Distributed Charging Control of Electric Vehicles Using Regret Minimization

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Abstract—This paper presents an electric vehicle (EV) charging algorithm based on regret minimization. While many distributed algorithms for EV charging have been proposed, many of them implicitly assume low-latency two-way communication between the distribution company and the EV customers. This assumption is limiting at present. Furthermore, transmission of information about total load and planned charging schedules also raises concerns of security and privacy. The proposed algorithm resolves these issues at the expense of slower convergence to the optimal solutions. The algorithm considers a relaxation of the problem in the sense that the charging profiles are sought to be optimized over the set of profiles that do not vary from one day to the next. The proposed method only requires communication in the form of the distribution company publishing the pricing profiles implemented for the previous days. We provide convergence results for the algorithm and illustrate the results through numerical examples.

I. INTRODUCTION

Demand response (DR) has now been accepted as an important capability for the next generation power grid that will allow the grid to operate at the maximum efficiency. It empowers the distribution company and its customers to decide collectively, but in a distributed manner, the best way to schedule energy usage; see e.g., [10], [21] and references therein. In this paper, we focus on DR for scheduling the charging of EVs at individual residences.

High penetration of EVs imposes a significant burden on the grid. Particularly, creation of new peaks, peak load amplification [13], voltage deviations [18], among other effects have been identified as major concerns. To cope with these issues, many charging control algorithms for EVs have been proposed, e.g., [5], [7], [20], [24].

We are particularly interested in the algorithms proposed in [8], [9], and [19], which provide an analytical guarantee on the convergence of the total load profile to a desired one (for instance, a valley-filling profile). The customers and the distribution company negotiate suitable charging profiles to optimize the total load profile by exchanging of price-like signals.

For a given day-ahead prediction of the base load profile, the algorithms in these references require multiple rounds of negotiations between the distribution company and its customers to obtain the optimal charging profiles for the next day. As the base load changes from day to day, the algorithms need to be executed daily to calculate the charging profiles. Since this negotiation needs to be completed before the EVs can begin charging, these algorithms implicitly assume the presence of a communication infrastructure and protocols that can support low latency two-way communication between the distribution company and the EV customers. Such infrastructure and protocols have not yet been deployed extensively. Moreover, the data that are exchanged between the distribution company and the customers during this negotiation encodes information that may raise concerns of security and privacy for both the distribution company and the customers, were this data to be accessed by an eavesdropper.

These constraints motivate us to consider a new algorithm for distributed charging control of EVs that is based on regret minimization. This algorithm requires only one-way communication from the distribution company to the customers. Furthermore, the communication needs not be in real-time and carries information about the pricing profiles that were realized on the previous days. Thus, the algorithm is free of the concerns identified above. The price that we pay for this freedom is slower convergence of the charging schedules to the optimal ones, and a relaxation of the problem in the sense that the charging profiles are sought to be optimized over the set of profiles that do not vary from day to day.

The regret minimization framework has gained tremendous popularity in the online optimization and machine learning community; see e.g., [3], [4], [6], [12], [22], [25], [26] and the references therein. Informally, the regret minimization framework operates as follows. Every decision maker is assigned a payoff that she wants to maximize. The payoff is a function of the environment and decisions made. The decisions are made repeatedly and the resulting payoff is used to iterate over the policies used by the decision makers. Thus, in every iteration, every participant makes a decision given the current policy and the realization of the environment. The environment is a priori uncertain and may vary from one iteration to the next. Given the decisions of the participants and the realization of the environment, the various decision makers obtain certain payoffs. The regret minimization algorithm provides an iterative way for every participant to update its policy, such that, at convergence, the policy is optimal in a suitably defined sense.

In our formulation, we model the distribution company and every EV customer as decision makers who wish to optimize their own utility functions. For the distribution company, the payoff is maximized if the total load profile over a day is
valley-filling. For the EV customer, the utility function is maximized if the cost of charging the EV over a day is minimized. By designing a suitable pricing policy, the distribution company aims at ensuring that the individual charging profiles followed by the customers aggregate to a valley-filling profile. No negotiations need to be carried out for selecting the charging profiles and the only communication that occurs is when the distribution company notifies every customer of the pricing profiles incurred over the previous days. This communication does not require infrastructure that can support low latency two-way communication, and can be done through, e.g., publishing the resulting total load for the previous day. Furthermore, since the customers do not need to transmit any data, concerns on security and privacy are mitigated. Finally, we consider the case in which the utility functions of various EV customers are not identical, or precisely known to the distribution company. This feature is of interest in modeling the response of diverse customer behavior in DR.

Some relevant references that apply regret minimization to DR are [14], [17] and [23]. In [14], real-time electricity pricing strategies for DR are designed using regret minimization. However, the focus of the reference is to optimize the utility function for the distribution company and the customer behavior is captured through a load-to-price sensitivity function such that the load change is linear in the price variation. Similar to [14], in [17], the authors use regret minimization to design dynamic pricing for DR. The objective of [17] is to design pricing policies for the retailer whose customers have price responsive loads. The exact demand function for current day is assumed to be unknown to the retailer. In [23], regret minimization is used to learn the charging behavior of the EV customers as the prices are varied. The price responsiveness for a community of customers is captured through a conditional random field model. The regret minimization algorithm is adopted to learn the parameters of the model.

The remainder of the paper is organized as follows. Section II presents the problem formulation. The regret minimization framework is discussed in Section III. Section IV considers the effect of the diverse customer behavior. The numerical examples can be found in Section V. Section VI concludes the paper.

II. PROBLEM FORMULATION

We consider a scenario in which $N$ customers schedule the charging of their EVs daily. The charging needs to be completed over the course of a day. The EV customers pose a load that can (at least potentially) be shaped by the distribution company through suitable incentives, so that the total of the EV load and the base (inflexible) load is valley-filling or some other desirable shape. To shape the EV load, the distribution company offers electricity at prices that vary with time or total usage. Every EV customer fixes the schedule at the beginning of the day based on the information about her own schedules and the pricing profiles that were charged over the previous days by the distribution company.

A price-sensitive EV customer seeks to minimize the total cost of charging by shaping her charging schedule suitably. The information flow topology of the system is as follows. We do not assume the presence of a communication infrastructure that can support two-way communication between the distribution company and the EV customers. Thus, the customers cannot negotiate directly with the distribution company. The distribution company monitors the total load and publishes the price profile according to a fixed (and known) pricing policy for the previous day. The customers decide on the schedule for the day with access to these profiles for all previous days. No other communication occurs between the distribution company and the customers.

We now introduce some notation. Let there be $T$ time slots in a day and denote the set of these time slots by $\mathcal{T} := \{1, \ldots, T\}$. In practice, a time slot could be an interval such as 15 minutes. Furthermore, denote the set of EVs by $\mathcal{N} := \{1, \ldots, N\}$. Denote the base load on day $k$ by $D^k(t), t \in \mathcal{T}$. We assume that this base load is unknown to all the EV customers and to the distribution company at the beginning of the $k$-th day, when the charging schedules are fixed. Furthermore, the base load may vary from one day to the next. Denote by $x^k_i(t)$ the charging rate of the $i$-th EV during the $t$-th time slot on the $k$-th day. For simplicity, we assume that the charging schedule for the day is piecewise constant such that the charging rates remain constant over at least one time slot. The charging profile of the $i$-th EV on the $k$-th day is denoted by a vector $x^k_i := (x^k_i(1), x^k_i(2), \ldots, x^k_i(T))$. The aggregated charging profile of all the customers is described by a vector $x^k := (x^k_1, \ldots, x^k_N)$. Let $x^{up}_i(t)$ denote the maximum charging rate for the $i$-th EV during the $t$-th time slot and $S_i$ denote the desired total charge for the $i$-th EV at the end of the $k$-th day;

$$S_i = \sum_{t=1}^{T} x^k_i(t),$$

for any $k$. The total load as seen by the distribution company is the sum of the base load and the charging rates adopted by all the EVs.

As stated earlier, the objective of the distribution company is to achieve a total load profile that is valley-filling while ensuring that both the base load and the EVs are supplied with the desired power. The base load is inflexible, while the EV charging profile may be shaped as long as the desired energy amount is provided by the end of the day. Following the discussion in [8], the goal of the distribution company can be described as obtaining the aggregated charging profile $x^k, k \in \mathbb{N}_{>0}$ that solves the system-level optimization problem,

$$\text{minimize } \sum_{t \in \mathcal{T}} U(D^k(t) + \sum_{i \in \mathcal{N}} x^k_i(t))$$

subject to $0 \leq x^k_i(t) \leq x^{up}_i(t), t \in \mathcal{T}, i \in \mathcal{N}$,

$$\sum_{t \in \mathcal{T}} x^k_i(t) = S_i, i \in \mathcal{N},$$

where $U(\cdot)$ is a convex function. For instance, [8], [9] proposed the function $U(x) = x^2$ to obtain a valley-filling total load profile. Let $c^k_u := \sum_{t \in \mathcal{T}} U(D^k(t) + \sum_{i \in \mathcal{N}} x^k_i(t))$ be the distribution company’s cost function on the $k$-th day. By solving (1), the distribution company can obtain a valley-filling total load profile and satisfy EV customers charging requests every day [8]. In order to ensure that lines
and transformers are not overloaded and voltage deviations are within limits, one can include the constraints in the formulation of alternating current optimal power flow into (1). For example, the convex relaxation proposed in [16] can be adopted to formulate the corresponding system-level optimization. In this paper, we focus on the design of an online algorithm to minimize regret.

On the other hand, the objective of the customers is to minimize their charging cost while ensuring that their EVs are charged up to the desired level every day. Thus, we formulate a customer-level optimization problem in which each customer \( i, i \in \mathcal{N} \) requires a charging profile \( x_i^k, k \in \mathbb{N}_{>0} \) that solves

\[
\begin{align*}
\text{minimize} & \quad c_i^k(x_i^k) \\
\text{subject to} & \quad 0 \leq x_i^k(t) \leq x_i^{up}(t), \quad t \in \mathcal{T}, \quad \sum_{t \in \mathcal{T}} x_i^k(t) = S_i, \quad k \in \mathbb{N}_{>0},
\end{align*}
\]

where \( c_i^k(.) \) is a convex function in \( x_i^k \), representing the cost that the \( i \)-th customer wants to minimize. In general, \( c_i^k(.) \) is a function of the base load \( D^k \) and other customer’s charging profiles as well.

In order to preserve security and privacy by avoiding the communication of the charging constraints of EVs, work such as [8], [9], [19] designed a charging algorithm to solve (1) in a distributed manner. However, these algorithms suffered from problem noted in Section I.

It is worth pointing out that it is not a priori obvious that the solution of (1) and (2) coincide. In fact, because the distribution company and its customers hold different objectives, in general, the solution of (1) will not solve the customer-level problem (2). In our problem formulation, we design algorithms to solve both (1) and (2) in a distributed manner by designing suitable cost functions \( c_i^k(.) \), \( i \in \mathcal{N}, \quad k \in \mathbb{N}_{>0} \).

III. REGRET MINIMIZATION FRAMEWORK

In this section, we consider a regret minimization framework to design the charging profile update daily. This framework is adopted to solve both the system-level problem (1) and the customer-level problem (2).

Before discussing the algorithm, we introduce some notation. Let \( L \) be a \( \lambda \)-strongly convex function with respect to a norm \( || \cdot || \). Let \( D_L(\cdot, \cdot) \) denote the Bregman divergence with respect to \( L \). \( || \cdot ||_L \) denotes the norm that is dual to \( || \cdot || \). \( \nabla L^{-1} \) denotes the inverse mapping of \( \nabla L \). For example, if \( L \) is selected to be the squared Euclidean norm, \( L(\cdot) = ||\cdot||^2 \), then the corresponding Bregman divergence \( D_L(x, y) \) is equal to the squared Euclidean distance \( ||x-y||^2 \), the inverse mapping \( \nabla L^{-1}(x) \) is equal to \( \frac{1}{2}x \), and the dual norm is the Euclidean norm.

Customer’s Perspective: For a price-sensitive customer, the objective is to minimize the cost for EV charging given the pricing profiles as provided by the distribution company daily and the price function realized over the previous days. The decisions to be made are the daily charging profiles.

For the \( i \)-th EV customer, the decision variable is the \( i \)-th customer’s charging profile on the \( k \)-th day \( x_i^k \). The feasible set of charging profiles is \( \mathcal{F}_i := \{x_i \in \mathbb{R}^T | 0 \leq x_i^k(t) \leq x_i^{up}(t), \quad t \in \mathcal{T}, \quad \sum_{t \in \mathcal{T}} x_i^k(t) = S_i \} \). Denote the solution of

\[
\begin{align*}
\min_{x_i \in \mathcal{F}_i} \sum_{k=1}^{K} c_i^k(x_i) \quad \text{by} \quad x^*.
\end{align*}
\]

The customer regret after \( K \) days is given by

\[
R_i(K) := \sum_{k=1}^{K} c_i^k(x_i^k) - \min_{x_i \in \mathcal{F}_i} \sum_{k=1}^{K} c_i^k(x_i).
\]

We adopt the Optimistic Mirror Descent (OMD) algorithm to minimize regret [22]. The OMD algorithm incorporates the prediction \( M^k \) for the gradient of the cost function \( \nabla c_i^k \) into the algorithm. For example, we can select the prediction \( M^k \) as the average of the gradients of the cost functions for the previous days, namely, \( M^k(x^k) = \frac{1}{K-1} \sum_{t=1}^{K} \nabla c_i^k(x_i^t) \).

The minimization of regret based on the OMD algorithm [22] leads to

\[
\begin{align*}
\lambda_{g+1}^k &= \nabla L_i^{-1} (\nabla L_i(h_i^k) - \eta_c \nabla c_i^k(x_i^k)), \\
x_i^{k+1} &= \arg \min_{x_i \in \mathcal{F}_i} \eta_c x_i^T M_i^{k+1} + D_L(x_i, h_i^{k+1}),
\end{align*}
\]

where \( \eta_c \in \mathbb{R} \) is an algorithm parameter, \( L_i \) is any \( 1 \)-strongly convex function, and \( h_i^k \) is an intermediate update of the charging profile. \( M_i^k \) is the prediction of the gradient of the cost function \( \nabla c_i^k \).

Distribution Company’s Perspective: The objective of the distribution company is to achieve a valley-filling total load profile and to charge all the EVs to the desired level daily.

For the distribution company, the decision variable is the aggregated charging profile on the \( k \)-th day \( x^k \). The feasible set of the aggregated charging profiles is \( \mathcal{F} := \mathcal{F}_1 \times \mathcal{F}_2 \times ... \times \mathcal{F}_N \). The solution of \( \min_{\mathcal{F} \sum_{k=1}^{K} c_i^k(x)} \) is denoted by \( x^* \).

The distribution company regret after \( K \) days is given by

\[
R_u(K) := \sum_{k=1}^{K} c_u^k(x^k) - \min_{x \in \mathcal{F}} \sum_{k=1}^{K} c_u(x).
\]

The minimization of regret based on the OMD algorithm [22] leads to

\[
\begin{align*}
\lambda_{u+1}^k &= \nabla L_u^{-1} (\nabla L_u(h_u^k) - \eta_u \nabla c_u(x^k)), \\
x^{k+1} &= \arg \min_{x \in \mathcal{F}} \eta_u x^T M_u^{k+1} + D_L(x, h_u^{k+1}),
\end{align*}
\]

where \( \eta_u \in \mathbb{R} \) is an algorithm parameter, \( L_u \) is any \( 1 \)-strongly convex function, and \( h_u^k \) is an intermediate update of the charging profile. \( M_u^k \) is the prediction of the gradient of the cost function \( \nabla c_u \).

An alternative notion that measures the performance of online algorithms is the so-called competitive ratio [3]. Unlike regret, competitive ratio compares the cost of an online algorithm and the cost of an optimal offline algorithm which does not require the optimal solutions in hindsight being identical for all the days. However, in general, there are no online algorithms that can minimize regret and competitive ratio simultaneously [3]. In this paper, we focus on the design of an online algorithm to minimize regret.
A. Convergence Results

The following results summarize the convergence of the regrets of the customers and the distribution company. The results can be proven using standard techniques [22].

Proposition 3.1: (Customer Regret): The iteration (4) converges in the sense that, for any $x^* \in \mathcal{F}$,

$$R_i(K) \leq \frac{1}{\eta_i} P_i + \eta_i \sum_{k=1}^{K} \frac{\eta_i}{2} \|\nabla c^k_i(x^k) - M^k_i\|^2,$$

(7)

where

$$P_i := \max_{x_i \in \mathcal{F}} L_i(x_i) - \min_{x_i \in \mathcal{F}} L_i(x_i).$$

(8)

In particular, if $\eta_i$ is chosen as $O\left(\frac{1}{\sqrt{K}}\right)$, then the average regret, i.e., $R_i(K)$, converges to zero as $K \to \infty$.

(Distribution Company Regret): The iteration (6) converges in the sense that, for any $x^* \in \mathcal{F}$,

$$R_d(K) \leq \frac{1}{\eta_d} P_d + \eta_d \sum_{k=1}^{K} \frac{\eta_d}{2} \|\nabla c^k_d(x^k) - M^k_d\|^2,$$

(9)

where

$$P_d := \max_{x_d \in \mathcal{F}} L_d(x_d) - \min_{x_d \in \mathcal{F}} L_d(x_d).$$

(10)

In particular, if $\eta_d$ is chosen as $O\left(\frac{1}{\sqrt{K}}\right)$, then the average regret, i.e., $R_d(K)$, converges to zero as $K \to \infty$.

Remark 3.1: The result about the convergence of the consumer’s regret and the distribution company’s regret provides a guarantee that as the number of days increases, the average performance of the charging profile generated by the above algorithm approaches the performance that is obtained by the optimal day-to-day invariant charging profile. Notice that the optimal day-to-day invariant charging profile can only be calculated in hindsight.

Remark 3.2: In general, the optimal day-to-day invariant charging profiles $x^*$ and $x^*_i$, $i \in \mathcal{N}$, do not solve the optimization problem (1) and (2), respectively since the optimal solutions of (1) and (2) may not be the same for different days.

Remark 3.3: In the sequel, for simplicity, $L^k_i$, $i \in \mathcal{N}$, for all $k$, is selected to be the squared Euclidean norm divided by 2, and the distribution company cost function $c^k_d(\cdot)$ is chosen as

$$c^k_d := \sum_{t \in T} (D^k(t) + \sum_{x^* \in \mathcal{F}} x^*_i(t))^2.$$  

(11)

The cost function (11) for the distribution company is also considered in [8] and [9].

B. Design of the Pricing Function

Notice that there is no a priori guarantee that the solutions $\{x^*_i\}$ of problem (2) solve the problem (1). In fact, unless the pricing function $c^k_d(\cdot)$ is carefully designed, these solutions will not be the same since the objectives of the distribution company and the EV customers are different. As shown in the following result, the natural choice of $c^k_d(\cdot)$ as

$$c^k_d = \left(\sum_{j=1}^{N} x^*_j + D^k\right)^T x^*_i$$

(12)

does not lead to solutions $(x^*_1, \ldots, x^*_N)$ that reduce the regret of the distribution company to zero. Notice that the cost function (12) is adopted in [8] and [9] to design a distributed charging algorithm.

Proposition 3.2: Consider the choice of $c^k_d(\cdot)$ as in (12). Let the customers adopt the regret minimization algorithm as in the iteration (4). Define the resulting aggregate profile as $x^k = (x^*_1, \ldots, x^*_N)$. The average regret of the distribution company as defined in (5) with these values of $x^k$ does not converge to zero as the total number of days goes to infinity.

We now propose a choice of $c^k_d$ to ensure that the objectives of the customers and the distribution company align.

Theorem 3.3: Consider the choice of $c^k_d$ as

$$c^k_d = \left(\frac{1}{2} x^*_i + \sum_{j \neq i}^{N} x^*_j + D^k\right)^T x^*_i, \quad i \in \mathcal{N}_p.$$  

(13)

Let the customers adopt the regret minimization algorithm as in the iteration (4). Define the resulting aggregate profile as $x^k = (x^*_1, \ldots, x^*_N)$. Select $\eta_u = \frac{1}{2}. \eta_d$. The average regret of the distribution company as defined in (5) with these values of $x^k$ converge to zero as the total number of days goes to infinity.

Note that to update the charging profile on day $k$, the $i$-th customer needs to know $2x^k_i + \sum_{j \neq i}^{N} x^k_j + D^k$ or $\sum_{j \neq i}^{N} x^k_j + D^k$ depending on whether the pricing function (12) or (13) is adopted, respectively. As the number of customer increases, the difference between $2x^k_i + \sum_{j \neq i}^{N} x^k_j + D^k$ and $\sum_{j \neq i}^{N} x^k_j + D^k$ will decrease. The distribution company can simply publish the information to inform the customers of the total load for the previous day.

IV. DIVERSE CUSTOMER BEHAVIOR

So far we have assumed that all customers are fully rational and minimize (2). In reality, customers may be lazy in the sense of charging their EVs with fixed inter-day profiles and not seek charging profiles that minimize (2). We now study the effect of a fraction of the customers being lazy.

A. Presence of Lazy Customers

Suppose that $N_l$ out of $N$ customers are lazy. Denote the set of lazy customers by $N_l$. Since for a lazy customer $i \in N_l$, she may charge her EVs with a fixed inter-day profile and not seek charging profiles that minimize (2), we now study the effect of a fraction of the customers being lazy.

The update of the charging profile for the lazy customer $i \in N_l$ (without the prediction, i.e., $M^k_i = 0$, $i \in \mathcal{N}$, $k \in \mathbb{N}_{>0}$) can be modeled as

$$h^{k+1}_i = \nabla L^k_i(\nabla L_i(h^k_i) - \eta_i (\nabla c^k_i(x^k_i) + \epsilon^k_i)),$$

$$x^{k+1}_i = \underset{x_i \in F_i}{\arg\min} D L_i(x_i, h^{k+1}_i),$$

(14)

where $\epsilon^k$ an error term that is used to model the lazy customer behavior. The error term explicitly quantifies the
inconsistency between the updates as designed by the distribution company for each customer to execute and the actual behavior (adopting a fixed inter-day charging profile) that the lazy customer has at the beginning of the day. For example, consider the cost function (13) for the customers. For a lazy customer \(i \in \mathcal{N}_l\) that executes a fixed inter-day charging profile daily, the error \(\epsilon^k_i \in \mathbb{R}^T\), \(k \in \mathbb{N}_{>0}\) is modeled by \(\epsilon^k_i = g^k_i\), where \(g^k_i = (\sum_{i=1}^N x^k_i + D^k_i)\), namely, the term \(g^k_i\) is equal to the total load profile on the \(k\)-th day.

Due to the presence of the lazy customers, the ability of the aggregate solution to be valley-filling and hence to minimize the cost function in problem (1) is decreased. The performance loss is given in the following result.

**Theorem 4.1:** Consider \(N\) EV customers out of which \(N_l\) are lazy. Let the lazy customers choose profiles as (4) (without the prediction) with cost function \(c^k_i = r_i\), \(i \in \mathcal{N}_l\), for any constant \(r \in \mathbb{R}\), the remaining customers as (4) (without the prediction) with cost function (13). The distribution company’s cost function is selected as (11). Select \(\eta_u = \frac{1}{2}\eta_l\). For any \(x^* \in \mathcal{F}\), the regret of the distribution company with these values of \(x^*\) is bounded as

\[
R_u(K) \leq \frac{1}{\eta_u} P_u + \eta_u \sum_{k=1}^{K} \left\| \left( \nabla c^k_u(x^k) + \epsilon^k \right) \right\|_2^2 + K \sum_{i \in \mathcal{N}_l} \left\| F_i \right\| \left\| \epsilon_i \right\|, \tag{15}
\]

where

\[
P_u := \max_{x \in \mathcal{F}} \left\{ L_u(x) - \min_{x \in \mathcal{F}} \left\{ L_u(x) \right\} \right\}, \tag{16}
\]

\[
\left\| F_i \right\| := \max_{x,y \in \mathcal{F}_i} \left\| x - y \right\|, \quad \left\| \epsilon_i \right\| := \max_k \left\| \epsilon^k_i \right\|. \tag{17}
\]

**Proof:** Following the proof of [22, Lemma 2], we have

\[
c^k_u(x^k) - c^k_u(x^*) \leq (x^k - x^*)^T \nabla c^k_u(x^k), \tag{18}
\]

and

\[
(x^k - x^*)^T \nabla c^k_u(x^k) \leq \frac{\eta_u}{2} \left\| \nabla c^k_u(x^k) \right\|_2^2 + \frac{1}{\eta_u} \left( D_{L_u}(x^*, h^k) - D_{L_u}(x^*, h^{k+1}) \right), \tag{19}
\]

where

\[
\nabla c^k_u(x^k) = \begin{bmatrix}
2(D^k + \sum_{i \in \mathcal{N}} x_i^k) \\
\vdots \\
2(D^k + \sum_{i \in \mathcal{N}} x_i^k)
\end{bmatrix}_{NT}. \tag{20}
\]

Note that for each \(i \in \mathcal{N}_l\), the cost function \(c^k_i\) is selected as a constant noted in Section IV-A. The corresponding gradient of the lazy customer’s cost function is zero, namely, for lazy customer \(i \in \mathcal{N}_l\),

\[
\nabla c^k_i(x^k) = (D^k + \sum_{i \in \mathcal{N}} x_i^k) + \epsilon^k_i = 0, \tag{21}
\]

where \(\epsilon^k_i = -(D^k + \sum_{i \in \mathcal{N}} x_i^k)\). Move \(\epsilon^k_i\), \(i \in \mathcal{N}_l\) to the right hand side of the inequality in (18). The reminder of the proof is followed by summing over \(k = 1, \ldots, K\) and collecting terms.

Note that, by choosing an appropriate choice of the parameter \(\eta_u\), the asymptotic average regret will converge to a constant, i.e., \(\lim_{K \to \infty} \frac{R_u(K)}{K} = \sum_{i \in \mathcal{N}_l} \left\| F_i \right\| \left\| \epsilon_i \right\|\). The size of the constant depends on the error terms \(\epsilon_i, i \in \mathcal{N}_l\) and the charging constraints of the lazy customers. The result explicitly quantifies the deviation from the desired performance in terms of the asymptotic average regret of the distribution company. Without lazy customers, the average regret will converge to zero.

**Remark 4.1:** For a lazy customer \(i \in \mathcal{N}_l\), the error term, \(\epsilon^k_i\), is equal to \(- \sum_{i=1}^N x_i^k + D^k_i\). The size of \(\left\| \epsilon_i \right\|\) can be bounded by the upper bounds of \(\left\| D^k_i \right\|\) and \(\left\| x_i^k \right\|\). The term \(\left\| F_i \right\|\) can be bounded by twice the radius of the smallest Euclidean ball containing \(\mathcal{F}_i\). Since the set \(\mathcal{F}_i\) is a polytope, the algorithm in [15] can be adopted to compute the radius of the smallest Euclidean ball that contains \(\mathcal{F}_i\).

After characterizing the effects of customer behavior to the system performance, we now introduce the notion of controllable customer to mitigate the effects to the desired total load profile caused by the errors in the updates. The idea of controllable customer along with the associated controllable load gives us the flexibility that the distribution company can design in order to make the total load profile converge to a desired one even with the presence of different customer behavior (for example, price-sensitive, lazy behavior, etc.).

### B. Controllable Customers

To compensate for the performance loss due to the presence of lazy customers, the distribution company may have some loads to be completely controllable, namely, the corresponding customers adopt the charging profiles assigned by the company. Moreover, the charging constraints of the controllable EV customers are known and can be adjusted by the distribution company. In practice, the distribution company can offer a contract to a subset of customers offering a special price to be a controllable load. This contract-based direct load control has been implemented by many distribution companies, e.g., [1], [2].

We modify the charging constraints of the controllable EV customers to overcome the performance degradation caused by the presence of lazy customers. The main idea is to allow the charging constraints of the controllable customers to be relaxed (thus enlarging the feasible set of charging profiles for the controllable customers).

Denote the set of controllable customers by \(\mathcal{N}_c\). For the controllable customer \(i \in \mathcal{N}_c\), the relaxed feasible set of charging constraints is defined as \(\tilde{\mathcal{F}}_i := \{ \tilde{x}_i \in \mathbb{R}^T | 0 \leq \tilde{x}_i(t) \leq \tilde{x}^{up}_i(t), \ t \in T, \sum_{t \in T} a_i x_i(t) = \tilde{S}_i \}\), where \(a_i = \{0, 1\}\). Denote the charging profile with respect to the set \(\tilde{\mathcal{F}}\) on the \(k\)-th day by \(\tilde{x}_i^k\). The solution of \(\min_{\tilde{x}_i \in \tilde{\mathcal{F}}_i} \sum_{k=1}^K c^k_i(\tilde{x}_i)\) is denoted by \(\tilde{x}^*_i\). Denote the corresponding aggregate profile as \(\tilde{x}^k := (\tilde{x}^*_1, \ldots, \tilde{x}^*_K)\). There are different ways to relax the feasible set of charging constraints for the controllable customers. For example, if we select \(a_i = 0\) and \(\tilde{S}_i = 0\), then the equality constraint is removed from the feasible set of charging profiles for the \(i\)-th customer, \(i \in \mathcal{N}_c\), namely, the \(i\)-th controllable customer removes her total charging sum requirement daily. We can also relax the charging deadline by adjusting \(\tilde{x}^{up}_i(t)\).
By enlarging the feasible set of charging constraints, for controllable customer $i \in \mathcal{N}_c$, the iteration (4) provides a low bound for the cumulative cost function value $\sum_{k=1}^{K} c_k(x_k^i)$ obtained by using the same iteration (4) without relaxation. Notice that in the customer regret (3), we compare the cumulative cost generated by the optimal charging profile. The cumulative cost of the optimal charging profile is computed with respect to the charging constraints without relaxation. Therefore, it is possible to obtain negative regret for controllable customers. Negative regret means that the cumulative cost generated by the online algorithm with the relaxed charging constraints is less than the cumulative cost of the optimal charging profile without the relaxation.

V. Numerical Examples

Consider a scaled-down model. Suppose that there are 20 customers. A time slot representing an interval of 30 minutes is used. There are $T = 24$ time slots. The starting time is set to 8:00 pm. For simplicity, we consider that all EV customers have charging constraints such that charging is allowed from the 9th to the 16th time slots. For the first day, the initial charging profiles are assumed to be uniformly distributed among time slots. The maximum charging rate is $x_i^w(t) = 2$ kW, $i \in \mathcal{N}$ and the desired sum $S_i = 10$ kW, $i \in \mathcal{N}$. The simulation is carried out for total $K = 200$ days. We first examine the results of the total load profile updated by regret minimization. Figure 1 provides the total load profiles at convergence, with various number of lazy customers. We can see in Figure 1 that as the total number of lazy customers increases, the variation of total load profile increases. The result illustrates that the effect of lazy customers perturbs the valley-filling load profile. Now suppose that there are 10 lazy customers and 10 customers with relaxed constraints. We compare two different relaxation strategies. Relaxation 1 represents the strategy that allows the EV to be charged over the entire times slots (rather than merely between the 9th and the 16th time slots), whereas Relaxation 2 represents the one extending the charging time slots to cover from the 8th to the 17th time slots. We can see in Figure 1 that as the allowed charging time slots are extended, the total load profile resembles a valley-filling profile. Considering the same example, Figure 2 presents the evolution of the average regret for the distribution company due to aggregating the profiles generated by the customers. We can see that in the presence of a sufficient number of customers with the relaxed constraints, the average cumulative cost approaches the cost with the optimal day-to-day invariant charging profile in hindsight. As noted in Section IV-B, we have negative regret as the charging constraints are relaxed.

In practice, there are extra costs that the distribution company pays to the customers to have relaxed constraints. Define a relaxation index $Q$ as the number of time slots by which the starting time slot is advanced and the final slot is postponed. Assume that the cost $c_{\text{cont}}$ to implement the relaxed constraints is linear in the relaxation index, namely, $c_{\text{cont}} = AQ$, $A \in \mathbb{R}$. We compare the sum of the average regret on the final day (the 200th day) and the cost $c_{\text{cont}}$ as a function of the relaxation index $Q$. Figure 2 shows that there is a minimum at $Q = 4$, which indicates that this relaxation index provides the best trade-off between the cost to implement the relaxed constraints and the performance of the average regret of the distribution company.

Finally, we compare the performance of the regret minimization algorithm with base load that varies from day to day. The other simulation parameters are the same as the previous example. Suppose that for all odd number days, the base load is realized by Base Load 1 in Figure 3, whereas for all even number days, the base load is realized by Base Load 2 in Figure 3. We can see in Figure 3 that despite the fact the base load is switching and the distribution company is not aware of the changes in the base load, the regret minimization algorithm still provides updates of the charging profiles having converging behavior of the average regret. It indicates that regret minimization successfully adapts to a base load that varies from one day to the next.

VI. Conclusions and Critique

We have designed a framework for distributed charging control of EVs based on regret minimization. The proposed distributed charging mechanism can be implemented without low-latency two-way communication and has the advantage in terms of preserving security and privacy.

This paper explores the potential of the use of regret minimization for power system and EVs. The results are preliminary. There are many interesting open issues summarized as follows.

\footnote{The problem formulation (2) allows EV customers to have different charging constraints. The initial charging time and final charging time mainly depend on the preferences of EV customers.}
As noted in Section II, in order to ensure that power lines and transformers are not overloaded and voltage deviations are within limits, one can include the constraints in alternating current optimal power flow into (1). However, because of the coupled non-convex constraints in alternating current optimal power flow, the OMD algorithm needs to be modified to converge to global optimum in a distributed manner. The convex relaxation of alternating current optimal power flow [16] and online alternating direction method of multipliers [25] may be used to address this issue.

Regret defined in (3) and (5) compares the cumulative cost function value generated by the online algorithm and the one obtained from the optimal day-to-day invariant charging profile. One may either modify the online algorithm to get a better regret bound as comparing to optimal day-to-day varying charging profiles or derive a regret bound as a function of the variation of the optimal day-to-day varying charging profile. The notion of tracking regret [11] may be used to address this issue.

Also, one of the weaknesses of the current problem formulation is that the charging constraints cannot vary from day to day, which is not realistic in practice. In regret minimization, the constraints are usually assumed to be invariant over the iterations of the online algorithms [6]. Therefore, within the framework of regret minimization, in order to include day-to-day variation in charging constraints, one needs to modify the OMD algorithm and provide a convergence proof for the corresponding average regret.

Finally, as noted in Section III, competitive ratio is an alternative notion to measure the performance of online algorithms. Regret minimization and competitive ratio minimization are based on different assumptions. In general, there are no online algorithms that can minimize regret and competitive ratio simultaneously. Therefore, a comparison between the use of regret and the use of competitive ratio for power system and EVs is necessary.

REFERENCES