Abstract—This paper presents a novel approach to fuel efficient vehicle speed control. Motivated by potential fuel economy improvements through periodic time-varying cruise control, a finite state machine structure is designed to control transitions between discrete states of acceleration, constant speed, and deceleration. The scheme accounts for changes in road grade, reduces fuel consumption, maintains drivability, and is easy to calibrate and implement. A method for optimizing the thresholds that control transitions between the states of the finite state machine is introduced and demonstrated. Results from simulation and vehicle testing are presented for a test route, demonstrating up to a 5.47% dynamic fuel consumption improvement potential in vehicle testing.

I. INTRODUCTION

As fuel economy standards become more stringent, innovative methods for improving fuel economy are being researched. Some methods focus on adaptation of, or implementation of, new technologies into the powertrain or vehicle. Others focus on optimizing the vehicle speed trajectory using preview-based electronic horizon (e-horizon) and commute learning techniques.

Cruise control strategies that produce fuel efficient trajectories for vehicles with traditional powertrains have been reported in [3], [13], [16], [17], see also references therein. Applications of model-predictive control for fuel efficient cruise have been considered in [1], [5], [7]. In [3], [17], periodic vehicle speed trajectories have been studied based on periodic optimal control theory. In [13], [14], periodic vehicle speed trajectories have emerged from a stochastic optimal control policy and have been shown to improve fuel economy as compared to constant speed driving in simulations and real vehicle tests.

In this paper we define a novel rule based cruise control (RBCC) strategy that generates a time-varying periodic vehicle speed profile near a prescribed setpoint that reduces fuel consumption. As we show based on results of simulations and experimental vehicle tests, RBCC improves fuel economy over constant speed trajectories. A comparison will be made in this paper between the results from the RBCC strategy and from stochastic dynamic programming strategies tested on the same platform, see [13], [14].

The RBCC strategy modifies vehicle speed as determined by a set of rules generated by a finite state machine described below. This paper presents the RBCC framework and demonstrates an optimization methodology for tuning state transition thresholds. The states of the machine are: normal cruise, glide, bleed, and recover, see Figures 1 and 2. These states were chosen because they replicate phases of the periodic speed trajectories observed in our previous publications [13], [14].

II. PROBLEM FORMULATION

For the purpose of optimizing vehicle cruise performance, we consider a fuel consumption model of the form,

$$f_c(k) = G(V_{driver}, \Delta V(k), \delta(k)), \quad (1)$$

where $f_c(k) \in \mathbb{R}$ is the instantaneous fuel consumption at time instant $k$, $V_{driver} \in \mathbb{R}$ is the driver selected cruise control setpoint, $\Delta V(k) \in \mathbb{R}$ is the modification to driver selected cruise control setpoint with the commanded speed as $V_{driver} + \Delta V(k)$, and $\delta(k) \in \mathbb{R}$ is the instantaneous road grade. We impose the constraint $\Delta V(k) \leq 0$ so that the commanded speed does not exceed $V_{driver}$. Furthermore, let $V(k)$ be the instantaneous vehicle speed. Note that the continuous dynamics are determined by the vehicle and the nominal cruise controller and we treat these dynamics as being discretely sampled.

For this paper, two baseline fuel consumption calculations are referenced. The first is the result of $\Delta V(k) = 0$, that is, $V(k) = V_{driver}$. The resulting fuel consumption over a route driven at a constant speed of $V_{driver}$ is $f_{c, driver}$. The second is a result of $V(k) = V_{ave}$ where $V_{ave}$ is the average speed of the trajectory from the RBCC strategy, with a resulting fuel consumption over a route of $f_{c, ave}$. The objective is to develop a control strategy that maximizes the cost function,

$$f_{c, ave} \frac{\sum_{k=0}^{n} f_c(k)}{f_{c, driver}} \times 100. \quad (2)$$

The cost function (2) quantifies the dynamic fuel consumption improvement (DFCI). Specifically, (2) is a percent improvement over the baseline $f_{c, driver}$ along a route with horizon $n$, but discounts fuel economy improvement due to a change in the average vehicle speed and indirectly penalizes large deviations of vehicle speed from the cruise control setpoint.

Note that if $f_{c, dyn} > 0$, the RBCC strategy is more fuel efficient than traveling at $V(k) = V_{ave}$. If, in addition, $f_{c, driver} \geq f_{c, ave}$, then the RBCC strategy is more fuel efficient than traveling at $V(k) = V_{driver}$. Also note that the distinction is made between $V_{ave}$ and $V_{driver}$. $V_{driver}$ is the...
maximum allowed speed set by the driver while \( V_{ave} \) is the average speed the vehicle travels using the RBCC strategy.

III. RULE BASED CRUISE CONTROL

In this section we describe the structure and state transitions of the finite state machine with outputs \( \Delta V(k) \), and neutral command \( n(k) \in \{0,1\} \), where \( n(k) = 0 \) indicates the vehicle is in gear at a time instant \( k \) and \( n(k) = 1 \) indicates the vehicle is in neutral. The finite state machine is shown graphically in Figure 1 with its four state transitions. Here \( A_\Delta \) is the normal to glide grade transition threshold, \( \delta_{NB} \) is the neutral to bleed grade transition threshold, and \( \delta_{BR} \) is the bleed to recovery state transition threshold. Furthermore, assume that \( \delta_{NG} \leq \delta_{NB} \) and \( V_0 < V_d \).

The mappings \( f(A_x(k), x(k)) \), \( h_1(x(k)) \), and \( h_2(x(k)) \), are defined for each state as follows:

A. Normal State - \( \mathcal{N} \)

The normal cruise state of the RBCC holds a speed setpoint determined by the driver. In the normal state, allowable state transitions are

\[
x(k+1) = f(A_\mathcal{N}, \mathcal{N}) = \begin{cases} 
\mathcal{G}, & \text{if } d_s = d_p = 1 \\
\mathcal{B}, & \text{if } d_s = d_{ph} = 1 \\
\mathcal{N}, & \text{if } d_s(1 - (1 - d_p)(1 - d_{ph})) = 0 
\end{cases},
\]

where \( A_\mathcal{N} = \{ V_r, \delta_{NG}, \delta_{NB} \} \),

\[
d_{pl} = \begin{cases} 
1, & \text{if } \delta(k) \leq \delta_{NG} \\
0, & \text{if } \delta(k) > \delta_{NG} 
\end{cases},
\]

\[
d_{ph} = \begin{cases} 
1, & \text{if } \delta(k) \geq \delta_{NB} \\
0, & \text{if } \delta(k) < \delta_{NB} 
\end{cases},
\]

\[
d_s = \begin{cases} 
1, & \text{if } V_{driver} - V(k) \leq V_r \\
0, & \text{if } V_{driver} - V(k) > V_r 
\end{cases},
\]

where \( \delta(k) \) is the average predicted grade over the horizon \( k, \ldots, k + r \). The method used in this paper for road grade prediction is presented in the Appendix. The alteration to the cruise control setpoint is

\[
\Delta V(k) = h_1(\mathcal{N}) = \Delta V(k-1),
\]

and the neutral command is

\[
n(k) = h_2(\mathcal{N}) = 0.
\]

B. Glide state - \( \mathcal{G} \)

The glide state allows the vehicle to naturally decelerate. In the glide state, allowable state transitions are

\[
x(k+1) = f(A_\mathcal{G}, \mathcal{G}) = \begin{cases} 
\mathcal{R}, & \text{if } d_s = 1 \\
\mathcal{G}, & \text{if } d_s = 0 
\end{cases},
\]

where \( A_\mathcal{G} = \{ V_d \} \), and

\[
d_s = \begin{cases} 
1, & \text{if } V_{driver} - V(k) \geq V_d \\
0, & \text{if } V_{driver} - V(k) < V_d 
\end{cases}.
\]

The alteration to the cruise control setpoint is

\[
\Delta V(k) = h_1(\mathcal{G}) = V(k) - V_{driver},
\]

where \( V_{\mathcal{G}} \) is a positive constant. The neutral command is

\[
n(k) = h_2(\mathcal{G}) = 1.
\]

C. Bleed - \( \mathcal{B} \)

The bleed state holds an engine torque level on high grade to reduce vehicle speed loss. In the bleed state, allowable state transitions are

\[
x(k+1) = f(A_\mathcal{B}, \mathcal{B}) = \begin{cases} 
\mathcal{N}, & \text{if } d_{sh} = 1 \\
\mathcal{G}, & \text{if } (1 - d_{ph})(1 - d_p) = 0 \\
\mathcal{B}, & \text{if } (1 - d_{sh})(1 - d_s)(1 - d_p) = 1 
\end{cases},
\]

where \( A_\mathcal{B} = \{ V_d, V_r, \delta_{BR} \} \),

\[
d_{sh} = \begin{cases} 
1, & \text{if } V_{driver} - V(k) < V_r \\
0, & \text{if } V_{driver} - V(k) \geq V_r 
\end{cases},
\]

\[
d_{sl} = \begin{cases} 
1, & \text{if } V_{driver} - V(k) \geq V_d \\
0, & \text{if } V_{driver} - V(k) < V_d 
\end{cases},
\]

and

\[
d_p = \begin{cases} 
1, & \text{if } \delta \leq \delta_{BR} \\
0, & \text{if } \delta > \delta_{BR} 
\end{cases}.
\]

The alteration to the cruise control setpoint is

\[
\Delta V(k) = h_1(\mathcal{B}) = F_1(F_{PI}(\bar{q})(\tau_B - \tau(k)) - F_2V(k)) + V(k) - V_{driver},
\]
IV. OPTIMIZATION

The state transitions in the finite state machine in Section III are dependent on six threshold parameters. These parameters are tuned to maximize the cost (2) subject to driver comfort considerations. The tuning guidelines and our off-line, model-based optimization procedure are now described.

The first parameter is the normal cruise set point speed, $V_{\text{driver}}$. The RBCC varies vehicle speed in a neighborhood below this value. As in conventional adaptive cruise control systems, $V_{\text{driver}}$ limits the maximum speed the vehicle will reach and is set according to speed limits or driver comfort.

The second parameter is the speed at which the vehicle speed is considered to be recovered, $V_r$. Its value is close to, but less than, $V_{\text{driver}}$, and is intended to stop the vehicle speed from overshooting $V_{\text{driver}}$. The value of $V_r$ is set manually depending on the performance of the nominal cruise controller observed in vehicle experiments.

The third parameter is the maximum allowed drift speed from the set point speed, $V_d$. Its value is determined as a result of the optimization but its range is restricted by considerations of the driver comfort. If the vehicle slows down too much with respect to the driver set point speed, the driver may become uncomfortable or the vehicle may start impeding the traffic flow.

The fourth, fifth, and sixth parameters are the road grade-related parameters $\delta_{NG}$, $\delta_{NB}$, and $\delta_{BR}$. These three parameters and $V_d$ are determined through the optimization.

The optimization problem is to determine the threshold parameter vector,

$$p = [V_d \delta_{NG} \delta_{NB} \delta_{BR}]^T,$$

such that, for given values of $V_r$ and $V_{\text{driver}}$, the cost function (2) is maximized. Since the route, grade, and initial state of the RBCC machine are not known a priori, several trajectory scenarios are considered, and the average value of the cost (2) is maximized.

Our approach to computing the averaged cost from the trajectory scenarios consists of three steps. In the first step, we use grade trajectories over sample routes to generate a statistical model of the grade in the form of a Markov Chain. In this paper a sample route based on M-39 highway in Michigan, which has a notable periodic grade, was used. Techniques exemplified in our previous work [6], [11], [14] were exploited to infer the Transition Probability Matrix (TPM) of the Markov Chain.

In the second step, we perform a set of $N_{\delta} \times N_{\delta} \times N_{MC}$ Monte Carlo simulations that are organized as follows. For each initial grade, $\delta(0)$, and for each initial state of the RBCC machine, $x(0)$, $N_{MC}$ grade trajectories are generated at random using the statistical model of the grade from the first step. Each simulation is run until a full cycle of state transitions of the RBCC machine is executed.

In the third step, the resulting values of the cost (2), denoted by $f_{c,\text{dyn},i}^{T}(p)$, $i = 1, \cdots, N_{MC}$, are first averaged to yield the expected value of the cost (2) conditional to
chosen $\delta(0)$, $x(0)$ and $p$:

$$f_{c,dyn}(p) = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \bar{f}_{c,dyn,i}(p),$$  \hspace{1cm} (5)

and then averaged with respect to $\delta(0)$ and $x(0)$. For the latter averaging, the distribution of $\delta(0)$ is inferred from the steady state grade distribution as implied by the grade statistical model, while the distribution of $x(0)$ is assumed to be uniform. This leads to expressions,

$$f_{c,dyn}(p) = \sum_{i=1}^{N_{x}} \bar{f}_{c,dyn,i}(p) \mathbb{P}(\delta(0) = \delta_i),$$  \hspace{1cm} (6)

where $\mathbb{P}(\delta(0) = \delta_i)$ denotes the probability that the initial grade $\delta(0)$ takes one of $N_{\delta}$ given values $\delta_i$, and

$$f_{c,dyn}(p) = \frac{1}{N_{x}} \sum_{i=1}^{N_{x}} \bar{f}_{c,dyn,i}(p),$$  \hspace{1cm} (7)

where $x_i$, $i = 1, \ldots, N_x$, denote the possible values of $x(0)$.

The average cost function $f_{c,dyn}(p)$ is maximized with respect to $p$ subject to range constraints on $p$. The optimization is performed using an adaptive mesh search algorithm. As a result, look up tables of parameters $V_d$, $\delta_{NG}$, $\delta_{NB}$, and $\delta_{BR}$ are produced as functions of the given $V_{\text{driver}}$ and $V_r$. Note that this optimization is performed off line and the on line implementation of the RBCC logic and these look-up tables is straightforward.

V. RESULTS

Results are now presented for the simulation and real vehicle testing. A model of the testing vehicle representing vehicle and engine dynamics was used to perform the threshold optimization and assess fuel consumption benefits. The test vehicle has a naturally aspirated engine and uses neutral and its highest gear during testing. The results presented are for the specific case where $V_{\text{driver}} = 55$ mph and $V_r = 54.5$ mph for the route along M-39 near Dearborn, MI.

Three test case studies are considered. Case study 1 is a validation test with parameter values chosen heuristically for initial testing of the RBCC structure. Case study 2 is based on the threshold optimization which was performed over the intervals $V_d \in [3, 4]$ mph, $\delta_{NG} \in [1, 2]$, $\delta_{NB} \in [2, 3]$, and $\delta_{BR} \in [2, 3]$. Case study 3 is based on threshold optimization over the intervals $V_d \in [3, 4]$ mph, $\delta_{NG} \in [1, 2]$, $\delta_{NB} \in [2, 3]$, and $\delta_{BR} \in [2, 3]$. Note that the interval for $\delta_{BR}$ was relaxed from case study 2 to case study 3 in an effort to improve results. Table I provides the values of the thresholds calculated and used for each case study.

In all three case studies for vehicle testing, the grade prediction method described in the appendix was used to generate grade predictions over a prediction horizon $r$. For the simulation results, a preview of the grade was fed into the algorithm as the predicted grade in order to produce a benchmark for the vehicle testing. The differences in vehicle testing and simulation results are due to model error.

Fig. 3. Simulation speed trajectory and state trajectory for case study 2.

A. SIMULATION RESULTS

The simulations were performed for case studies 2 and 3. Case study 1 was not evaluated in the simulations and was only used to validate the RBCC strategy in the vehicle. For case study 2, an average speed of 52.38 mph was observed with a DFCI of 3.30%. For case study 3, the average speed was 52.15 mph and the DFCI was 3.54%. As can be observed, changing the intervals over which the optimization is performed did not produce significantly different simulated DFCI results, though this was not confirmed during the vehicle testing.

Figures 3 and 4 show the speed trajectories of the vehicle under the RBCC scheme for case studies 2 and 3 as well as the RBCC machine states, where 10 corresponds to the normal cruise state, 20-25 are glide states, 30 is bleed state, and 40 is recover state. We note that for the implementation, the glide state was broken into a set of sub-states to generate a smooth transition in and out of neutral and, as such, states 20-25 correspond to the entire glide state.

Figures 5 and 6 provide examples of the grade profile and RBCC speed trajectory appropriately overlaid with the threshold values for case study 2. In Figure 6, it is seen that the vehicle speed drops below $V_{\text{driver}} - V_d$. This is because $V_{\text{driver}} - V_d$ is the lowest commanded speed, however, due to the dynamics of the nominal speed controller the vehicle may drift below the lowest commanded speed when it is passing from glide to recover. This phenomenon was also observed during the vehicle testing.

B. TEST VEHICLE RESULTS

Vehicle tests were performed during the Fall of 2013 along the M-39 route. At the beginning of each test the vehicle was driven long enough to sufficiently warm the engine and transmission. Because M-39 has an average positive grade

| Table I: The case studies and thresholds used. |
|---|---|---|---|---|
| Case study | $V_d$ | $\delta_{NG}$ | $\delta_{NB}$ | $\delta_{BR}$ |
| 1 | 6 | -1.5 | 0.5 | -1 |
| 2 | 3.58 | 1.67 | -1.68 | 1.5 |
| 3 | 3.9 | 1.4 | -1.4 | 2.5 |
in the northbound direction, baseline drives were conducted in both directions at 55 mph to eliminate any effects due to the direction.

The vehicle testing results from all three case studies are summarized in Table II. In Table II, “NB” denotes the northbound direction and “SB” denotes the southbound direction. As is observed, the RBCC strategy in case study 2 performs better than the RBCC strategy in case study 1, though it still maintains the primarily one-directional benefit observed with the heuristic threshold parameters. Case study 3 was then performed to eliminate this one-directional benefit. Though case study 3 eliminates the one-directional benefit, the overall benefit in the northbound direction is reduced.

In [13], a stochastic dynamic programming (SDP) is used to develop a control policy that produces oscillations in the vehicle speed similar to the RBCC strategy. The SDP strategy was implemented on the same testing platform and tests were conducted on the same stretch of M-39. The SDP strategy produced an average DFCI of 2.74% for all vehicle tests. This is compared to an average of 2.99% in case study 2 and an average of 2.52% in case study 3 for RBCC. The results are of similar magnitude in both cases, and improved upon in case study 2, while also benefiting from a reduction in the overall complexity required to develop and implement the strategies. Note that four test drives were conducted for each case study to test RBCC and twelve test drives were conducted to produce the SDP strategy average.

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VI. CONCLUSION

A rule based cruise control (RBCC) system was developed, optimized using a Monte Carlo simulations based method for fuel efficiency, and successfully tested in simulations and in-vehicle. The RBCC system, modeled as a finite state machine, exploits periodically varying vehicle speed setpoint to maximize fuel economy in the presence of changing driving conditions such as road grade.

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APPENDIX

Grade Prediction

In this section we describe a prediction algorithm for forecasting road grade, \( \delta(k) \), \( r \)-steps into the future. Assume the road grade propagates according to

\[
\delta(k+1) = \theta(k) \phi(k),
\]

where \( \theta(k) \in \mathbb{R}^{1 \times n} \) is a vector of model parameters, and \( \phi(k) \in \mathbb{R}^n \) is a vector of previous grades from time \( k-n+1 \) to \( k \).

Next, let \( r \), a positive integer, be the width of the prediction horizon. Then the measurement \( \hat{\delta}(k+r) \), is obtained in state space form as

\[
\begin{bmatrix}
\delta(k+1) \\
\vdots \\
\delta(k-n+2)
\end{bmatrix} = A(k) \phi(k),
\]

where

\[
A(k) = \begin{bmatrix}
\theta(k) \\
\lambda_{12}
\end{bmatrix} \in \mathbb{R}^{n \times n},
\]

reflects the measurement dynamics, and

\[
\lambda_{12} = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{(n-1) \times n}.
\]

Next, it follows from (8) that the road grade at \( k+r \) is

\[
\delta(k+r) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \prod_{i=1}^{r} A(k+i-1) \phi(k).
\]  

Assuming that \( A(k) \) evolves slowly over time, i.e., for \( i = 1, \ldots, r \), \( A(k+i) \approx A(k) \), and (9) is rewritten as

\[
\hat{\delta}(k+r) \approx \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \hat{A}^r(k) \hat{\phi}(k).
\]

The objective is then to obtain an estimate, \( \hat{A}(k) \), of \( A(k) \) in order to calculate the estimate \( \hat{\delta}(k+r) \) of the future road grade measurement \( \delta(k+r) \).

Let \( \hat{\delta}(k) \) be an estimate of the road grade \( \delta(k) \), then

\[
\hat{\delta}(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \hat{A}^r(k) \hat{\phi}(k),
\]

where

\[
\hat{A}(k) = \begin{bmatrix} \hat{\theta}(k) \\
\hat{\lambda}_{12}
\end{bmatrix} \in \mathbb{R}^{n \times n},
\]

and \( \hat{\theta}(k) \in \mathbb{R}^{1 \times n} \) is an estimate of \( \theta(k) \). We compute \( \hat{\theta}(k) \) using the recursive least squares update

\[
\hat{\theta}(k) \triangleq \hat{\theta}(k-1) + \beta(k)[\hat{\theta}(k-1)\phi(k-1) - \delta(k-1)]
\]

\[
\cdot \phi^T(k-1)P(k-1)\phi(k-1) + \lambda^{-1}
\]

\[
\cdot \phi^T(k-1)P(k-1),
\]

where \( P(0) \in \mathbb{R}^{n \times n} \) is symmetric positive definite, \( \lambda \in (0,1) \) is the forgetting factor, \( \beta(k) \) is either 1 or 0 to enable or disable updates of the form

\[
P(k) = \lambda^{-1}P(k-1) - \beta(k)\lambda^{-1}P(k-1)\phi(k-1)
\]

\[
\cdot \phi^T(k-1)P(k-1)\phi(k-1) + \lambda^{-1}
\]

\[
\cdot \phi^T(k-1)P(k-1).
\]

We initialize \( P(0) = \alpha I \), where \( \alpha > 0 \). See Figure 7 for an example of the grade prediction accuracy.