Abstract—An $H^\infty$ filtering problem for a system driven by uncertain previewable signals is discussed, and a design method of an $H^\infty$ filtering law is presented. The problem pertains to generalized $H^\infty$ filtering with previewable reference signals or disturbances. The solvability condition is characterized by the solutions of differential/algebraic Riccati equations, and some interpretations are provided for the equivalent class of $H^\infty$ problems. The features of the filtering law are examined using numerical examples.

I. INTRODUCTION

Preview control plays a key role in improving the transient response of controlled systems. The advantage of preview action has been studied from various points of view [3], [15], [13], [10], [8], [2], [14], [9], [5], [6], [7]. In [3], [15], a fundamental feature of preview action was investigated based on finite-horizon optimal control problems, $H^\infty$ preview control in the min-max setting was investigated in [13], and $H^2$ optimal preview control was discussed in [10], [8], [2]. The solutions to $H^\infty$ preview control problems have been also studied in [14], [9], [5], [6], [7], and it has been shown that the solvability condition and control laws are characterized using Hamiltonian matrices defined by the generalized plant.

In the literature on the preview controller design, it is generally assumed that the shape of previewable signal is accurate, and the achievable performance attained by the preview action has been discussed. However, in the broad range of applications such as system control with photovoltaic power generation forecasts or tracking control of man-machine interactive systems, the previewable information inherits uncertainty, and the preview control system is required to appropriately address the uncertainty of preview information.

In this paper, we focus on an $H^\infty$ filtering problem with uncertain preview information and present a design method of the filtering law. The problem is illustrated in Fig. 1. Our objective here is to determine the achievable $H^\infty$ performance based on estimation of the partial state of $\Sigma$ and the future information $w$. Employing the state-space approach on an appropriate signal space, we characterize the solvability condition using differential/algebraic Riccati equations and provide a design method of the filtering law. Consistent with the state-space approach discussed in [5], [6], [7], a broad class of disturbances that lie in the function space is introduced to describe the perturbed previewable information. The corresponding $H^\infty$ filtering problem is solved using the operator Riccati equation approach.

This paper is organized as follows. In Section II, an $H^\infty$ filtering problem with uncertain previewable signals is formulated, and fundamental results for the system description are presented. In Section III, the $H^\infty$ filtering problem is solved, and the solvability condition and filtering law are characterized based on the properties of differential/algebraic Riccati equations. In Section IV, the features of the proposed $H^\infty$ filtering law are examined using numerical examples, and the fundamental properties of the filtering system are described. The conclusions of this paper are presented in Section V.

II. FORMULATION AND PRELIMINARIES

Focus on the $H^\infty$ filtering problem defined by the system $\Sigma$ (Fig. 1):

$$\Sigma: \begin{align*}
\dot{x}(t) &= Ax(t) + Bw(t-h) \\
z^0(t) &= C_1 x(t) \\
z^1(t, \beta) &= E_1(\beta) w(t + \beta) \\
y^0(t) &= C_2 x(t) + Dv^0(t) \\
y^1(t, \beta) &= E_2(\beta) w(t + \beta) + F(\beta) v^1(t, \beta)
\end{align*}$$

$$(-h \leq \beta \leq 0).$$

In the formulation $\Sigma$, $w(t) \in \mathbb{R}^l$ is the previewable disturbance, $(y^0(t), y^1(t, \cdot)) \in \mathbb{R}^{p_0} \times L_2(-h, 0; \mathbb{R}^{p_1})$ is the measurement, $(v^0(t), v^1(t, \cdot)) \in \mathbb{R}^{p_0} \times L_2(-h, 0; \mathbb{R}^{p_1})$ is the measurement noise, and $(z^0(t), z^1(t, \cdot)) \in \mathbb{R}^{p_0} \times L_2(-h, 0; \mathbb{R}^{p_1})$ is the generalized state of $\Sigma$ to be estimated. It is noted that the system dynamics is driven by the delayed disturbance $w(t-h)$, and on the other hand, up to current information $w(t + \beta)$ ($-h \leq \beta \leq 0$) is included in the measurement $y^1(t, \beta)$. Thus the formulation $\Sigma$ enables to deal with the filtering problem with the preview information.
w. For the system Σ, we will derive an $H^\infty$ preview filtering law based on the uncertain measurement (1d), (1e).

In the filtering problem Σ, the previewable information $w(t+\beta)$ ($-h \leq \beta \leq 0$) is regarded as a part of internal state, and the input/output signals are defined on product function spaces. For the simplicity of the description, we introduce the following notation for the signals $(v^0, v^1)$, $(z^0, z^1)$, and $(y^0, y^1)$.

\[ \hat{v}(t) = \begin{bmatrix} v^0(t) \\ v^1(t) \end{bmatrix} \in V := \mathbb{R}^{m_0} \times L_2(-h, 0; \mathbb{R}^{m_1}) \]

\[ v^1(\beta) := v^1(t, \beta), \quad -h \leq \beta \leq 0 \]  

\[ \hat{z}(t) = \begin{bmatrix} z^0(t) \\ z^1(t) \end{bmatrix} \in Z := \mathbb{R}^{p_0} \times L_2(-h, 0; \mathbb{R}^{p_1}) \]

\[ z^1(\beta) := z^1(t, \beta), \quad -h \leq \beta \leq 0. \]  

\[ \hat{y}(t) = \begin{bmatrix} y^0(t) \\ y^1(t) \end{bmatrix} \in Y := \mathbb{R}^{q_0} \times L_2(-h, 0; \mathbb{R}^{q_1}) \]

\[ y^1(\beta) := y^1(t, \beta), \quad -h \leq \beta \leq 0. \]  

For a given $\gamma > 0$, the $H^\infty$ preview filtering problem (6) is solvable iff the condition (A) is satisfied.

\[ \text{Denote the estimation of } \hat{z}(t) = (z^0(t), z^1(t)) \text{ by} \]

\[ \hat{u}(t) = \begin{bmatrix} u^0(t) \\ u^1(t) \end{bmatrix} \in Z = \mathbb{R}^{p_0} \times L_2(-h, 0; \mathbb{R}^{p_1}) \]

\[ u^1(\beta) := u^1(t, \beta), \quad -h \leq \beta \leq 0. \]

We will solve the $H^\infty$ preview filtering problem defined as follows.

**$H^\infty$ Preview Filtering Problem:** For a given $\gamma > 0$, find a causal filtering law $\hat{u} = \Phi(\hat{y})$ such that the inequality

\[ J := \sup_{\beta \in L_2} \left\| \hat{z} - \hat{u} \right\|_{L_2(0;\infty; Z)}^2 < \gamma^2 \]  

holds.

The $H^\infty$ preview filtering problem covers the estimation of the previewable signal $w(t+\beta)$ ($-h \leq \beta \leq 0$) as well as the partial state $C_1 \hat{x}(t)$ and, further, enables to clarify the estimation mechanism under the worst case uncertainties. We will make the following assumptions:

(H1) $C_2, A$ is detectable,

(H2) $E_1(\cdot), E_2(\cdot), F(\cdot)$ are bounded piecewise continuous functions.

(H3) $D$ is full column rank, and $F(\beta) F^T(\beta) > I$ ($\epsilon > 0$) holds for $\forall \beta \in [-h, 0]$, and prepare an auxiliary system description on appropriate function space.

For the filtering problem Σ, introduce a state-space $\mathcal{X} := \mathbb{R}^n \times L_2(-h, 0; \mathbb{R}^\ell)$ endowed with the inner product:

\[ \langle \psi, \phi \rangle := \psi^T \phi^0 + \int_{-h}^0 \psi^T(\beta) \phi^1(\beta) d\beta \]

\[ \psi = (\psi^0, \psi^1) \in \mathcal{X}, \quad \phi = (\phi^0, \phi^1) \in \mathcal{X}. \]  

On the state-space $\mathcal{X}$, the system $\Sigma$ is described by the following evolution equation [11]:

\[ \dot{\Sigma}: \quad \begin{cases} \dot{x}(t) = A x(t) + B w(t) \\ \dot{z}(t) = C_1 x(t) \\ \dot{y}(t) = C_2 x(t) + D \hat{v}(t) \end{cases} \]  

where the state is denoted by

\[ \dot{x}(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \in \mathcal{X} := \mathbb{R}^n \times L_2(-h, 0; \mathbb{R}^\ell) \]

\[ w(t) := w(t+\beta), \quad -h \leq \beta \leq 0. \]  

The operator $\mathcal{A}$ is an infinitesimal generator defined by

\[ \mathcal{A} \phi = \begin{bmatrix} A \phi^0 + B \phi^1(-h) \\ \phi^{1T} \end{bmatrix}, \]

\[ D(\mathcal{A}) = \{ \phi \in \mathcal{X} : \phi^1 \in W^{1,2}(-h, 0; \mathbb{R}^\ell), \phi^1(0) = 0 \} \]  

where $W^{1,2}(-L, 0; \mathbb{R}^\ell)$ is the Sobolev space of $\mathbb{R}^\ell$-valued, absolutely continuous functions with square integrable derivatives on $[-h, 0]$. The adjoint operator of $\mathcal{A}$ is given by

\[ \mathcal{A}^* \psi = \begin{bmatrix} A^T \psi^0 \\ \psi^{1T} \end{bmatrix}, \]

\[ D(\mathcal{A}^*) = \{ \psi \in \mathcal{X} : \psi^1 \in W^{1,2}(-h, 0; \mathbb{R}^\ell), \psi^1(-h) = B^T \psi^0 \} \]  

Let $\mathcal{V}^* := D(\mathcal{A}^*)$ be a subspace of $\mathcal{X}$. Then $D_V(\mathcal{A}) = \mathcal{X}$ holds, and $\mathcal{X}, \mathcal{V}$ are with continuous, dense injections satisfying $\mathcal{X} \subset \mathcal{V}$ ([11], Remark 2.6). The operators $\mathcal{B} \in L(\mathcal{L}(\mathcal{F}, \mathcal{V}), C_1 \in L(\mathcal{X}, \mathcal{Z}), C_2 \in L(\mathcal{X}, \mathcal{Y}), D \in L(\mathcal{V}, \mathcal{Y})$ are defined as follows:

\[ B \psi = \psi^1(0) \in \mathcal{R}^\ell, \quad \psi = (\psi^0, \psi^1) \in \mathcal{V}^*, \]

\[ C_1 \phi = \begin{bmatrix} (C_1 \phi)^0 \\ (C_1 \phi)^1 \end{bmatrix}, \quad \phi = (\phi^0, \phi^1) \in \mathcal{X}, \]

\[ (C_1 \phi)^0 = C_1 \phi^0, \quad (C_1 \phi)^1(\beta) = E_1(\beta) \phi^1(\beta), \]

\[ C_2 \phi = \begin{bmatrix} (C_2 \phi)^0 \\ (C_2 \phi)^1 \end{bmatrix}, \quad \phi = (\phi^0, \phi^1) \in \mathcal{X}, \]

\[ (C_2 \phi)^0 = C_2 \phi^0, \quad (C_2 \phi)^1(\beta) = E_2(\beta) \phi^1(\beta), \]

\[ D \psi = \begin{bmatrix} D \psi^0 \\ D \psi^1 \end{bmatrix}, \quad \psi = (\psi^0, \psi^1) \in \mathcal{V}, \]

\[ (D \psi)^0 = D \psi^0, \quad (D \psi)^1(\beta) = F(\beta) \psi^1(\beta), \quad (\beta \in [-h, 0]). \]  

The infinite-dimensional system $\hat{\Sigma}$ is in the class of Pritchard-Salamon systems [11], [12], and the solution of the $H^\infty$ filtering problem $\Sigma$ is characterized by corresponding operator Riccati equation [16]:

\[ \mathcal{P} \mathcal{A}^* \phi + \mathcal{A} \mathcal{P} \phi + \frac{1}{\gamma} \cdot \mathcal{P} C^1 \mathcal{C}_1 \mathcal{P} \phi \]

\[ - \mathcal{P} C^2 (D D^*)^{-1} C_2 \mathcal{P} \phi + B_1 B_1^* \phi = 0, \quad \phi \in \mathcal{V}^*. \]  

In the sequel, we will solve the operator Riccati equation (13) and establish a design method of the $H^\infty$ filtering law which attains the performance (6). The solution of the $H^\infty$ filtering problem $\Sigma$ is formally characterized by the following proposition.

**Proposition 1 ([16]):** For a given $\gamma > 0$, the $H^\infty$ preview filtering problem (6) is solvable iff the condition (A) is satisfied.
(A) The operator Riccati equation (13) has a stabilizing solution $P \geq 0$ such that $A + \frac{1}{\gamma^2} \cdot PC_1 C_1 - PC_2 (D D^T)^{-1} C_2$ generates an exponentially stable semigroup on $\mathcal{X}$, $\mathcal{Y}$.

If (A) holds, an $H^\infty$ filtering law which attains (6) is given by

$$\dot{\hat{x}}(t) = C_1 \hat{x}(t) \quad (14a)$$

$$\dot{\hat{y}}(t) = A \hat{y}(t) + PC_2 (D D^T)^{-1} (\hat{y}(t) - C_2 \hat{x}(t)). \quad (14b)$$

III. MAIN RESULTS

Based on the preliminary results stated in Section II, we will clarify the solvability condition (A) in Proposition 1 and provide a design method of the filtering law (14). The operator Riccati equation (13) has a special structure and enables to characterize a disturbance attenuation problem where the distributed disturbance $v^1(t, \cdot)$ is imposed on the generalized system measurement $y^1(t, \cdot)$.

The following theorem clarifies the solvability condition, and the analytic solution of (13) by introducing auxiliary differential/algebraic Riccati equations.

**Theorem 2:** For a given $\gamma > 0$, the condition (A) and (a1), (a2) are equivalent.

(a1) The differential equation

$$- \Pi'(\beta) = \frac{1}{\gamma^2} \cdot \Pi(\beta) E_1^T(\beta) E_1(\beta) \Pi(\beta)$$

$$- \Pi(\beta) E_2^T(\beta) (F(\beta) F^T(\beta))^{-1} E_2(\beta) \Pi(\beta), \quad (15a)$$

$$\Pi(0) = I \quad (15b)$$

has a bounded solution $\Pi(\beta)$ ($-h \leq \beta \leq 0$) such that

$$\exists \epsilon > 0: \Pi(\beta) > \epsilon \cdot I, \quad \forall \beta \in [-h, 0] \quad (16)$$

holds.

(a2) The algebraic Riccati equation

$$PA^T + AP + \frac{1}{\gamma^2} \cdot PC_1^T C_1 P$$

$$- PC_2^T (D D^T)^{-1} C_2 P + B \Pi(-h) B^T = 0 \quad (17)$$

has a stabilizing solution $P \geq 0$ such that $A + \frac{1}{\gamma^2} \cdot PC_1 C_1 - PC_2 (D D^T)^{-1} C_2$ is stable.

If (A) or (a1), (a2) are satisfied, the stabilizing solution $P \geq 0$ of (13) is given as follows:

$$P = \begin{bmatrix} P & 0 \\ 0 & \Pi \cdot I \end{bmatrix}. \quad (18)$$

**Proof:** (a1), (a2) $\Rightarrow$ (A): Under (a1), (a2), we first show that (18) meets a positive semi-definite stabilizing solution of (13). By (a1), (a2), it is verified that (18) is positive semi-definite. The following equalities are obtained for the terms in the left hand side of (13):

$$\langle \psi, PA^T \phi \rangle + C_1 \phi = \left( \begin{bmatrix} \psi_0 \\ 0 \end{bmatrix}, \begin{bmatrix} P & 0 \\ 0 & \Pi \cdot I \end{bmatrix} \begin{bmatrix} A^T \psi \phi \\ \phi \psi \end{bmatrix} \right) = \psi^0 T P A^T \phi^0 - \int_{-h}^0 \psi_{1T}^T (\beta) \Pi(\beta) \phi_1^T (\beta) d\beta \quad (19)$$

$$\langle \psi, A \phi \psi \rangle + C_1 \phi = \langle PA^T \psi, \phi \rangle \psi \psi \quad (20)$$

Summing up the equalities (19)-(22), then substituting (15a), (15b), (17), and $\psi_1(-h) = B^T \psi, \psi^1(-h) = B^T \psi$ which are obtained from $\psi, \phi \in \mathcal{V}^*$, we finally obtain the equality:

$$\langle \psi, (PA^T + A \phi \psi) \psi \rangle + \frac{1}{\gamma^2} \cdot \langle \psi, PC_1^T C_1 \phi \psi \rangle + \frac{1}{\gamma^2} \cdot \langle \psi, BB^T \phi \psi \rangle$$

$$= - \psi^0 T B \Pi(-h) B^T \phi^0 + \psi_{1T}^0 (0) \phi_1^0 \quad (23)$$

Thus (18) is a positive semi-definite solution of (13).

Since the closed loop system $\hat{x}(t) = \{ A + \frac{1}{\gamma^2} \cdot PC_1^T C_1 - PC_2^T (D D^T)^{-1} C_2 \} \hat{x}(t), \hat{x}(0) \in \mathcal{X}$ is expressed as

$$\hat{x}(t) = \{ A + \frac{1}{\gamma^2} \cdot PC_1^T C_1 - PC_2^T (D D^T)^{-1} C_2 \} \hat{x}(t), \hat{x}(0) \in \mathcal{X}$$

$$+ Bx^1(t, -h) \quad (24)$$

$$\frac{\partial}{\partial t} x^1(t, \beta) = \frac{\partial}{\partial \beta} x^1(t, \beta) + K(\beta) x^1(t, \beta), \quad x^1(t, 0) = 0 \quad (25)$$

$$K(\beta) := \frac{1}{\gamma^2} \cdot \Pi(\beta) E_1^T(\beta) E_1(\beta)$$

$$- \Pi(\beta) E_2^T(\beta) (F(\beta) F^T(\beta))^{-1} E_2(\beta) \quad (26)$$

the solution is bounded in $0 \leq t \leq h$ and, for $t > h$, the equality $x^1(t, \beta) = 0$ ($-h \leq \beta \leq 0$) holds. Hence by (a2), the operator (18) meets the stabilizing solution of (13).

(A) $\Rightarrow$ (a1), (a2): For a given $\gamma > 0$, we will prove by contradiction that the condition (A) derives (a1) and (a2). It is noted that, if (A) holds for $\gamma > 0$, the equation (13) has a uniformly bounded stabilizing solution $P \geq 0$ in the interval $\gamma \in [\gamma_*, \infty)$.

Suppose that (a1) holds only for $\gamma \in (\gamma_1, \infty)$, and (a2) is satisfied for $\gamma \in (\gamma_1, \infty)$. In other words, assume that the solution of (15): $\Pi(\beta) (-h \leq \beta \leq 0)$ is bounded for $\gamma = \gamma_1$. If $\gamma_1 \geq \gamma_*$, it is derived that the stabilizing solution (18) is not uniformly bounded in the interval $\gamma \in (\gamma_1, \infty)$. This fact contradicts the assumption that the stabilizing solution $P \geq 0$ of (13) is uniformly bounded in $\gamma \in [\gamma_*, \infty)$. Thus it is shown that the condition (A) requires (a1).
Suppose that (a2) holds only for $\gamma \in (\gamma_2, \infty)$, and (a1) is satisfied for $\gamma \in (\gamma_2, \infty)$. If $\gamma_2 \geq \gamma^*$, it is shown by \[1\] that $\|P\| \to \infty$ ($\gamma \to \gamma_2 + 0$) holds, and the solution (18) is not uniformly bounded in the interval $\gamma \in (\gamma_2, \infty)$. Thus it is shown that (A) is satisfied (a2).

By Theorem 2, it is shown that the $H^\infty$ performance (6) is characterized based on the solutions of differential/algebraic Riccati equations (15), (17). The following lemma further simplifies the expression (15).

**Lemma 3:** Define an auxiliary differential equation:

$$\Delta^1(\beta) = \frac{1}{\gamma^2} : E_d^T(\beta)E_1(\beta)$$

$$E_2^T(\beta)(F(\beta)F^T(\beta))^{-1}E_2(\beta),$$

$$\Delta(0) = 0.$$ \hspace{1cm} (27a)

The differential equation (15) has a bounded solution $\Pi^1(\beta)$ ($-h \leq \beta \leq 0$) satisfying (16) iff (27) has a bounded solution $\Delta^1(\beta)$ ($-h \leq \beta \leq 0$) such that

$$\exists \varepsilon > 0 : \Delta^1(\beta) > (\varepsilon - 1) \cdot I, \forall \beta \in [-h, 0].$$ \hspace{1cm} (28)

If exists, the solution of (15) is expressed as

$$\Pi^1(\beta) = (I + \Delta^1(\beta))^{-1}. \hspace{1cm} (29)$$

**Proof:** Suppose (15) is a stabilizing solution $\Pi^1(\beta)$ ($-h \leq \beta \leq 0$) satisfying (16). Substituting (29) and

$$\Pi^1(\beta) = -(I + \Delta^1(\beta))^{-1}\Delta^1(\beta)(I + \Delta^1(\beta))^{-1}$$

(30)

to (15a), the equation (27a) is obtained. Furthermore (15b), (16), and (29) yield (27b), (28).

Conversely suppose (27) has a bounded solution $\Delta^1(\beta)$ ($-h \leq \beta \leq 0$) satisfying (28). Multiplying both sides of (27a) by $(I + \Delta^1(\beta))^{-1}$, then substituting (29), (30), the equation (15a) is obtained. The equality (15b) and the positive semi-definiteness (16) follow from (27b), (28), and (29).

By Lemma 3, the equation (15) is characterized by the first order differential equation (27). This fact enables us to solve (17), and the analytic solution is expressed as follows:

$$\Pi(\beta) = 1 + \int_\beta^0 \{ - \frac{1}{\gamma} : E_d^T(\tau)E_1(\tau)$$

$$+ E_2^T(\tau)(F(\tau)F^T(\tau))^{-1}E_2(\tau) \} d\tau \].$$ \hspace{1cm} (31)

In the general cases such that $E_1(\cdot) \neq 0$ holds, the boundedness of the solution (31) is not guaranteed, and (28) characterizes the limitation of achievable $H^\infty$ performance caused by the measurement noise $u^1(t, \cdot)$. While in case of $E_1(\cdot) = 0$ which excludes the estimation of $z^1$, the solution satisfying (15), (16) always exists, and the $H^\infty$ performance is directly characterized by the condition (a2) in Theorem 2 with $\Pi(-h)$ given by (31).

Based on Theorem 2, Lemma 3, the solution of the $H^\infty$ preview filtering problem is obtained by the following theorem.

**Theorem 4 (Main results):** For a given $\gamma > 0$, the $H^\infty$ preview filtering problem (6) is solvable iff the conditions (a1), (a2) in Theorem 2 are satisfied. Furthermore, the $H^\infty$ preview filtering law is given by

$$u^0(t) = C_1z^0(t)$$

$$u^1(t, \beta) = E_1^1(\beta, \gamma)$$

$$\dot{z}^0(t) = A_2z^0(t) + Bz^1(t, -h)$$

$$+ G(y^1(t) - Cz^0(t))$$

$$\frac{\partial}{\partial \beta} z^1(t, \beta) = \frac{\partial}{\partial \beta} z^1(t, \beta)$$

$$H(\beta) := \Pi^1(\beta)(F(\beta)F^T(\beta))^{-1}$$

(32c)

where $\Pi(\beta) > 0 (-h \leq \beta \leq 0)$ and $P \geq 0$ are defined by (15), (16), and (17).

**Proof:** By Theorem 2, Lemma 3, the solvability condition is characterized by (a1), (a2).

Rewriting the control law (14b) in the following form:

$$\dot{\tilde{z}}^0(t) = A_2\tilde{z}^0(t) + f(t)$$

$$f(t) = PC^1(DD^2)^{-1}((y(t) - Cz^0(t)),$$

we first derive (32c), (32d). Since $\langle \psi, \tilde{z}(t) \rangle_{V^*, V} = \langle \psi, A_2\tilde{z}^0(t) + f(t) \rangle_{V^*, V}$, $\forall \psi \in V^*$ holds, the following representation is obtained for $\tilde{z}(t) := (\dot{z}^0(t), z^1(t, \cdot)) \in \mathcal{X}$,

$$f^0(t) = G(y^0(t) - Cz^0(t))$$

$$f^1(t, \beta) = H(\beta)(y^1(t, \beta) - E_2(\beta)z^1(t, \beta))$$

$$\dot{z}^0(t) = A_2z^0(t) + Bz^1(t, -h) + f^0(t)$$

$$\frac{\partial}{\partial \beta} z^1(t, \beta) = \frac{\partial}{\partial \beta} z^1(t, \beta) + f^1(t, \beta).$$

Thus (32c), (32d) are derived from (34a)-(34d). The expressions (32a), (32b) are directly derived from (14a).

By Theorems 2, 4, and Lemma 3, the solvability of the $H^\infty$ filtering problem is characterized by the solutions of differential and algebraic Riccati equations (15), (17). Under the existence of uniformly bounded solution $\Pi(\beta)$ ($-h \leq \beta \leq 0$) in (31), the solvability condition is further transformed to the standard $H^\infty$ filtering problem:

$$\dot{\bar{z}}(t) = A\bar{z}(t) + \Pi^{1/2}(-h)Bw(t)$$

$$\bar{z}(t) = C_1\tilde{x}(t)$$

$$y(t) = C_2\tilde{x}(t) + Dw(t)$$

which requires the $H^\infty$ filtering performance:

$$\bar{J} := \sup_{w, \dot{w} \in L_2} \frac{\|w - \theta\|^2_{L_2}}{\|\dot{w}\|^2_{L_2} + \|w\|^2_{L_2}} < \gamma^2$$

by a causal filtering law $u = \Phi(y)$ (see also [17]).

The $H^\infty$ preview filtering law (34) is constructed based on a standard observer (32c) with an infinite-dimensional system (32d) which inherits a dynamics of transport systems. From the practical point of view, some discretization technique
such as average approximation will be employed for the realization of the filtering law (32). The feature of the filtering law is illustrated in the numerical example.

IV. EXAMPLE

Focus on the preview filtering problem defined by

\[
\begin{align*}
x(t) &= \begin{bmatrix} -0.5 & 0 \\ 1 & -0.2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t - h) \quad (36a) \\
z_0(t) &= \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} x(t) \quad (36b) \\
z^1(t, \beta) &= E_1(\beta) w(t + \beta) \quad (36c) \\
y^0(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + 0.1 \cdot v^0(t) \quad (36d) \\
y^1(t, \beta) &= w(t + \beta) + F(\beta)v^1(t, \beta) \quad (36e) \\
E_1(\beta) &= 1/\sqrt{h} \cdot E_{10} \quad (36f) \\
F(\beta) &= 1/\sqrt{h} \cdot F_0 \quad (36g)
\end{align*}
\]

where \( E_1(\cdot) \) is the weight for the estimation of preview information \( w \), and \( F(\cdot) \) is for the uncertainty of the measurement \( y^1 \). In this formulation, the functions \( E_1(\cdot), F(\cdot) \) are normalized by (36f), (36g), and the achievable \( H^\infty \) performance (6) is investigated in terms of the length of preview time \( h \).

For the following cases:

- (Case 1) \( E_{10} = 1 \),
- (Case 2) \( E_{10} = 0 \),

we will investigate the achievable \( H^\infty \) performance \( \gamma > 0 \) defined by (6). Case 1 (\( E_{10} = 1 \)) deals with a general filtering problem where the estimation of preview information is included in the performance (6). While Case 2 deals with the estimation of the partial state \( C_1 x(t) \) and enables to illustrate the standard filtering problem with the effect of preview information.

In Case 1, the relation between the optimal filtering performance \( \gamma_{opt} \) (see (6)) and the preview time \( h \) is summarized by Fig. 3 (\( F_0 = 0.1 \sim 5.0 \)). For each weight \( F_0 \) reflecting the uncertainty of the measurement \( y^1(t, \cdot) \), it is observed that the filtering performance is improved as the preview time \( h \) increases. However, in the case the preview information includes large uncertainty (e.g. \( F_0 = 5.0 \)), the filtering performance is not significantly recovered even if long preview time is employed.

In Case 2, the relation between the optimal filtering performance for the partial state \( z_0^0(t) \) is investigated, and the relations between \( \gamma_{opt} \) and the preview time \( h \) are summarized by Fig. 4. In like manner of Case 1, the filtering performance is improved as the preview time increases. For the filtering law (32) which is designed by \( \gamma = 1.05 \times \gamma_{opt} \) with \( h = 1.0 \), the \( \sigma \)-plots from the disturbance \((\bar{v}, w)\) to the estimation error \( z^0 - u^0 \) are obtained by Fig. 5. It is observed that the peak gain is decreasing as the uncertainty of the measurement is reduced. Furthermore, in this example,

\[ \text{Fig. 2. } H^\infty \text{ estimation problem for a preview system.} \]

\[ \text{Fig. 3. } H^\infty \text{ performance vs. preview time (Case 1: } E_1 \neq 0). \]

\( \text{Fig. 4. } H^\infty \text{ performance vs. preview time (Case 1: } E_1 \neq 0). \]

the filtering performance is improved in a broad range of frequency as the uncertainty decreases.

V. CONCLUSION

An \( H^\infty \) filtering problem with uncertain preview information was discussed, and based on the operator Riccati equation approach, the solvability condition and the filtering law were determined. The solvability condition is characterized by the solutions of differential/algebraic Riccati equations. The \( H^\infty \) filtering law was obtained by combining the standard observer and the transport system dynamics, which updates the estimation of the previewable exogenous signals. The features of the filtering law were investigated using numerical examples.

The filtering law obtained here provides a basis for solving generalized \( H^\infty \) preview control problems which deal with uncertain reference signals, and makes it possible to obtain solutions using the state-space approach [4]. Development of a design method for generalized uncertain preview systems will be the focus of future research.

ACKNOWLEDGMENT

This work was supported by JST CREST and JSPS under Grant-in-Aid for Scientific Research (C) 26420423.
Fig. 4. $H^\infty$ performance vs. preview time (Case 2: $E_1 = 0$).

Fig. 5. $\sigma$-plots of the filtering system (Case 2: $E_1 = 0$).

REFERENCES