Global Exponential Stabilization of Neural Networks with Time Delay via Impulsive Control

Wu-Hua Chen Xiaomei Lu Wei Xing Zheng

Abstract—The problem of global exponential stabilization of discrete-time delayed neural networks (DDNNs) via impulsive control is addressed in this paper. A novel time-varying Lyapunov functional is proposed to capture the dynamical characteristic of discrete-time impulsive delayed neural networks (DIDNNs). In conjunction with the convex combination technique, new conditions in the form of linear matrix inequalities are established for global exponential stability of DIDNNs. The derived stability conditions are dependent upon the lengths of impulsive intervals but independent of the size of time delay. This paves the way for designing the impulsive controller for impulsive stabilization of DDNNs. The applicability of the developed global exponential stabilization conditions is validated by numerical results.

I. INTRODUCTION

Nowadays artificial neural networks have found extensive applications in numerous areas such as associative memory, pattern recognition, model identification, signal processing, intelligent control and so on. Commonly used neural network models, for example, Hopfield neural network [1], Cohen-Grossberg neural networks [2], cellular neural networks [3], are described by either nonlinear ordinary differential equations or nonlinear delay differential equations. Due to its practical importance, the stability problem of neural networks has received considerable attention over the years. Some recent results on the stability of neural networks may be found in [4]–[12] and the references therein. In particular, it was reported in [13] that the discrete-time cellular neural networks can do something that the continuous-time cellular neural networks cannot. Hence, the theory of discrete-time neural networks has assumed the same importance as that of continuous-time neural networks [14]–[17]. On the other hand, being an efficient control methodology, impulsive control strategy has been successfully utilized to synchronize coupled continuous-time neural networks in several works (see, e.g., [18]–[22]).

However, up to now only a few results have been available for the stability problem of discrete-time impulsive delayed neural networks (DIDNNs). In [23]–[26], the stability problem of discrete-time delayed neural networks (DDNNs) with destabilizing impulses was considered and some criteria that maintain the stability property of the original DDNNs were established. In [27], by using the contraction mapping theorem and inequality techniques, sufficient conditions for the existence and global exponential stability of periodic solution for a class of cellular neural networks with delays and impulses were obtained. In [28], the impulsive stabilization problem for DDNNs was proposed and an exponential stability criterion for the impulsive controlled DDNNs was presented. In [29], the Lyapunov functional approach was applied to investigate the multistability of discrete-time Hopfield-type neural networks with distributed delays and impulses. Recently, the robust stability of uncertain DDNNs with stabilizing/destabilizing impulses was studied in [30] by using Lyapunov functions together with Razumikhin technique and some delay-independent stability criteria were derived. It should be pointed out that in the models of DIDNNs considered in [23]–[29], the values of state at impulse instants are completely determined by the difference equations describing state jump. That is, the dynamical property of the DDNNs involved in the DIDNNs does not affect the values of state at impulse instants.

The purpose of this paper is to take a new look at the problem of global exponential stabilization of DDNNs via impulsive control. [23]–[29] Unlike the time-invariant Lyapunov function/functional based methods used in [23]–[30], we employ the time-varying Lyapunov functionals to investigate the stability of DIDNNs. This time-varying property of the new Lyapunov functionals enables to capture more dynamical characteristic of the DIDNNs, which is one main novelty of the paper. By utilizing the new time-varying Lyapunov functionals in combination with the convex combination technique, we obtain new stability results of DIDNNs. The derived stability conditions are dependent upon the lengths of impulsive intervals but independent of the size of delay. The newly obtained stability criteria are then applied to derive sufficient conditions on the existence of linear state feedback impulsive controllers. A numerical example is presented to substantiate the theoretical predications.

Notation. The notation $M > (\geq) 0$ is used to denote a symmetric positive-definite (positive-semidefinite) matrix. $I$ stands for an identity matrix of appropriate dimension. $\mathbb{N}$ represents the set of positive integers. Define

$$N_0 = \{0\} \cup \mathbb{N}.$$ 

For any two integers $a, b$, define

$$N(a) = \{a, a+1, \ldots\},$$

$$N(a, b) = \{a, a+1, \ldots, b\}.$$
For $x \in \mathbb{R}^n$, $\|x\|$ denotes the Euclidean norm of vector $x$.

II. PRELIMINARIES
Consider the DDNN described by the following delay difference equations:

$$
\begin{align*}
\dot{u}(t) &= Du(t-1) + A_0g_0(u(t-1)) \\
&\quad + A_1g_1(u(t-1-\tau)) + I_v, \quad t \in \mathbb{N}, \\
\end{align*}
$$

(1)

where $u = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^n$ is the state vector associated with the neurons; $D = \text{diag} \{d_1, d_2, \ldots, d_n\}$ is a diagonal matrix with $d_i$ being the self-regulating parameters of the neurons; $A_0$ and $A_1$ are the interconnection weight matrix and the delayed connection weight matrix, respectively; and $I_v$ is an external input vector.

$$
g_i(u) = [g_{i1}(u_1), g_{i2}(u_2), \ldots, g_{in}(u_n)]^T, \quad i = 0, 1
$$

are the nonlinear functions, where $g_{ij} : \mathbb{R} \to \mathbb{R}$, $i = 0, 1$, $j = 1, 2, \ldots, n$ are the neuron activation functions. The scalar $\tau$ is a non-negative integer representing the time delay, and

$$
\phi : N(-\tau, 0) \to \mathbb{R}^n
$$

is the initial function.

We make the following assumption.

(H) Each function $g_{ij}$ is continuous, and there exist scalars $l_{ij}^-$ and $l_{ij}^+$ such that for any $\alpha, \beta$, $\alpha \neq \beta$,

$$
l_{ij}^- \leq \frac{g_{ij}(\alpha) - g_{ij}(\beta)}{\alpha - \beta} \leq l_{ij}^+, \quad i = 0, 1, \ j = 1, 2, \ldots, n.
$$

As usual, a constant vector $u^* \in \mathbb{R}^n$ is called an equilibrium point of DDNN (1) if it satisfies

$$
u^* = Du^* + A_0g_0(u^*) + A_1g_1(u^*) + I_v.
$$

Moreover, we assume that some additional conditions are satisfied so that system (1) admits an equilibrium point $u^*$.

The objective of this paper is to investigate the global impulsive stabilization problem of certain equilibrium point $u^*$. It is well known that for continuous-time systems, the impulsive control law usually takes the following form

$$
\Delta x(t_k) \triangleq x^+(t_k) - x^-(t_k) = E_k(x^-(t_k)), \quad k \in \mathbb{N},
$$

(2)

where $\{t_k\}$ is the impulse time sequence, $E_k$ is the impulsive operator at impulse time $t_k$, and $x^-(t_k)$ and $x^+(t_k)$ denote the left-limit and right-limit of the state $x(t)$ at impulse time $t_k$, respectively. The above impulsive control law describes the instantaneous change of the state $x(t)$ at impulse instants $t_k$. It is worth mentioning that for discrete-time functions, there are no concepts of left-limit and right-limits. So, in order to apply the impulsive control scheme to stabilize discrete-time systems, it is necessary to make some modifications on the form of impulsive control law (2) such that it is suitable to discrete-time systems.

In this paper, the impulsive control law for the equilibrium $u^*$ is given by

$$
\Delta u(t_k) = BK(u(t_k - 1) - u^*), \quad k \in \mathbb{N},
$$

(3)

where $B \in \mathbb{R}^{n \times m}$ denotes the impulsive input matrix, $K \in \mathbb{R}^{m \times n}$ is the impulsive control gain matrix to be designed,

$$
\Delta u(t_k) = u(t_k) - u(t_k - 1),
$$

and $t_k \in \mathbb{N}$ are the impulse instants satisfying

$$
0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k \to \infty} t_k = +\infty.
$$

Define

$$
x(t) = u(t) - u^*.
$$

Then the closed-loop system given by (1) and (3) can be rewritten as

$$
\begin{align*}
x(t) &= Dx(t-1) + \sum_{i=0}^{1} A_if_i(x(t-1-\tau_i)), \quad t \neq t_k, \\
\Delta x(t) &= BKx(t-1), \quad t = t_k, \\
x(t) &= \phi_1(t) \triangleq \phi(t) - u^*, \quad t \in N(-\tau, 0),
\end{align*}
$$

(4a)

(4b)

(4c)

where

$$
\tau_0 = 0, \quad \tau_1 = \tau, \\
f_i(x) = [f_{i1}(x_1), f_{i2}(x_2), \ldots, f_{in}(x_n)]^T
$$

with

$$
f_{ij}(x_j) = g_{ij}(x_j + u_j^*) - g_{ij}(u_j^*), \quad i = 0, 1, \ j = 1, 2, \ldots, n.
$$

For illustration convenience, (4a) is referred to as the discrete-time dynamics, while (4b) is referred to as the resetting law. From condition (H) and the relationship between $f(x)$ and $g(u)$, it is easy to see that functions $f_i(x)$, $i = 0, 1$, satisfy the following condition:

(H1) Each function $f_{ij}$ is continuous, and there exist scalars $l_{ij}^-$ and $l_{ij}^+$ such that for any $\alpha, \beta$, $\alpha \neq \beta$,

$$
l_{ij}^- \leq \frac{f_{ij}(\alpha) - f_{ij}(\beta)}{\alpha - \beta} \leq l_{ij}^+, \quad i = 0, 1, \ j = 1, 2, \ldots, n.
$$

Hence, the impulsive stabilization problem of the equilibrium $u^*$ reduces to the one of finding some conditions on the impulsive control gain matrix $K$, and the impulse time sequence $\{t_k\}$ such that the zero solution of the impulsive controlled system (4) is globally exponentially stable.

To deal with the impulse time sequences with non-equidistant intervals, we introduce the notation $S(\delta_1, \delta_2)$ to denote the class of impulse time sequences $\{t_k\}$ satisfying

$$
\delta_1 \leq t_k - t_{k-1} \leq \delta_2, \quad k \in \mathbb{N},
$$

where $\delta_1$ and $\delta_2$ are positive scalars.
Definition 1: For given positive integers $\delta_1$ and $\delta_2$ satisfying
\[ \delta_1 \leq \delta_2, \]
the zero solution of system (4) is said to be globally uniform exponentially stable (GES) over $S(\delta_1, \delta_2)$, if there exist scalars $a \in (0, 1)$ and $b > 0$ such that
\[ \|x(t)\| \leq ba^t \max_{s \in N(-\tau, 0)} \|\phi_1(s)\|, \quad t \in \mathbb{N} \]
for any impulse time sequence $\{t_k\} \in S(\delta_1, \delta_2)$.

The following notation will be used in the subsequent sections:
\[
L_{i1} = \text{diag} \left\{ l_{i1}^+ t_{i1}^+, l_{i2}^+ t_{i2}^+, \ldots, l_{in}^+ t_{in}^+ \right\}, \quad i = 0, 1, \\
L_{i2} = \text{diag} \left\{ l_{i1}^+ + \frac{1}{2}, l_{i2}^+ + \frac{1}{2}, \ldots, l_{in}^+ + \frac{1}{2} \right\}, \quad i = 0, 1.
\]

III. GLOBAL EXPONENTIAL STABILIZATION

In this section, we will investigate the global exponential stability of the DIDNN (4) using the time-varying Lyapunov functional based method. For this purpose, we first introduce several discrete-time functions associated with the impulse time sequences.

For given impulsive time sequence $\{t_k\} \in S(\delta_1, \delta_2)$, we define a piecewise linear function $\rho: \mathbb{N}_0 \to \mathbb{R}^+$ as follows:
\[
\rho(t) = \begin{cases} 
\frac{t_{k-1}-t}{t_k-1-t_{k-1}}, & \text{if } t \in N(t_{k-1}, t_k-1), \quad k \in \mathbb{N}, \\
1, & \text{if } t \in N(-\tau, -1). 
\end{cases}
\]

It can be seen that
\[
\rho(t_{k-1}) = 1, \quad \rho(t_k) = 0, \quad \text{and } \rho(t) \in [0, 1] \text{ for } t \in N(t_k, t_{k+1} - 1), \quad k \in \mathbb{N}. \tag{5}
\]

Moreover, for $t \in N(t_{k-1} + 1, t_k - 1), \quad k \in \mathbb{N},$
\[
\rho(t-1) = \frac{t_k-t}{t_k-1-t_{k-1}} = \rho(t) + \rho_1(t), \tag{6}
\]
where
\[
\rho_1(t) = \frac{1}{t_k-1-t_{k-1}}, \\
t \in N(t_{k-1}, t_k-1), \quad k \in \mathbb{N}.
\]

For $t \in \mathbb{N}_0$, set
\[
\rho_2(t) = \begin{cases} 
\frac{1}{t_{k-1}-t_{k-2}} - \frac{\rho_2(t)}{t_{k-1}-t_{k-2}}, & \text{if } \delta_1 < \delta_2, \\
0, & \text{if } \delta_1 = \delta_2.
\end{cases}
\]

Noticing that $\{t_k\} \in S(\delta_1, \delta_2)$, it follows that
\[
0 \leq \rho_2(t) \leq 1, \quad \rho_1(t) = \frac{\rho_2(t)}{\delta_2 - 1} + 1 - \frac{\rho_2(t)}{\delta_1 - 1}, \quad \text{for all } t \in \mathbb{N}_0. \tag{7}
\]

Set $\tilde{\rho}(t) = 1 - \rho(t), \quad \tilde{\rho}_2(t) = 1 - \rho_2(t)$.

Theorem 1: Consider the DIDNN (4) satisfying (H1).

Given positive integers $\delta_1$ and $\delta_2$ satisfying
\[ 2 \leq \delta_1 \leq \delta_2, \]
the zero solution of the DIDNN (4) is GUES over $S(\delta_1, \delta_2)$, if for a prescribed scalar $\mu \in (0, 1]$, there exist $n \times n$ matrices $P_i > 0, Q_l > 0, n \times n$ diagonal matrices $\Lambda_{ijtl} > 0, i, j, l = 1, 2, h = 0, 1$, such that the following linear matrix inequalities (LMIs) (8) (given at the top of the next page) and (9) hold:
\[
\begin{bmatrix}
-\mu P_1 + Q_2 & (I + BK)^T P_2 \\
-P_2 & -P_1
\end{bmatrix} \leq 0, \tag{9}
\]
where
\[
\Omega_{ijl} = \frac{1}{\mu} Q_i - \mu \frac{1}{\delta_j - 1} \left( P_i + \frac{1}{\delta_j - 1} (P_2 - P_1) \right) - \Lambda_{ijtl} L_{01}.
\]

Proof: For $t \in \mathbb{N}_0$, set
\[
\rho_3(t) = \rho(t - \tau), \quad \tilde{\rho}_3(t) = 1 - \rho_3(t).
\]

By the Schur complement condition (8) implies that there exist small enough positive scalars $\theta \in (0, 1)$ and $\sigma$ such that
\[
\Xi_{ijl} \leq -\sigma I, \quad i, j, l = 1, 2, \tag{10}
\]
where $\Xi_{ijl}$ is given at the top of the next page with
\[
A = [D \quad 0 \quad 0 \quad A_1], \\
\Phi_{ijl} = (1 - \theta)^T Q_l - \Lambda_{ijt1} L_{11}, \\
\tilde{\Omega}_{ijl} = \frac{1}{\mu} Q_l - \left( P_l + \frac{1}{\delta_j - 1} (P_2 - P_1) \right) - \Lambda_{ijtl} L_{01}.
\]

It follows that
\[
\Xi(t) \triangleq \tilde{\rho}(t) \Xi_1(t) + \rho(t) \Xi_2(t) \leq -\sigma I, \tag{11}
\]
where
\[
\Xi_1(t) = \tilde{\rho}_2(t) (\tilde{\rho}_3(t) \Xi_{111} + \rho_3(t) \Xi_{112}) + \rho_2(t) (\tilde{\rho}_3(t) \Xi_{121} + \rho_3(t) \Xi_{122}).
\]

Again, applying the Schur complement to (9) yields
\[
(I + BK)^T P_2 (I + BK) + Q_2 \leq \mu P_1. \tag{11}
\]

Define
\[
\varphi(t) = \mu^{1 - \rho(t)}
\]
for $t \in \mathbb{N}_0$. Choose a time-varying Lyapunov functional candidate for system (4) as follows
\[
V(t, x_t) = \varphi(t) x^T(t) P(t) x(t) \\
+ \sum_{s=t-\tau}^{t-1} (1 - \theta)^{t-s-1} x^T(s) Q(s) x(s), \tag{12}
\]
where

\[
P(t) = \hat{\rho}(t)P_1 + \rho(t)P_2,
\]

\[
Q(t) = \hat{\rho}(t+1)Q_1 + \rho(t+1)Q_2.
\]

Then we can establish that for \( t \in N(t_k + 1, t_{k+1} - 1) \),

\[
\Delta V(t) \leq -\theta V(t) + \varphi(t)\xi^T(t) [\hat{\rho}(t) (\Xi(t) + \Xi_2(t)) + \rho(t) \Xi_2(t)] \xi(t)
\]

\[
= -\theta V(t-1) + \varphi(t)\xi^T(t) [\hat{\rho}(t) \Xi_1(t) + \rho(t) \Xi_2(t)] \xi(t) 
\]

\[
\leq -\theta V(t-1).
\]

It follows that

\[
V(t) \leq (1 - \theta)^{t-t_k} V(t_k), \quad t \in N(t_k + 1, t_{k+1} - 1). \quad (13)
\]

Now we give an estimate of \( V(t_k) \), \( k \in \mathbb{N} \). Noticing that \( \varphi(t_k) = 1 \) and \( \varphi(t_k - 1) = \mu \), it follows from (4) and (11) that

\[
V(t_k) = \varphi(t_k)x^T(t_k)P(t_k)x(t_k)
\]

\[
+ \sum_{s=t_k-\tau}^{t_k-1} (1 - \theta)^{t_k-s-1}x^T(s)Q(s)x(s)
\]

\[
\leq \varphi(t_k - 1)x^T(t_k - 1)P(t_k - 1)x(t_k - 1)
\]

\[
+ \sum_{s=t_k-\tau-1}^{t_k-2} (1 - \theta)^{t_k-s-2}x^T(s)Q(s)x(s)
\]

\[
= V(t_k - 1). \quad (14)
\]

Combining (13) and (14) together gives

\[
V(t) \leq (1 - \theta)^{(1-1/\delta_t)(t-t_0)} V(t_0), \quad t \in \mathbb{N}_0,
\]

which means that

\[
\|x(t)\| \leq \sqrt{\frac{\lambda_2}{\mu \lambda_1}} (1 - \theta)^{(1-1/\delta_t)(t-t_0)} \max_{s \in N(-\tau,0)} \|\phi_1(s)\|, \quad t \in \mathbb{N}_0,
\]

where

\[
\lambda_1 = \min \{\lambda_{\min}(P_i), i = 1, 2\},
\]

\[
\lambda_2 = \max \{\max \{\lambda_{\max}(P_i), \lambda_{\max}(Q_i)\}, \quad i = 1, 2\}.
\]

Therefore, by Definition 1, we conclude that the zero solution of the DDNN (4) is GUES over \( S(\delta_1, \delta_2) \).

**Remark 1:** Unlike the existing time-invariant Lyapunov function based methods, the stability criterion in Theorem 1 is achieved by applying the novel time-varying Lyapunov functional introduced in (12). It is worth mentioning that an important dynamical characteristic of the DIDNN (4) is the sudden change of system structure at impulse instants, while the novelty of the Lyapunov functional (12) lies in that its construction is associated with impulse time sequence. Thus, compared with the time-invariant Lyapunov function based methods, the proposed time-varying Lyapunov functional based method is more suitable to capture the dynamical characteristic of the DIDNN (4), which can effectively reduce the conservatism of the resulting stability criterion.

**Remark 2:** In the existing works concerning impulsive stabilization of DDNNs, all the results require

\[
\rho(I + BK) < 1, \quad (15)
\]

where \( \rho(\cdot) \) denotes the spectral radius of a square matrix (see, e.g., [25], [28], [30]). However, as will be shown in next section, the restriction (15) is unnecessary in our stability criterion. Hence, our new results remove the restriction (15) on the impulsive control gain matrix \( K \) required by the existing results, which is a significant improvement.

Next, we present a solution to the impulsive stabilization problem of the DDNN (1) by using the impulse control law (3) under the assumption that

\[
\tilde{t}_{ij} \leq 0 \leq l_{ij}^+ + 1, \quad i = 0, 1, \quad j = 1, 2, \ldots, n. \quad (16)
\]

Note that under the assumption (16), the notation

\[
\tilde{L}_{i1} \equiv (L_{i1})^+, \quad i = 0, 1,
\]

are well-defined.

**Theorem 2:** Given a class \( S(\delta_1, \delta_2) \) of impulse time sequences in which

\[
2 \leq \delta_1 \leq \delta_2,
\]

consider the DDNN (1) satisfying (H1) and (16). Assume that

\[
I_v(t) \equiv I_v
\]
and the DDNN (1) possesses an equilibrium $u^*$. Set
\[ \bar{L}_{i1} \triangleq (L_{i1})^{1/2}, \quad i = 0, 1. \]

The state feedback impulsive law (3) globally exponentially stabilizes the equilibrium $u^*$ over $S(\delta_1, \delta_2)$, if for prescribed scalars $\mu \in (0, 1]$, $\epsilon_{jl} > 0$, $j, l = 1, 2$, there exist $n \times n$ matrices $X_j > 0$, $Q_j > 0$, $n \times n$ diagonal matrices $\Lambda_{ijth} > 0$, $i, j, l = 1, 2, h = 0, 1$, and an $m \times n$ matrix $K$, such that the following LMIs (17), (18) (given at the top of the next page) and (19) hold:
\[
\begin{bmatrix}
\Xi_{ijl} & T^T_j X_1 & T^T_j X_1 \bar{L}_{01} \\
\ast & -\mu \bar{Q}_1 & 0 \\
\ast & 0 & -\Lambda_{ij0}
\end{bmatrix} < 0, \quad j, l = 1, 2,
\]
\[
\begin{bmatrix}
-\mu X_1 & X_1 + K^T B^T & X_1 \\
\ast & -X_2 & 0 \\
\ast & 0 & -\bar{Q}_1
\end{bmatrix} \leq 0,
\]
where $\Xi_{ijl}$ is given at the top of the next page, and
\[
\bar{I}_1 = [I 0 0 0 0 0], \\
\bar{\Omega}_{1jl} = \frac{1}{\delta_j - 1} \mu^{-\gamma_{j-1}} (-\delta_j - 2 + 2\epsilon_{jl})X_1 + \epsilon_{jl}^2 X_2, \\
\bar{\Omega}_{2jl} = -\frac{\delta_j}{\delta_j - 1} \mu^{-\gamma_{j-1}} X_2.
\]

Moreover, the state feedback impulsive gain is given by
\[ K = \bar{K} X_1^{-1}. \]

Due to the limited space, the proof of Theorem 2 is omitted.

IV. NUMERICAL EXAMPLE

In this section, we will validate the obtained theoretical results by one numerical example.

Example 1: Consider a two-neuron DIDNN (4) with
\[
D = \begin{bmatrix}
\frac{2.42}{1 + \sqrt{2}} & 0 \\
0 & \frac{2.42}{3}
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
0.005 & 0.005 \\
0.002 & 0.004
\end{bmatrix},
\]
\[ A_0 = 0, \quad f_0(\alpha) = f_1(\alpha) = \tanh(\alpha). \]

It is easy to verify that
\[ L_{01} = L_{11} = 0 \quad \text{and} \quad L_{02} = L_{12} = 0.5 I_2. \]

It can be seen that when there are no impulsive actions, the state variable $u_1(t)$ is divergent as shown in Fig. 1(a). Since the stability conditions in [23], [25], [26] require that the neural networks should be globally asymptotically stable when there are no impulsive actions, the stability criteria in [23], [25], [26] are not applicable to this example.

Let $B = \begin{bmatrix}
1 \\
0
\end{bmatrix}$. We consider the problem of designing the linear state feedback impulsive controller with the following form
\[ \Delta u(t_k) = BK u(t_k - 1), \quad \{t_k\} \in S(\delta_1, \delta_2), \quad k \in \mathbb{N}. \]

Since $B \in \mathbb{R}^{2 \times 1}$, it is easy to see that
\[ \rho(I + BK) \geq 1, \quad \forall K \in \mathbb{R}^{1 \times 2}. \]

Thus, all the results in [28], [30]–[32] cannot be used to design the above impulsive controller. Now we apply our Theorem 2 to the problem of designing the impulsive controller. For illustration convenience, we only consider the case of \[ \delta_1 = \delta_2 = \delta. \]

By applying our Theorem 2 with the tuning parameters $\mu$ and $\epsilon_{jl}$, it has been found that for any $\delta \in N(2, 149)$, the DIDNN can be impulsively stabilizable over $S(\delta, \delta)$. For example, for both the case of $\delta = 4$ and $\delta = 10$, applying our Theorem 2 with the choice of $\mu = 0.98$ and $\epsilon_{jl} = 1, j, l = 1, 2$, it has been found that the LMIs in Theorem 2 are feasible. The corresponding impulsive control gain matrix is given by
\[ K = [-1 - 0.0001]. \]

For $\tau = 5$, the time evolutions of the state trajectory of the resulting closed-loop system with $\delta = 4$ and $\delta = 10$ are depicted in Fig. 1(b) and Fig. 1(c), respectively. The simulation results show that two state variables asymptotically converge to the origin under the impulsive actions for both cases.

V. CONCLUSION

In this paper, we have investigated the problem of global exponential stabilization for discrete-time impulsive delayed neural networks. We have established the new criteria for global exponential stability of DDNNs by means of a time-varying Lyapunov functional associated with the impulse time sequence. The new stability criteria are less conservative because of exploitation of the length information of the impulsive intervals. Furthermore, we have presented the conditions for constructing feedback impulsive controllers. Finally, we have shown the workability of the derived results by the numerical example.
\[
\begin{bmatrix}
\sum_{j} X_{ijl} & T_{ijl} X_{2} & T_{ijl} X_{2} L_{01} & X_{2} \\
* & -\mu Q_{ijl} & 0 & 0 \\
* & 0 & -\bar{\Lambda}_{ijl0} & 0 \\
* & 0 & 0 & -(\delta_{j} - 1)\mu X_{1} \\
\end{bmatrix} < 0, \quad j, l = 1, 2,
\]

(18)

References


