} Q & R \\ 0 & X \end{bmatrix} X - \begin{bmatrix} X^T \Theta Q, 0 \end{bmatrix}^T X
$$

$$
+ p^T \Theta_V V + \nu^T [A_X, A_U] X
$$

$$
+ \nu^T A_v V - \nu^T b
$$

where $\nu$ is the Lagrange multiplier associated with equality constraint (9c). The KKT conditions of the lower problem can be written as:

$$
\begin{align*}
Q X - Q X_{set} + \Theta_V X + A^T V \nu &= 0 \\
R U + \Theta_V U + A^T U \nu &= 0 \\
-\nu + \nu &= 0
\end{align*}
$$

which concludes the following relations for $\nu$ and $V$:

$$
\nu = p \quad V = b - (A_X X + A_U U)
$$

In other words, the Lagrange function of the lower level problem can be stated as:

$$
\mathcal{L}(\bar{Z}) = \mathcal{L}(Z) = \mathcal{L}_Z + p^T \mathcal{L}_p
$$

where

$$
\mathcal{L}_Z = \frac{1}{2} Z^T \Upsilon Z - [X^T \Theta Q, 0] Z
$$

$$
\mathcal{L}_p = [\Theta_X X + A_X \nu + A_U \nu] Z - b
$$

$$
\Upsilon = \begin{bmatrix} Q \\ 0 \\ 0 \\ R \end{bmatrix}
$$

The KKT conditions of the lower problem can be written in the following format:

$$
\nabla_Z \mathcal{L} = \frac{d \mathcal{L}_Z}{dZ} + p^T \frac{d \mathcal{L}_p}{dZ} = 0
$$

$$
[A_X, A_U] Z + V - b = 0
$$

Accordingly, the bilevel problem (9) can be written as a single layer optimization problem as:

$$
\min_{p,\bar{Z}} - J_D(p, \bar{Z})
$$

$$
\nabla \mathcal{L} = \frac{d \mathcal{L}_Z}{dZ} + p^T \frac{d \mathcal{L}_p}{dZ} = 0
$$

$$
[A_X, A_U] Z + V - b = 0
$$
Solving for \( Z \) and \( V \), the closed-loop solution of the lower problem w.r.t the current value of the price vector, provided by the coordinator is derived as:

\[
Z^*(p) = Y^{-1} \left( \begin{bmatrix} Q_X \set 0 \\ 0 \end{bmatrix} - [\Theta_X + A_X, \Theta_U + A_U]^T p \right)
= \left[ \begin{bmatrix} X_{\text{set}} \\ 0 \end{bmatrix} - \left[ -Q^{-1}(\Theta_X + A_X)^T \\ -R^{-1}(\Theta_U + A_U)^T \right] p \right]
\]
(17a)

\[V^*(p) = b - [A_X, A_U]Z^*(p)
\]
(17c)

The following theorem, expresses the nature of the closed form solution (17b) as a predictor-corrector term.

**Theorem 2.1:** The closed-loop solution \( Z^*(p) \) can be written as a predictor-corrector format around the non-coordinated trajectory, namely “\( p = 0 \)”: \( \)

\[
Z^*(p) = Z_{\text{predictor}} + Z_{\text{corrector}} p
\]
(18a)

\[
= Z(0) + \frac{dZ}{dp}|_{p=0}
\]
(18b)

**Proof:** The non-coordinated trajectory is derived directly from KKT conditions (15) by taking “\( p = 0 \)”: \( \)

\[
\nabla Z \bar{L}(0) = \frac{\partial Z}{\partial Z} = 0
\]
(19a)

then it can be shown that:

\[
YZ(0) - [X_{\text{set}}^T Q, 0]^T = 0
\]
(19b)

which means:

\[
Z(0) = Y^{-1} \left( \begin{bmatrix} Q_X \set 0 \\ 0 \end{bmatrix} \right) = \left[ X_{\text{set}} \right]
\]
(19c)

In order to find \( \frac{dZ}{dp} \), \( \nabla Z_p \bar{L} \) is calculated:

\[
\nabla Z_p \bar{L} = \frac{d}{dp} \left( \frac{dZ}{dp} \right) + \frac{dL_p}{dp}
\]

\[
= \frac{dZ}{dp} \left( \frac{d^2Z}{dp^2} + p^T \frac{d^2L_p}{dp^2} \right) + dL_p
\]

(20a)

Thus \( \frac{dZ}{dp} \) can be derived as:

\[
\left( \frac{dZ}{dp} \right)^T = - \frac{dL_p}{dp} \left( \frac{d^2Z}{dp^2} + p^T \frac{d^2L_p}{dp^2} \right)^{-1}
\]
(20b)

which completes the proof:

\[
\left. \frac{dZ}{dp} \right|_{p=0} = -Y^{-1} \left[ -Q^{-1}(\Theta_X + A_X)^T \\ -R^{-1}(\Theta_U + A_U)^T \right]
\]
(20c)

Parametrizing \( Z^* \) as a function of the price vector \( p \) and \( V^* \) as a function of \( Z^* \), using (17b) and (17c), the single layer optimization (16) can be converted into an unconstrained optimization problem w.r.t the price vector as:

\[
\min_p -J_D(p, Z^*(p))
\]
(21)

Similar to (18b), the Lagrange function of the overall problem can be stated as a quadratic function of the price vector around the non-coordinated trajectory.

**Theorem 2.2:** The Lagrange function of the single layer problem (21) can be written as a quadratic function of “\( p \)” around the non-coordinated trajectory, namely “\( p = 0 \)”:

\[
\bar{L}(p, Z^*(p)) = \bar{L}(p) = \frac{1}{2} p^T \nabla^2_{p,p} \bar{L}(p)|_{p=0} p + \nabla_p \bar{L}(p)|_{p=0} p + \bar{L}(0)
\]
(22)

**Proof:** According to (21), the Lagrangian of the unconstrained problem is derived as:

\[
\bar{L}(p, Z^*(p)) = -J_D(p, Z^*(p))
\]
(23a)

Based on the convex nature of the lower level problem, the equality constraint (16c) is always satisfied, i.e. \([A_X, A_U]Z^* + V^* - b = 0\). Therefore, penalizing this constraint using \( \nu \) at \( Z^*(p) \) and adding it to the right-hand-side (RHS) of (23a) would result in:

\[
\bar{L}(p, Z^*(p)) = -J_D(p, Z^*(p)) - \nu([A_X, A_U]Z^* + V^* - b)
\]
(23b)

in other words, at \( Z^*(p) \) we have:

\[
\bar{L}(p, Z^*(p)) = -L(p, Z^*(p))
\]
(23c)

Substituting (17b) into \( \bar{L}(p, Z^*, p) \) yields:

\[
\bar{L}(p) = -\frac{1}{2} \left( \nabla^{-1} \left( -[G_X, G_U]^T p + \left[ Q_X \set 0 \right] \right) \right)^T
\]

\[
\times \nabla \left( \nabla^{-1} \left( -[G_X, G_U]^T p + \left[ Q_X \set 0 \right] \right) \right)
\]

\[
+ [X_{set} Q, 0] \nabla^{-1} \left( -[G_X, G_U]^T p + \left[ Q_X \set 0 \right] \right)
\]

\[
p^T \left( [G_X, G_U] \nabla^{-1} \left( -[G_X, G_U]^T p + \left[ Q_X \set 0 \right] \right) \right)
\]

\[
+p^T b
\]

\[
= \frac{1}{2} b^T \left( [G_X, G_U] \nabla^{-1} \left( [G_X, G_U]^T p \right)\right.
\]

\[
- \left( [X_{set} Q, 0] \nabla^{-1} [G_X, G_U]^T p \right)
\]

\[
+ \left. \frac{1}{2} [X_{set} Q, 0] \nabla^{-1} \left[ Q_X \set 0 \right] \right)
\]
(24a)

but,

\[
\nabla^2_{p,p} \bar{L}(p)|_{p=0} = \frac{d^2L}{dp^2} \left|_{p=0} = [G_X, G_U] \nabla^{-1} [G_X, G_U]^T \right.
\]
(24b)

\[
\nabla_p \bar{L}(p)|_{p=0} = \bar{L}_p(0) = -[X_{set} Q, 0] \nabla^{-1} [G_X, G_U]^T + b^T
\]
(24c)

\[
\bar{L}(0) = L_Z(0) = \frac{1}{2} [X_{set} Q, 0] \nabla^{-1} \left[ Q_X \set 0 \right]
\]
(24d)

and the proof is complete.
As a result of (21) and (23c), the corresponding KKT condition would be:
\[ \frac{dL(Z^*, p)}{dp} = -\frac{dL(Z^*, p)}{dp} \]  
(25a)
\[ \frac{dL(Z^*, p)}{dp} = -\left( \frac{dL_p(Z^*)}{dz^*} + p^T \frac{dL_p(Z^*)}{dz^*} \right) dz^* + \mathcal{L}^T_p(Z^*) \]  
(25b)
but,
\[ \frac{dL(Z^*)}{dz^*} + p^T \frac{dL_p(Z^*)}{dz^*} = 0 \]  
(25c)
according to (15); this is equivalent to:
\[ \frac{dL(Z^*, p)}{dp} = \mathcal{L}^T_p(Z^*(p)) = 0 \]  
(25d)

To solve the CDMPC problem using conventional methods, (25d) together with (16c) are calculated numerically. Given sensitivity information of the local controllers (20b), the numerical coordinator uses an iterative gradient-based procedure such as Newton’s method to update the price vector. Thus, in order to converge to the optimal plant-wide solution, the coordinator and local controllers communicate, in coordination cycles, until the iteration constraint (7c) is satisfied [11–4, 7].

In the proposed approach, the KKT condition (25d) is solved analytically to obtain the optimal price vector, based on the parametrized values of \( Z^* \) and \( V^* \), explained in (17b)-(17c). Using (14c) and (17b), the price vector can be derived as the solution to (25d):
\[ [G_X, G_U]^T p \begin{bmatrix} -Q^{-1}(G_X)^T p + X_{set} \\ -R^{-1}(G_U)^T p \end{bmatrix} = b \]  
(26a)
From (26a), it can be obtained that:
\[ p = ([G_X, G_U]^T)^{-1} \begin{bmatrix} G_X \end{bmatrix} X_{set} - b \]  
(26b)
where
\[ G_X = \Theta_X + A_X \]  
(26c)
\[ G_U = \Theta_U + A_U \]  
(26d)
The next two lemmas are required to prove (26b) is not singular.

**Lemma 2.1:** A real symmetric matrix \( M \) is positive definite if and only if a real non-singular matrix \( Y \) exists such that
\[ M = YY^T \]  
(27)

**Lemma 2.2:** Summation preserves the positive definite property of matrices.

Next, we provide a theorem to show why (26b) always exists.

**Theorem 2.3:** The analytic solution to the price vector (26b) is not singular.

**Proof:** Let us expand the quadratic coefficient in equation (26b)
(28a)
Since the weighing matrices \( Q \) and \( R \) are positive definite, their inverse matrices \( Q^{-1} \) and \( R^{-1} \) are also positive definite. According to Lemma 2.1, \( Q^{-1} \) and \( R^{-1} \) can be written in terms of non-singular matrices \( Y_Q \) and \( Y_R \) as follows:
\[ Q^{-1} = Y_Q Y_Q^T \]  
(28b)
\[ R^{-1} = Y_R Y_R^T \]  
(28c)
Hence, the RHS of (28a) can be written as:
\[ G_X Q^{-1} G_X^T + G_U R^{-1} G_U^T = G_X Y_Q Y_Q^T G_X^T + G_U Y_R Y_R^T G_U^T \]  
(28d)
\[ = (G_X Y_Q)(G_X Y_Q)^T + (G_U Y_R)(G_U Y_R)^T \]  
(28e)
Since \( G_X \) and \( G_U \) are also non-singular matrices [4], \( (G_X Y_Q)(G_X Y_Q)^T \) and \( (G_U Y_R)(G_U Y_R)^T \) are positive definite matrices. Therefore, according to Lemma 2.2, left-hand-side (LHS) of (28a) is positive definite, and (26b) is not singular.

Finally, the closed-loop solution to the unconstrained CDMPC problem can be derived, as a result of (17b), (17c) and (26b):
\[ Z^*(p) = \begin{bmatrix} X_{set} \\ 0 \end{bmatrix} + \begin{bmatrix} -Q^{-1}(\Theta_X + A_X)^T \\ -R^{-1}(\Theta_U + A_U)^T \end{bmatrix} \times \left( \begin{bmatrix} G_X, G_U \end{bmatrix} Y^{-1} \begin{bmatrix} G_X, G_U \end{bmatrix}^T \right)^{-1} \left( \begin{bmatrix} G_X \end{bmatrix} X_{set} - b \right) \]  
(29a)
and
\[ V^*(p) = b - [A_X, A_U] \begin{bmatrix} X_{set} \\ 0 \end{bmatrix} + \begin{bmatrix} -Q^{-1}(\Theta_X + A_X)^T \\ -R^{-1}(\Theta_U + A_U)^T \end{bmatrix} \times \left( \begin{bmatrix} G_X, G_U \end{bmatrix} Y^{-1} \begin{bmatrix} G_X, G_U \end{bmatrix}^T \right)^{-1} \left( \begin{bmatrix} G_X \end{bmatrix} X_{set} - b \right) \]  
(29b)

### III. APPLICATION TO A CHEMICAL PROCESS

The simulation studies were performed on a 2.4 GHz Intel® Core 2 Duo processor with 8 GB of memory. For the numerical calculations, we used MATLAB® 2012a. The forced-circulation evaporator system [12] is used as a benchmark to show the efficiency of the distributed MPC strategy. This system is illustrated
in Fig. 2. The dynamics of the forced-circulation evaporator consists of three measured states ($L_2$: separator level [m], $X_2$: product composition [%], and $P_2$: operating pressure [kPa]), three input variables ($F_2$: product flowrate [kg/min], $P_{100}$: steam pressure [kPa], and $F_{200}$: cooling water flowrate [kg/min]). In Fig. 2, $T_2$ is product temperature [c], $T_{100}$ is steam temperature [c], $Q_{100}$ is heater duty [kW], $F_{100}$ is steam flowrate [kg/min], $T_3$ is vapour temperature, $F_3$ is vapour temperature [c], $Q_{200}$ is condenser duty [kW], $T_{201}$ is cooling water outlet temperature [c], and $F_5$ is condensate flowrate [kg/min]. The following two sub-systems exist in the overall process model:

$$X^T = ([x_1, x_2]|x_3)^T = ([L_2, X_2]|P_2)^T$$

$$U^T = ([u_1, u_2]|u_3)^T = ([F_2, P_{100}]|F_{200})^T$$

(30)

(31)

The discretized state-space model for a sampling time of 1[min] can be described as:

$$X(k + 1) = \begin{bmatrix} 1 & 0.0975 & 0 \\ 0 & 0.9048 & 0 \\ -0.1050 & 0.3795 & 0.0014 \\ -0.0952 & 0 & 0 \\ 0.0005 & 0.0359 & -0.0073 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ -0.0096 \\ -0.9467 \\ 0 \\ 0 \end{bmatrix} U(k)$$

(32)

In the simulations, $H_p = 30$ and $H_u = 5$. The centralized trajectory is calculated according to (7) and the actions are applied to the continuous system. In Fig. 3, the trends of the overall objective functions of the analytic CDMPC ($J_D$) formulation is compared to the centralized objective function ($J_P$). As can be seen, the analytic CDMPC can perfectly track the centralized performance. In addition, the corresponding computational cost is compared to the numerical CDMPC [7] via the CPU time taken to run the algorithms, in Fig. 4. Since the numerical method is based on iterative coordination cycles, significant improvement can be achieved using the proposed analytic approach.

IV. CONCLUSIONS

An analytic approach to coordinate distributed MPC controllers, using price-driven strategy, is proposed. The scheme is capable of achieving the optimal plant-wide performance in real-time. Unlike the numerical counterparts, the analytic CDMPC is less computationally demanding and eliminates any coordination cycles due to numerical approximations.

REFERENCES


