A Dual Decomposition Approach to Complete Energy Management for a Heavy-Duty Vehicle

T.C.J. Romijn, M.C.F. Donkers, J.T.B.A. Kessels and S. Weiland

Abstract—In this paper, we will propose a scalable and systematic procedure to solve the complete vehicle energy management problem, which requires solving a large-scale optimization problem subject to a large number of constraints. We consider a case study of a hybrid heavy-duty vehicle, equipped with an electric machine, a high-voltage battery pack and a refrigerated semi-trailer. The procedure is based on the application of the dual decomposition to the energy management problem. This dual decomposition allows the large-scale optimization problem to be solved by solving several smaller optimization problems, which gives favourable scalability properties. To efficiently decompose the problem, we will decompose the objective function of the optimization problem, being the fuel consumption, into a sum of functions each representing ‘energy losses’. Using the case study, we will compare the novel methodology based on the dual decomposition with dynamic programming, showing the benefits in terms of computational efficiency of the novel solution strategy. Moreover, we will show the benefits in terms of fuel consumption of complete vehicle energy management.

I. INTRODUCTION

Improving vehicle energy efficiency has always been an important topic of research for the automotive industry and has led to the introduction of several new technologies in vehicles in the last three decades. To properly manage the increased complexity of the vehicle’s powertrain and auxiliary systems, an increasingly complex supervisory control system is needed so that the synergy between all subsystems is maximized. The introduction of hybrid electric vehicles (HEVs), which offered the freedom to select the powersplit ratio between the internal combustion engine and the electric machine, has led to the development of more systematic methods to design supervisory control systems for vehicles [1]–[3]. At the same time, it became clear that optimization of individual systems will not automatically guarantee global efficiency at a vehicle level and that energy management has to be done on a complete vehicle level: complete vehicle energy management (CVEM) [4].

Current research on supervisory control systems for automotive applications focusses mainly on the integration of additional energy storage buffers and control inputs into the classical energy management problem. For instance, inclusion of start-stop functionality and incorporation of gear shifting strategy in the energy management system is discussed in [5], thermal management of the engine and the aftertreatment system is considered in [6], the integration of a waste-heat recovery system is considered in [7], and thermal management of batteries and the impact of battery temperature on the cycle life is discussed in [8], [9].

While each of these energy management problems is interesting and shows the potential of looking at CVEM, the methodology used to solve the actual design of the supervisory controller is often based on standard tools from optimal control theory. In particular, in case information about the future drive cycle is given, dynamic programming (DP) can be used, see, e.g., [1], or a large static optimization problem can be formulated, see, e.g., [10], and in case the future drive cycle is unknown, feedback strategies known as equivalent cost-minimisation strategies (ECMS) can be used, see, e.g., [2]. All of the aforementioned papers use one of these techniques. ECMS does not guarantee optimal performance, while the complexity of DP typically scales exponentially with the number of components [11], due to gridding of the state space. Therefore, besides extending the number of components in the energy management system, also a scalable method for solving the optimization problem is needed to arrive at an implementable CVEM strategy.

In this paper, we will propose a scalable and systematic procedure to solve the CVEM problem. For ease of exposition, we simplified the problem of CVEM by considering a hybrid heavy-duty vehicle, equipped with an electric machine, a high-voltage battery pack and a refrigerated semi-trailer, even though the proposed method is in principle scalable in the sense that more components could be added. The methodology is based on (static) distributed optimization. In particular, we apply a dual decomposition [12], [13] to the (discrete-time) dynamic optimization problem, where the solutions of the system dynamics are incorporated in the constraints to arrive at a static optimization problem. The dual decomposition allows the large-scale optimization problem resulting from the CVEM problem to be solved by solving several smaller optimization problems, which gives more favourable scalability properties than DP. In order to solve the optimization problem for CVEM, we propose to decompose the fuel consumption, which is the objective function of the optimisation problem in CVEM into multiple objective functions each representing ‘energy losses’ in each subsystem. Using the case study, we will show the benefits of having a supervisory control system that includes variable electric machine power and variable cooling power in terms of fuel consumption. More importantly, we will illustrate
the computationally favourable properties of the proposed solution strategy based on the dual decomposition.

II. TOPOLOGY AND COMPONENT MODELING

In this section, we will present the models needed to formulate (a simplified version of) the CVEM problem in Section 3. We focus on a case study of a configuration that includes an internal combustion engine (ICE), an electric machine, a high-voltage battery pack and a refrigerated semi-trailer. The topology is schematically shown in Fig. 1. The problem is relatively simple, yet interesting enough from an energy management point-of-view. Namely, the configuration has two energy buffers, i.e., the battery and the refrigerated semi-trailer. The battery can store energy that can be re-used later, while the refrigerated semi-trailer can only consume energy. Still, the refrigerated semi-trailer can temporarily lower its temperature, so that it can be turned off for a certain period of time, while not exceeding its maximum allowable temperature.

A. Internal Combustion Engine Model

The internal combustion engine model is based on a Euro- VI 12.9L Diesel engine. The fuel mass flow into the engine is approximated quadratically using a Willans lines model (see, e.g., [3])

\[ \dot{m}_t = \alpha_2(\omega_p)P_p^2 + \alpha_1(\omega_p)P_p + \alpha_0(\omega_p), \]

where \( P_p \) is the output power at the crankshaft, \( \omega_p \) is the engine speed and \( \alpha_i(\omega_p), i \in \{0, 1, 2\} \), are positive speed-dependent coefficients. The chemical power which the fuel produces is given by

\[ P_t = H_0 \dot{m}_t, \]

where \( H_0 \) denotes the lower heating value of the fuel. The engine power satisfies

\[ P_p \in \mathcal{U}_p = [P_{p,\text{min}}(\omega_p), P_{p,\text{max}}(\omega_p)], \]

where \( P_{p,\text{max}}(\omega_p) \) is the speed-dependent maximum engine output power and \( P_{p,\text{min}}(\omega_p) \) is the speed dependent minimum engine output power given by the engine brake torque when no fuel is injected.

B. Electric Machine Model

The electric machine is also modeled using a quadratic function (see, e.g., [3])

\[ P_e = \gamma_2(\omega_{em})P_{em}^2 + \gamma_1(\omega_{em})P_{em} + \gamma_0(\omega_{em}), \]

where \( P_e \) is the electric power, \( P_{em} \) is the mechanical power and \( \gamma_i(\omega_{em}), i \in \{0, 1, 2\} \) are positive speed-dependent efficiency coefficients. The electric machine power satisfies

\[ P_{em} \in \mathcal{U}_{em} = [P_{em,\text{min}}, P_{em,\text{max}}], \]

where \( P_{em,\text{max}} \) and \( P_{em,\text{min}} \) denotes the maximum and minimum mechanical power of the electric machine, respectively.

C. Battery Model

The battery is modeled with quadratic power losses as function of the stored battery power \( P_s \), see, e.g., [3], where negative values of \( P_s \) reflect charging of the battery and positive values discharging. This allows the battery output power \( P_b \) to be given by

\[ P_b = P_s - \beta_{bt}P_s^2, \]

where \( \beta_{bt} \) is the quadratic power loss coefficient given by

\[ \beta_{bt} = \frac{R_t}{U_s}, \]

which can be derived from an equivalent battery circuit, see, e.g., [3], with internal resistance \( R_t \) and open circuit voltage \( U_s \). The coefficient \( \beta_{bt} \) is constant, assuming that the internal voltage \( U_s \) and internal resistance \( R_t \) are independent of battery temperature and stored energy in the battery. The battery storage power satisfies

\[ P_s \in \mathcal{U}_s = [P_{s,\text{min}}, P_{s,\text{max}}], \]

where \( P_{s,\text{min}} \) and \( P_{s,\text{max}} \) are the minimum and maximum storage power, respectively. The battery allows storing energy, which can be represented by a discrete-time linear time-invariant system

\[ E_s(k + 1) = A_sE_s(k) + B_sP_s(k), \]

with \( k \in \mathbb{N} \), in which \( E_s \) is the state-of-energy, with some given initial condition \( E_s(0) \), \( A_s = 1 \) and \( B_s = -T_s \), where \( T_s \in \mathbb{R}_+ \), is the step size. The state-of-energy satisfies

\[ E_s \in \mathcal{X}_s = [E_{s,\text{min}}, E_{s,\text{max}}], \]

where \( E_{s,\text{min}} \) and \( E_{s,\text{max}} \) are the lower and upper bound of the energy storage capacity of the battery, respectively.

D. Refrigerated Semi-Trailer Model

The dynamics of the refrigerated semi-trailer are assumed to satisfy an energy balance given by

\[ \frac{d}{dt}T = \frac{1}{C_t}(P_t - h(T - T_{amb})), \]

where \( C_t \) is the heat capacity of the refrigerated semi-trailer and its contents, \( P_t \) is the thermal power where negative values indicate cooling, \( h \) is the heat transfer coefficient between the semi-trailer and the environment, \( T \) is the temperature inside the semi-trailer, where the initial condition \( T(0) \) is assumed to be given, and \( T_{amb} \) is the ambient temperature (which is assumed to be constant). The required electric
power $P_l$ is assumed to depend quadratically on the thermal power $Pt$ and linearly on the coefficient of performance ... the drive cycle is assumed to be given) and can be removed from
the optimization problem without changing its solution.

The temperature in the refrigerated semi-trailer can also be represented by a discrete-time linear time-invariant system, by making a forward Euler approximation of (10). This results in

$$ T(k + 1) = AT(k) + B_1 u_k + B_2 P_l(k), $$

with $k \in \mathbb{N}$, in which $u_k = \frac{k T_{amb}}{C_l}$ is the disturbance due to heat loss to the environment, and $A = 1 - \frac{T_k}{T_s}$, $B_1 = T_s$ and $B_2 = \frac{C_l}{T_s}$. Similar to the battery, we can represent the refrigerated trailer model in terms of stored energy by defining the thermal energy relative to the ambient temperature $E_t = C_l(T_{amb} - T)$. By doing so, (13) can be represented by a discrete-time linear time-invariant system given by

$$ E_t(k + 1) = A_t E_t(k) + B_t P_t(k), $$

for some given maximum thermal energy $E_{t,max} = C_l(T_{amb} - T_{min})$ and minimum thermal energy $E_{t,min} = C_l(T_{amb} - T_{max})$, which typically depend on the product transported in the refrigerated semi-trailer.

### III. PROBLEM FORMULATION

After presenting the necessary models, we can now formalize the CVEM problem. In CVEM, the main objective is to minimize the cumulative fuel consumption

$$ J = T_s \sum_{k=1}^{N} \dot{m}_f(P_p(k)), $$

over a drive cycle subject to a charge sustaining requirement on the battery, i.e., $E_s(0) - E_s(N) = \sum_{k=1}^{N} P_s = 0$, the dynamic systems (8), (14) and constraints (3), (5), (7), (9), (12), (15) and the power balances

$$ P_t(k) - P_p(k) - P_{em}(k) = P_{br}(k) \leq 0 \quad (17a) $$

$$ P_s(k) - P_b(k) - P_f(k) = 0, \quad (17b) $$

defined by the vehicle topology as indicated in Fig. 1, where we used the fact that braking power is always nonpositive to arrive at an inequality constraint. In (16), $T_s \in \mathbb{R}_+$ is the step size which we take $T_s = 1$, for simplicity, and $N \in \mathbb{N}$ indicates the length of the drive cycle. Note that this objective function only depends on the power of the engine $P_p$, which indirectly depends on the power of the other components which are in this case the electric machine, the high-voltage battery pack and the refrigerated semi-trailer.

### A. Decomposition of the Objective Function

To arrive at an efficient solution for the minimization of (16), we propose to decompose the objective function into separate energy losses related to each component. The rationale behind this proposal is that, ultimately, all energy that the engine delivers by combusting fuel is lost through heat generated in the brakes, lost to the environment through the refrigerated semi-trailer, and so on. This idea inspired us to rewrite the cumulative chemical power as in (2) as

$$ H_0 \sum_{k=1}^{N} \dot{m}_f(P_p(k)) = J_1(P_p) + J_2(P_e, P_g) + J_3(P_b, P_s) + J_4(P_f) + J_5(P_{br, P_l}), $$

where each $J_i, i \in \{1, \ldots, 5\}$ is related to energy losses in a component. Eqn. (18) reflects that all dissipation in the system is assumed to be known and modeled. For the engine, the total losses are given by the cumulative difference between the chemical power and the mechanical output power at the crankshaft, i.e.,

$$ J_1(P_p) = \sum_{k=1}^{N} P_t(k) - P_p(k). \quad (19) $$

For the electric machine, the losses are given by the cumulative difference between electrical power and the mechanical power, i.e.,

$$ J_2(P_e, P_{em}) = \sum_{k=1}^{N} P_e(k) - P_{em}(k). \quad (20) $$

The cost function for the battery represents the cumulative difference between storage power and effective battery power, i.e.,

$$ J_3(P_b, P_s) = \sum_{k=1}^{N} P_b(k) - P_s(k). \quad (21) $$

The cost function of the refrigerated semi-trailer is the cumulative electric power consumed, as all the electrical power going into the system is converted into thermal power and cannot be converted back to useful electrical power. Hence,

$$ J_4(P_f) = \sum_{k=1}^{N} -P_f(k). \quad (22) $$

The last term in (18) is related to the losses caused by braking and those related to the requested power, e.g., air drag

$$ J_5(P_{br}) = \sum_{k=1}^{N} P_{br}(k) - P_{br}(k). \quad (23) $$

Due to the power balances (17) and the charge sustaining requirement $E_s(0) - E_s(N) = \sum_{k=1}^{N} P_s = 0$, we have that (18) holds. Hence, we can write the objective function (16) equivalently as the sum of energy losses. Note that the energy losses in (18) as function of the requested power $P_r$, cannot be influenced by any of the control variables (as the drive cycle is assumed to be given) and can be removed from the optimization problem without changing its solution.
B. Vector and Matrix Representation

To formulate the optimization problem of CVEM as a static optimization problem, we introduce the notation

\[
P_i = [p_i(0), \ldots, p_i(N-1)]^T \in \mathbb{R}^N
\]

\[
\alpha_j = [\alpha_j(\omega_p(0)), \ldots, \alpha_j(\omega_p(N-1))]^T \in \mathbb{R}^N
\]

\[
\gamma_j = [\gamma_j(\omega_p(0)), \ldots, \gamma_j(\omega_p(N-1))]^T \in \mathbb{R}^N
\]

\[
E_m = [E_m(1), \ldots, E_m(N)]^T \in \mathbb{R}^N,
\]

for \( i \in \{p, em, e, b, l, s, t\} \), \( j \in \{0, 1, 2\} \) and \( m \in \{s, t\} \). Using this notation, we can write the solutions of (8) and (14) as

\[
E_m = \Phi_mE_m(0) + \Gamma_mP_m,
\]

with

\[
\Phi_m = \begin{bmatrix} A_m \\ A_m^2 \\ \vdots \\ A_m^N \end{bmatrix}, \quad \Gamma_m = \begin{bmatrix} B_m & 0 & \cdots & 0 \\ A_mB_m & B_m & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_m^{N-2}B_m & A_m^{N-3}B_m & \cdots & B_m \end{bmatrix},
\]

for \( m \in \{s, t\} \). Furthermore, the notation (24) allows the constraints (9) and (15) to be written as a vector of linear inequalities, given by

\[
1E_{m,\text{min}} \leq \Phi_mE_m(0) + \Gamma_mP_m \leq 1E_{m,\text{max}},
\]

for \( m \in \{s, t\} \). Finally, the notation (24) allows the components of the objective function (18) to be written in a compact form. In particular, substituting (2), (4), (6) and (11) into (19), (20), (21) and (22), respectively, yields

\[
J_1(P_p) = H_0(P_p^T \text{diag}(\alpha_2)P_p + \alpha_1^T P_p + 1^T \alpha_0) - 1^T P_p,
\]

\[
J_2(P_{em}) = P_{em}^T \text{diag}(\gamma_2)P_{em} + \gamma_1^T P_{em} + 1^T \gamma_0 - 1^T P_{em},
\]

\[
J_3(P_s) = \beta_0P_s^TP_s,
\]

\[
J_4(P_t) = \frac{1}{\text{COPT}}(\beta_{\text{ref}}P_t^TP_t - 1^T P_t).
\]

Note that each of the costs in (18) is a quadratic function of its argument. These compact notations allow us to formally formulate the CVEM problem for the considered case study.

C. Static optimization problem

Using the result of the previous subsection, we can rewrite the minimisation of (16) subject to the charge sustaining constraint \( E_s(0) = E_s(N) \), the dynamic systems (8) and (14) and the constraints (3), (5), (7), (12), as the following static optimization problem:

\[
\min_{P_p, P_{em}, P_s, P_t} J_1(P_p) + J_2(P_{em}) + J_3(P_s) + J_4(P_t),
\]

subject to

\[
P_e - P_p - P_{em} \leq 0
\]

\[
P_e - P_b - P_t = 0.
\]

which follow from the power balances (17), and subject to

\[
P_e = f_e(P_{em}) = \text{diag}(\gamma_2) \text{diag}(P_{em})P_{em}
\]

\[
+ \text{diag}(\gamma_1)P_{em} + \gamma_0
\]

\[
P_b = f_b(P_s) = P_s - \beta_{bs} \text{diag}(P_s)P_s
\]

\[
P_t = f_t(P_t) = \frac{1}{\text{COPT}}(P_t - \beta_{\text{ref}} \text{diag}(P_t)P_t),
\]

which follow from the component models (4), (6), (11), and subject to (3), (5), (7), (12) and (27), which we will write, with slight abuse of notation, as

\[
P_p \in \mathcal{U}_p, P_{em} \in \mathcal{U}_{em}, P_s \in \mathcal{U}_s
\]

\[
P_t \in \mathcal{U}_t, E_s \in \mathcal{X}_s, E_t \in \mathcal{X}_t.
\]

The optimization problem (29) is a quadratically constrained quadratic optimization problem (QCQP). To solve the QCQP, we propose a computationally efficient solution using a dual decomposition approach [12], [13].

IV. SOLUTION VIA DUAL DECOMPOSITION

The Lagrangian of problem (29) is usually obtained by augmenting the objective function in (29a) with the weighted sum of all the constraints in (29b), (29c) and (29d). For decomposition, it is more effective to first decompose the problem with the constraints that are defined in decision variables for more than one component, and pass the local constraints, defined in decision variables for one component only, directly to the decomposed problem. With slight abuse of terminology, we define a partial Lagrangian given by

\[
L(P, \lambda_1, \nu_1) = J_1(P_p) + J_2(P_{em}) + J_3(P_s)
\]

\[
+ J_4(P_t) + \lambda_1^T (P_s - P_p - P_{em}) + \nu_1^T (P_e - P_b - P_t),
\]

with \( P = [P_p, P_{em}, P_s, P_t, P_e, P_b, P_t] \in \mathbb{R}^{N \times 7} \), and in which \( \lambda_1, \nu_1 \in \mathbb{R}^N \) are Lagrange multipliers. Note that (30) is separable in variables related to each component. The partial Lagrange dual function of this problem is

\[
g(\lambda_1, \nu_1) = \min_p L(P, \lambda_1, \nu_1)
\]

\[
= g_1(\lambda_1) + g_2(\lambda_1, \nu_1) + g_3(\nu_1) + g_4(\nu_1),
\]

with

\[
g_1(\lambda_1) = \min_{P_p} \{ J_1(P_p) - \lambda_1^T P_p \mid P_p \in \mathcal{U}_p \}
\]

\[
g_2(\lambda_1, \nu_1) = \min_{P_{em}, P_s} \{ J_2(P_{em}) - \lambda_1^T P_{em} + \nu_1^T P_e \mid P_e \in \mathcal{U}_e, P_s \in \mathcal{X}_s \}
\]

\[
g_3(\nu_1) = \min_{P_s} \{ J_3(P_s) - \nu_1^T P_s \mid P_s \in \mathcal{U}_s \}
\]

\[
g_4(\nu_1) = \min_{P_t} \{ J_4(P_t) - \nu_1^T P_t \mid P_t \in \mathcal{U}_t, E_t \in \mathcal{X}_t \}.
\]

Note that (32) are linearly constrained quadratic programs, which can be solved efficiently. The Lagrange dual function yields a lower bound on the optimal value \( p^* \) of the problem (29). Namely, for any \( \lambda_1 \geq 0 \) and \( \nu_1 \), we have

\[
g(\lambda_1, \nu_1) \leq p^*.
\]
The best lower bound that can be obtained from the Lagrange dual function is given by
\[ d^* = \sup_{\lambda \geq 0, \nu} g(\lambda, \nu) \]  
(34)

Given the notion of the refined Slater’s condition for affine inequality constraints [12], strong duality holds for this problem, and the best lower bound will be the optimal value, i.e., \( d^* = p^* \). The problem of finding the best lower bound is called the Lagrange dual problem associated with problem (29). We now propose to use the dual ascent method to solve the dual problem (see, e.g., [12], [13]). The dual ascent method is an iterative method, in which we iteratively find the optimal \( P^* = [P^* p, P^* em, P^* b, P^* s, P^* e, P^* t] \in \mathbb{R}^{N \times 7} \), for given \( \lambda, \nu \) by solving (32), followed by an update of the Lagrange multipliers \( \lambda, \nu \) using a ‘steepest ascent’ method resulting in an update of \( \lambda, \nu \) by \( \lambda_1 + \alpha_1 \frac{\partial g}{\partial \lambda_1} \) and \( \nu_1 + \alpha_2 \frac{\partial g}{\partial \nu_1} \), for some suitably chosen step sizes \( \alpha_1 \) and \( \alpha_2 \). This iterative procedure is implemented in the algorithm shown in Table I. The stopping criteria can be either a maximum number of iterations or can be based on the constraint residuals, dual variables and/or convergence of the lower bound \( g(\lambda, \nu) \). The dual ascent method will always converge with a sufficiently small step size \( \alpha_i \), \( i \in \{1, 2\} \). Convergence is slow when the step size is chosen too small. When the step size is chosen too large, the dual variables start oscillating, slowing down the convergence. A solution is to start with a small step size and update the step size at each iteration based on the behaviour of the dual variables, e.g., whether they are oscillating or not. How to systematically determine suitable step sizes \( \alpha_i \), \( i \in \{1, 2\} \) is a topic of future research.

The dual decomposition algorithm, as given in Table I, requires an initial guess of the dual variables \( \lambda_1 \) and \( \nu_1 \). From (32) we can express the optimal engine powers \( P_p^* \) and optimal electric machine powers \( P_{em}^* \) as function of the dual variables \( \lambda_1 \) and \( \nu_1 \), i.e.,
\[ P_p^*(\lambda_1) = \arg \min_{P_p} \{ J_1(P_p) - \lambda_1^T P_p | P_p \in \mathcal{U}_p \} \]
\[ P_{em}^*(\lambda_1, \nu_1) = \arg \min_{P_{em}} \{ J_2(P_{em}) - \lambda_1^T P_{em} + \nu_1^T P_e \} \]  
(35)
\[ P_e = f_e(P_{em}), P_{em} \in \mathcal{U}_{em}. \]

Suppose we can estimate the optimal powers of the major components, i.e., the engine powers \( \hat{P}_p \) and the electric machine powers \( \hat{P}_{em} \). Then we need to find the dual variables for which the solution to (32) yield this estimated value for \( \hat{P}_p \) and \( \hat{P}_{em} \), i.e., \( \lambda_1 = \{ \lambda_1 | P_p^*(\lambda_1) = \hat{P}_p \} \) and \( \nu_1 = \{ \nu_1 | P_{em}^*(\lambda_1, \nu_1) = \hat{P}_{em} \} \). These values exist and are uniquely given by
\[ \lambda_1 = 2H_0 \text{diag}(\alpha_2) \hat{P}_p + H_0 \alpha_1 - 1 \]
\[ \nu_1 = \frac{\lambda_1 - 2 \text{diag}(\gamma_2) \hat{P}_{em} - \gamma_1 + 1}{\gamma_1 + 2 \text{diag}(\gamma_2) \hat{P}_{em}}. \]  
(36)

We can estimate the optimal engine powers \( \hat{P}_p \) and the optimal electric machine powers \( \hat{P}_{em} \), knowing that regenerative brake energy is always recovered and by assuming that the electric machine supplies an average power \( \hat{P}_{em} \) for propulsion. Because of this we can write
\[ \hat{P}_{em} = \begin{cases} \hat{P}_{em} & P_t > P_{p,\min} + \hat{P}_{em} \\ P_t - P_{p,\min} & P_t \leq P_{p,\min} + \hat{P}_{em}, \end{cases} \]  
(37)
and
\[ \hat{P}_p = \begin{cases} P_t - \hat{P}_{em} & P_t > P_{p,\min} \\ P_{p,\min} & P_t \leq P_{p,\min}, \end{cases} \]  
(38)
subject to (5) and (3), respectively. Hence, (36) can be used to initiate the algorithm in Table I.

V. NUMERICAL EXAMPLE

To demonstrate the computational efficiency of the proposed algorithm, we will solve (29) for a small part of a Pan-European driving cycle with length \( N = 739 \). To benchmark the performance of the algorithm, the problem is also solved with the generic Dynamic Programming (DP) Matlab function presented in [11]. To show that minimizing fuel consumption, as in (16), is equivalent to minimizing the sum of energy losses as in (29), the DP algorithm solves the problem by minimizing the fuel consumption, while the Dual Decomposition (DD) algorithm solves the problem by minimizing the sum of energy losses.

A. Scenarios

To illustrate how the dual decomposition-based approach scales with complexity when compared to dynamic programming, we consider the following three scenarios, ordered with increased complexity:

- **Scenario S1**: In this scenario, the battery is not used, i.e., \( P_b = P_s = 0 \). Moreover, the temperature in the refrigerated semi-trailer is fixed at a constant temperature (namely, its upper bound) requiring a constant thermal power of \( P_t = h(T_{\text{max}} - T_{\text{amb}}) \). Note that this scenario does not require an optimization problem to be solved as the fuel consumption is analytically given by solving (1), (4), (11) and (17).
- **Scenario S2**: In this scenario, one decision variable is added with respect to Scenario S1, namely the battery storage power \( P_s \). The temperature in the refrigerated semi-trailer is still fixed at the upper bound. This scenario has one state, being the battery state-of-energy \( E_s \) and one decision variable, being the storage power \( P_s \).
The power and state trajectories for a simulation with scenario S3 are given in Fig. 2. In this figure, the battery storage powers $P_s$, the battery state-of-energy $E_s$, thermal powers $P_t$ and the temperature inside the refrigerated semi-trailer $T$ are given as function of time, both for DD and DP. It is shown that the trajectories are almost equivalent. Differences can be caused by gridding for DP and the stopping criteria for DD. The difference in fuel consumption for both algorithms is negligible.

The simulation results show that adding the battery storage power as a decision variable (Scenario S2) results in 11.01% fuel consumption reduction compared to Scenario S1, as expected, because regenerative braking energy can be recovered, stored and re-used. Adding the thermal power as a decision variable as well (scenario S3) and storing energy in the refrigerated semi-trailer instead of the battery, causes another (but very small) 0.14% reduction in fuel consumption. Although, the fuel consumption reduction is not significant in Scenario S3 compared to Scenario S2, it does show that the refrigerated semi-trailer is used as a storage device to temporary store energy.

The most significant differences between the solutions found through the DD algorithm and the DP algorithm is the time needed to compute the solution. With one control input and one state (Scenario S2), DD takes 65 seconds (45 iterations) while DP takes 158 seconds. When we add an extra decision variable and state (Scenario S3), a significant difference in execution time can be observed. The DD algorithm takes 90 seconds (45 iterations) while DP takes 8004 seconds. It is interesting that the increase in time is not caused by more iterations but by the amount of time each iteration takes due to an extra optimization. As such, the scalability of the proposed DD algorithm seems therefore to be a lot better than DP, which is favourable for CVEM in which ultimately, the number of states and control inputs will be even larger.

VI. CONCLUSIONS

In this paper, we have proposed a scalable and systematic procedure to solve the CVEM problem. We have considered a case study of a hybrid heavy-duty vehicle, equipped with an electric machine, a high-voltage battery pack and a refrigerated semi-trailer. The procedure was based on the application of the dual decomposition, which allowed the large-scale optimization problem to be solved by solving several smaller optimization problems, which gives favourable scalability properties. To efficiently decompose the problem, we decomposed the objective function of the optimization problem, being the fuel consumption, into a function representing a ‘sum of energy losses’. The numerical example showed i) that minimizing fuel consumption is equivalent to minimizing the ‘sum of energy losses’, ii) that the novel methodology outperforms dynamic programming in terms of computation time and iii) the fuel savings benefit of the considered energy management problem.

REFERENCES