A Novel Bounded Cooperative Multi-Rate Current-Sharing Control for Parallel Charging System with Directed Communication*

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Abstract—In order to address the output current imbalanced problem of a parallel charging system with directed communication for energy storage type light rail vehicles, a cooperative current-sharing scheme is proposed in this paper. This parallel charging system is a non-identical nonlinear multi-agent systems with bounded input constraints by considering each charger as an agent. Each charger transmits the current state information to its neighbors via communication network, which can be modeled by a directed graph. Input-output feedback linearization is introduced to convert the current-sharing control of non-identical nonlinear parallel charging system into a first-order multi-rate integrator current consensus-tracking problem. To satisfy the boundedness of control input, a general saturation function is put forward to design the bounded cooperative current-sharing control law based on the nearest neighborhood rule. A novel Lyapunov candidate function depending on the communication topology is proposed to rigorously prove the cooperative current-sharing stability of closed-loop system, which is under the fixed directed topology provided that the digraph only has a directed spanning tree. Several charging schemes are compared and analyzed. The output current of parallel chargers can be balanced by adopting the proposed approach, which is verified by simulating a parallel charging test system.

I. INTRODUCTION

As a new type of electric traction light rail transportation system, the energy storage type light rail vehicle adopts super-capacitors as its power supply. With this energy storage technology, there is no need to construct a traction power grid and it is possible to recover the energy of regenerative braking. Energy storage type light rail vehicles need to be charged in seconds by the charging system when it parks at the platform. Therefore, the charging system should provide a large enough output power to shorten the charging process. An effective method to solve this problem would be by connecting several chargers in parallel with each other to increase system capacity [1]. However, the challenge for the charging system is how to balance the charging current between the chargers. If the charging current is not balanced, the charger with a higher output current has to bear a greater output power, which may lead to a large thermal stress and degrade the reliability and performance of the whole charging system. It is therefore necessary to design an effective current-sharing control strategy to balance the output current for charging system.

Several conventional approaches exist for current-sharing problems, such as central current-sharing control method, droop control method [2][3], master-slave method [4][5],etc. However, such methods are undistributed, restrictive and have some limitations, which are not the appropriate approach for the charging system of energy storage type light rail vehicles when the chargers parallel with each other. Over the last one decade, distributed consensus and cooperative control of multi-agent systems have been studied extensively [6], giving results in the areas such as flocking [7], formation control [8], distributed mobile sensor network [9], rendezvous in space, and autonomous vehicles [10]. Distributed cooperative control has also been recently introduced in power systems to regulate the output power of multiple photovoltaic generators [11], which has also been adopted to synchronize the output voltage and frequency of distributed generators in microgrids and shipboard power system [12]-[13].

Taking each charger in the charging system as an agent in multi-agent systems, the current-sharing control design resembles a current consensus tracking problem, where the charger’s current tracks the reference. Several chargers are connected in parallel with each other in the charging system of an energy storage type light rail vehicle. The adjacent chargers interact with each other and exchange the state information such as the charging current via communication network. This method is distributed, each charger only requires its own information and the information of some neighbors, and one faulty node cannot cause the collapse of the whole charging system, which is an appropriate method for the charging system to solve the current-sharing problem.

While the charging system has some physical constraints, the input duty ratio is bounded. And there are inevitably component error and manufacturing error in each charging system, the dynamics of the chargers in the charging system is non-identical. The charger has intrinsic nonlinear characteristics because of Buck DC/DC circuit’s principle and super capacitor’s feature. The state information can be exchanged via industrial bus in a directed way. This paper seeks to address the challenge of how to design a bounded cooperative multi-rate current-sharing control law under directed communication and take into account both the charger’s non-identical and nonlinear features.

The rest of this paper is organized as follows: current-sharing problems of the charging system are set out in Section II, the current-sharing control strategy based on
distributed cooperative control of multi-agent systems is proposed in Section III, and cooperative stability is proved in Section IV. The proposed current-sharing strategy is verified in Section V. Section VI concludes this paper.

II. CURRENT-SHARING PROBLEM FORMULATION

In this section, the problem to be solved for the charging system is set out in details.

As shown in Fig. 1, the charging system of an energy storage type light rail vehicle consists of 10kV AC supply grid, 10kV/900V AC converter, and several parallel charger subsystems. Each charger subsystem exchange state information with the aid of a communication network. The charger subsystem mainly consist of a three-phase bridge rectifier circuit and chopper BUCK DC / DC circuit.

Considering each charger as an agent, with the help of average value modeling method [14], the nonlinear parallel chargers system with non-identical nodes shown in Fig. 1 can also be uniformly described by

\[
\begin{align*}
\dot{x}_i &= f_i(x_1, x_2, \cdots, x_n) + g_i(x_i)u_i \\
y_i &= h_i(x_i)
\end{align*}
\]

(1)

where \( i = 1, 2, \cdots, n \), \( x_i = \begin{bmatrix} I_i \\ \frac{U}{U_c} \end{bmatrix} \) is the \( i \)-th charger’s state, \( u_i(t) \in R \) is the control input, \( f_i(x_1, x_2, \cdots, x_n), g_i(x_i) \) are bounded Lipschitz continuous function, and given below.

\[
\begin{align*}
f_i(\cdot) &= \left[ \begin{array}{c}
-\frac{L}{T} I_i - \frac{U_c}{T} \\
\frac{1}{c_0 + c_i} \sum_{k=1}^{n} I_k
\end{array} \right] \\
h_i(x_i) &= I_i
\end{align*}
\]

(2)

where \( I_i \) is the output current by each charging sub-system, \( U_c \) is super-capacitor’s voltage, \( U_{d} \) is the DC input voltage which is obtained by three-phase bridge rectifier. \( L_i \) is the flux for the energy storage inductor, \( r_i \) is the equivalent resistance of the circuit, \( c_i \) is the super capacitor’s capacitance. The capacitance \( c_0 \) and voltage-dependent capacitance \( c_i \) are parameters values of the super capacitor, which are related with the super capacitor’s terminal voltage \( U_c \).

When the energy storage rail vehicle reaches the platform, it must be fully charged by the charging system in seconds so that it can run to the next station with enough power to be recharged again. Because manufacture error inevitably exists in the AC adapter, the DC input voltage \( U_{d} \) can not be exactly the same. There are also component errors in each charger sub-system. Thus, each charger is non-identical and has intrinsic nonlinear feature due to the complex circuit and super-capacitor’s principle. In addition, the line connection of the circuit can not be guaranteed to be completely symmetrical. Such reasons above will result in that the output current of each charger is not balanced, seriously affecting charging performance.

In this paper, in order to carry out current-sharing objective, we should design a bounded cooperative control \( u_i(t) \) such that

\[
\lim_{t \to +\infty} |y_i(t) - y_0| = 0
\]

(3)

where \( y_0 \) is the reference current.

Definition 2.1: The reference current \( y_0 \) can be computed by equation below:

\[
y_0 = \frac{I_c}{n} = \frac{1}{n} C_{sc}(0)(U_c(t_d) - U_c(0))
\]

(4)

where \( U_c(t_d) \) is the desired voltage within the required charging time \( t_d \), \( C_{sc}(0) \) is the initial capacitances and \( U_c(0) \) is the initial residual voltage of the super-capacitor, which depend on the physical design of the charging system and are given in advance. \( I_c \) represents the total charging current, which can be computed in advance according to the above parameters \( U_c(t_d), t_d, U_c(0), C_{sc}(0) \) and equation (4). \( n \) is the number of parallel chargers involved in the charging process, which can be known by communication.

The output current balance problem i.e. current-sharing control design resembles a consensus tracking problem of nonlinear and non-identical multi-agent systems with bounded input constraint. The current-sharing strategy based on distributed cooperative control will be proposed in the next section.
III. Multi-rate Current-sharing strategy based on distributed cooperative control

In this section, a preliminary of graph theory is firstly presented. Then, a cooperative current-sharing controller is designed under fixed topology based on the nearest neighbor principle by using input-output feedback linearization.

A. Preliminary of Graph Theory

The communication topology of the parallel charging system can be modeled by a graph [15]. A graph is usually denoted as $G(v, e, A)$ with a nonempty finite set of $n$ nodes $v = \{ v_1, v_2, \ldots, v_n \}$, a set of edges or arcs $e \subseteq v \times v$, and the associated adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$.

**Definition 3.1:** A graph without loops is called a simple graph [7] i.e. the adjacency matrix diagonal elements $a_{ii} = 0$ and its Laplacian matrix is denoted as $L = [l_{ij}]_{n \times n}$.

$$l_{ij} = \begin{cases} \sum_{k=1\atop k \neq i}^n a_{ki}, & j = i \\ -a_{ij}, & j \neq i \end{cases}$$ (5)

All row sums of $L$ are equal to zero, that is $L1_n = 0$, $1_n$ is a column vector with $n$ elements equal to 1.

A directed path from node $i$ to node $j$ is a sequence of edges, expressed as $\{ (v_i, v_{i1}), (v_{i1}, v_{i2}), \ldots, (v_{im}, v_j) \}$. A digraph is said to have a spanning tree, if there is a node $i$ (called the root), such that there is a directed path from the root to every other node in the graph.

B. Current-Sharing Controller design under Directed Communication Topology

It is not easy to design the controller and analyze the cooperative stability because the parallel charging system considered in this paper is nonlinear and non-identical. Input-output feedback linearization can be adopted to facilitate the current-sharing strategy design [16].

It follows that the time derivative of the output $y_i$ is

$$\dot{y}_i = \frac{\partial h_i(x_i)}{\partial x_i} \dot{x}_i = \frac{\partial h_i(x_i)}{\partial x_i} f_i(x_i, x_2, \ldots, x_n) + g_i(x_i)u_i$$

$$= L_{jh}h_i + L_{gh}g_iu_i$$ (6)

where $L_{jh}h_i = \nabla h_i f_i = \frac{\partial h_i(x_i)}{\partial x_i} f_i(x_i, x_2, \ldots, x_n)$, $L_{gh}g_i = \nabla h_i g_i = \frac{\partial h_i(x_i)}{\partial x_i} g_i(x_i)$.

Define an auxiliary control $\vartheta_i$ as follows:

$$\vartheta_i = L_{jh}h_i + L_{gh}g_iu_i$$ (7)

Substituting (7) into (6) and with the help of input-output feedback linearization, the parallel charging system can be decoupled into the first-order integrator multi-agent systems as follows:

$$\dot{y}_i = \vartheta_i$$ (8)

Since the model of the charger is based on converter by using average value modeling method, the duty cycle of Insulated Gate Bipolar Transistor is bounded. Therefore, we introduce a saturation function $\phi(\cdot)$ in the proposed cooperative control to guarantee the boundedness of the control input.

**Assumption 3.1:** A general saturation function $\phi(\cdot)$ satisfy:

1. $\phi(\cdot)$ is Lipschitz continuous,
2. $\phi(z) = 0 \Leftrightarrow z = 0$,
3. $\phi(z) > 0, \forall z \neq 0$,
4. $\phi_{\text{min}} \leq \phi(z) \leq \phi_{\text{max}}, \forall z \in \mathbb{R}$.

**Remark 3.1:** One possible choice of saturation functions is hyperbolic tangent function as used in [17]. Nonetheless, any function satisfying Assumption 3.1 would work. To this end, once $\vartheta_i$ is bounded, the original control input $u_i$ is also bounded based on inverse transformation of (7) since $f_i(\cdot)$ and $g_i(\cdot)$ are bounded.

In order to make the current-sharing controller designing much more flexible and satisfy the charging requirement for each non-identical charging subsystem, we choose different coupling strength $c_i$ to design a cooperative multi-rate current-sharing control law for each sub-system instead of using the traditional common coupling strength $c$ [18]. Therefore, the proposed auxiliary control $\vartheta_i$ can be designed with the aid of the saturation function $\phi(\cdot)$ and coupling strength $c_i$ based on the nearest neighborhood rule as follows:

$$\vartheta_i = c_i\phi(\sum_{j \in N_i} a_{ij}(y_j - y_i) + \rho_i(y_0 - y_i))$$ (9)

where $\phi(z)$ satisfy Assumption 3.1, $N_i = \{ v_j \in v : (v_j, v_i) \in e \}$ is the set of neighbors of $v_i$, $a_{ij} > 0$ means that for the $i_{th}$ charger, it can receive the information of the $j_{th}$ charger. $c_i > 0$ is the coupling strength. $\rho_i$ is the pinning gain and $\rho_i > 0$ for at least one $i$, which means that at least one charger knows the reference charging current. From (9), we know that the current-sharing control law $\vartheta_i$ only based on the $i_{th}$ charger’s current information and the information of its neighbors.

**Remark 3.2:** The coupling strength $c_i$ in (9) is not exactly the same, which doesn’t depend on the whole network topology. Therefore, the transformed and decoupled system (8) is a multi-rate first-order integrator system. The convergence speed can be improved by tuning $c_i$ appropriately.

When the communication topology is fixed, the final cooperative current-sharing controller is formulated as follows:

$$u_i = c_i\phi(\sum_{j \in N_i} a_{ij}(y_j - y_i) + \rho_i(y_0 - y_i)) - L_{jh}h_i$$

$$u_i = \frac{L_{gh}g_i}{L_{jh}}$$ (10)

As the communication between each charger is implemented by CAN Bus communication protocol in a directed way [19], the communication network can be modeled by a directed graph.

**Assumption 3.2:** The digraph $G(v, e, A)$ has a spanning tree and $\rho_i > 0$ for at least one root node, i.e. at least one charger can know the reference current information. \hspace{1cm} \diamond

The local neighborhood current tracking error can be defined below

$$e_i = \phi(\sum_{j \in N_i} a_{ij}(y_i - y_j) + \rho_i(y_i - y_0))$$ (11)
And then the auxiliary control $\vartheta_i$ can be rewritten as

$$\vartheta_i = c_i \phi \left( \sum_{j \in N_i} a_{ij} (y_j - y_i) + \rho_i (y_0 - y_i) \right) = -c_i e_i \quad (12)$$

To this end, by substituting (10) into (6), we have the closed-loop sub-system dynamics

$$\dot{y}_i = c_i \phi \left( \sum_{j \in N_i} a_{ij} (y_j - y_i) + \rho_i (y_0 - y_i) \right) = -c_i e_i \quad (13)$$

From (11), the global current tracking error vector is written as

$$e = \phi ((L + G) (Y - Y_0)) = \phi ((L + G) \delta) \quad (14)$$

where the global current vector is defined as $Y = [y_1 \ y_2 \ \cdots \ y_n]^T$, $e = [e_1 \ e_2 \ \cdots \ e_n]^T$, $Y_0 = 1_n \otimes y_0$, $\otimes$ is the Kronecker product. $G = \text{diag} \{ \rho_i \} \in R^{n \times n}$ is a diagonal matrix. $\delta = [y_1 - y_0 \ y_2 - y_0 \ \cdots \ y_n - y_0]^T$ is the global disagreement vector.

Furthermore, the closed-loop overall system dynamics can be gained with aid of Laplacian matrix as follows

$$\dot{Y} = -C \phi ((L + G) (Y - Y_0)) \quad (15)$$

where $C = \text{diag} \{ c_i \} \in R^{n \times n}$ is a coupling strength matrix.

IV. COOPERATIVE CURRENT-SHARING STABILITY ANALYSIS

In this section, the cooperative current-sharing stability analysis of the parallel charging system under the fixed directed communication topology with the aid of the Lyapunov function integrating LaSalle Invariant Principle is given.

**Theorem 1:** Consider the parallel charging system (1) with fixed communication topology. Under the Assumption 3.2 that the digraph $G(v,e,A)$ has a spanning tree and $\rho_i > 0$ for at least one root node, then the current-sharing objective below can be achieved by the distributed cooperative current-sharing control law (10) and (9)

$$\lim_{t \to +\infty} |y_i(t) - y_0| = 0 \quad (16)$$

i.e. the current state of the all chargers can ultimately be consensus and track the desired current. Furthermore, the overall closed-loop system (15) is asymptotically stable.

**Proof:** For the closed-loop system (15), since $L + G$ is a positive definitive matrix when the directed graph has a spanning tree [18], the Lyapunov candidate function can be chosen as follows:

$$V = \frac{1}{2} \delta^T (L + G)^T \delta \quad (17)$$

Differentiate the Lyapunov function $V$ along the system’s trajectory with respect to time $t$, and then substitute the state equation of the closed-loop system into the derivative function $\dot{V}$, we can have

$$\dot{V} = \delta^T (L + G)^T \dot{\delta}$$

$$= \delta^T (L + G)^T \dot{Y}$$

$$= -\delta^T (L + G)^T C \phi ((L + G) \delta)$$

$$= -\delta^T (L + G)^T C \phi ((L + G) \delta) \leq 0 \quad (18)$$

Since $(L + G) \delta$ and $\phi ((L + G) \delta)$ have the same sign component-wise, $C > 0$ is a positive weighted matrix, we get $\dot{V} \leq 0$. To this end, the global disagreement vector $\delta$ is stable.

Note that $\dot{V} \equiv 0$ implies that $\delta = 0$ when the associated directed graph has a spanning tree ($(L + G) > 0$ and $C > 0$), which, in turn, implies that $\dot{\delta} = 0$. By LaSalles Invariance principle, it follows that $\delta(t) \to 0$ asymptotically as $t \to +\infty$, i.e. $\lim_{t \to +\infty} |\delta(t)| = 0$. That is to say, the global disagreement vector $\delta$ is asymptotically stable. Furthermore, we can obtain $\lim_{t \to +\infty} |y_i(t) - y_0| = 0, \forall i \neq j$ since the fact is that $\dot{\delta}_i = y_i - y_0,

To this end, the current state of all the chargers can ultimately be consensus and track the desired current. Furthermore, the overall closed-loop system (15) is asymptotically stable.

This completes the proof.

**Remark 4.1:** The proposed novel Lyapunov candidate function matrix $(L + G)^T$ is only related with the directed communication topology among the parallel chargers. The stability analysis is also effective to the parallel charging systems when the coupling gains $c_i$ are all chosen to be the same. Although all chargers are connected in parallel with each other physically, the communication among them is implemented by CAN Bus communication protocol in cyber layer [19].

V. CASE STUDIES

In this section, we use the charging test system shown in Fig. 1 to validate the feasibility of the proposed cooperative control scheme. Several different cases below are considered and compared in this section.

A. Simulation Results For Different Charging Case

Case I: The charging current is chosen only to be 360A in the whole charging process.

Case II: The whole charging process includes two sequential phases, namely a fast charging phase and a trickle charging phase. At the fast charging phase, the charging current is chosen to be 360A. At the trickle charging phase, the charging current is chosen to be 80A.

In our case studies, 5 nonidentical chargers parallel with each other and consist of the charging system. The main physical parameters for this 5 nonidentical chargers systems are given in Table 1. The given communication topology between each charging sub-system is shown in Fig. 2, which has a directed spanning tree obviously.

![Fig. 2](chart.png)

The simulation parameters for the super capacitor are $R_1 = 5.6m\Omega, C_0 = 92.3F, C_v = 0.0747F/V$. Only one charger (No.1) has access to the reference current, i.e. $\rho_1 = 1, \rho_2 = \cdots \cdots \rho_5 = 0$. The given directed communication topology for the parallel chargers system

![Chart](chart.png)
$\rho_3 = \rho_4 = \rho_5 = 0$; In Case I and Case II, the coupling strengths $c_1 = 1.4, c_2 = 1.8, c_3 = 2.2, c_4 = 2.6, c_5 = 3$. The initial current $y_i(0) = 0A, i = 1, 2, 3, 4, 5$, the super-capacitor’s initial voltage $U_c(0) = 500V$ in all Cases.

Table 1  The main physical parameters for the charging systems model

<table>
<thead>
<tr>
<th>Charger $i$</th>
<th>$L_i$ (mH)</th>
<th>$r_i$ ($\Omega$)</th>
<th>$U_d$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.05</td>
<td>0.0035</td>
<td>1335</td>
</tr>
<tr>
<td>2</td>
<td>3.12</td>
<td>0.0031</td>
<td>1272</td>
</tr>
<tr>
<td>3</td>
<td>2.95</td>
<td>0.0029</td>
<td>1295</td>
</tr>
<tr>
<td>4</td>
<td>3.01</td>
<td>0.0040</td>
<td>1371</td>
</tr>
<tr>
<td>5</td>
<td>2.98</td>
<td>0.0030</td>
<td>1301</td>
</tr>
</tbody>
</table>

The current-sharing curve for Case I and Case II under the directed communication topology in Fig. 2 are plotted in Fig. 3 and Fig. 4 respectively, which illustrates that the current has been balanced and the consensus and cooperative objective has been achieved. As shown in Fig. 4, the charging current ascends to be 360A from 0A within 5s, and then it goes to be constant current charging stage until the super-capacitor is charged to be about 870V. Since the super-capacitor’s voltage will decline at the end of charging procedure when we always choose the large charging current due to the super-capacitor’s intrinsic feature, the charging current is then reduced to 80A. As Fig. 5(b) shows, the voltage is charged fully from 500V to about 900V finally, illustrating that voltage objective has been achieved by adopting the proposed control strategy.

Comparing Case I with Case II, as shown in Fig. 5, the super-capacitor’s voltage will decline a little at the end of charging procedure by using Case I when the charging current is chosen only to be 360A. On the contrary, when we choose 360A as the charging current at the first stage and 80A at the last stage by using Case II, the super-capacitor can be charged fully to be 900V finally.

B. The proposed cooperative current-sharing approach with respect to failures

When a charger (No.5) is faulty in the parallel charging system, such fault can be tolerant under the condition that the communication topology for the remaining chargers has a directed spanning tree. The corresponding current-sharing curve is plotted in Fig. 6, the remaining 4 chargers’ output current converge to the new cooperative objective (450A) automatically after the charger 5 failure.

When communication link failures occur inevitably in the parallel charging system, the cooperative objective can still be achieved provided that the union of the time-varying communication topologies have a directed spanning tree frequently enough as the system evolves by. However, as shown in Fig. 7, the convergence time with link failures is much longer than that of the case without link failures in Fig. 3. In addition, the system performance is degraded to some extent due to the link failures.
In this paper the output current imbalance problem of parallel charging system with directed communication for energy storage type light rail vehicles is successfully addressed by a cooperative current-sharing scheme. Such current-sharing strategy is implemented based on the concept of the nonlinear and non-identical multi-agent systems with the bounded input constraints. The proposed approach is distributed, each charger only needs its own current information and the information of its neighbors. Only one charger has access to the reference current, all chargers can converge to the cooperative object via communication network that is a directed graph. The current-sharing control of the nonlinear and non-identical parallel charging system is transformed into a first-order multi-rate integrator consensus tracking problem with bounded input constraints. A general saturation function satisfying several assumption conditions can maintain the boundedness of control input. The proposed novel Lyapunov candidate function is only related with the graph Laplacian and pinning gain matrix, which is found to prove the stability of the whole closed-loop charging system provided that the directed graph has a spanning tree. The convergence speed can be tuned by choosing the controller parameters appropriately in a flexible way. Case studies results show that our current-sharing approach can balance the output current of each parallel charger.

VI. CONCLUSIONS

In this paper the output current imbalance problem of parallel charging system with directed communication for energy storage type light rail vehicles is successfully addressed by a cooperative current-sharing scheme. Such current-sharing strategy is implemented based on the concept of the nonlinear and non-identical multi-agent systems with the bounded input constraints. The proposed approach is distributed, each charger only needs its own current information and the information of its neighbors. Only one charger has access to the reference current, all chargers can converge to the cooperative object via communication network that is a directed graph. The current-sharing control of the nonlinear and non-identical parallel charging system is transformed into a first-order multi-rate integrator consensus tracking problem with bounded input constraints. A general saturation function satisfying several assumption conditions can maintain the boundedness of control input. The proposed novel Lyapunov candidate function is only related with the graph Laplacian and pinning gain matrix, which is found to prove the stability of the whole closed-loop charging system provided that the directed graph has a spanning tree. The convergence speed can be tuned by choosing the controller parameters appropriately in a flexible way. Case studies results show that our current-sharing approach can balance the output current of each parallel charger.

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