Cooperative Line-flow Power Electronics Control for Transient Stabilization

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Abstract—Transient stabilization of power systems in response to large disturbances is a challenging problem of high complexity. The complexity arises from the large number of system states and the nonlinearities in system dynamics which evolve at various time scales. The first contribution of this paper is a two-level approach to multi-time scale modeling of power system dynamics which includes dynamics of power-electronically controlled devices, such as Flexible AC Transmission Systems (FACTS) and High Voltage Direct Current (HVDC) lines. This approach proposes an interaction variable-based power system model suitable for the design of the FACTS controllers. The second contribution is an entropy-based cooperative controller for FACTS capable of transiently stabilizing generator dynamics and extending the critical clearing time of a fault.

I. MOTIVATION

Interconnected power systems are composed of many different components. Each component contributes to the interconnected system dynamics at multiple time scales and with naturally diverse dynamic behavior, resulting in a complex and nonlinear response of the system to large disturbances. Stability of such large and dynamically complex systems is hard to guarantee without a systematic approach to modeling and control design.

Standard practice in today’s power system industry is to use excitation controllers for transient stabilization [1], [2]. The main limitation of using these is the saturation of the exciter voltage, which is generally reached quickly during large disturbances. The exciter saturation has been known to limit controller performance.

One possible new technology which could help overcome transient stability problems during large disturbances is very fast power-electronically-switched control of large reactive components. These controllers are generally known as FACTS devices. The investments in power-electronic switching technologies have increased in recent years. However, systematic integration of power-electronics for the dynamic stabilization of interconnected power systems during large disturbances remains an open question. The main challenge is modeling and control design with provable performance.

In this paper, a systematic approach for power system modeling for dynamic stabilization using FACTS is proposed. In particular, a detailed power system model which spans over multiple time scales is taken as the starting point. This model is then reduced by applying a multi-time scale singular perturbation method. Finally, the newly obtained reduced-order model is used to design a FACTS controller which stabilizes generators during large disturbances.

A. Problem Formulation and Literature Overview

Transient stability is defined as the ability of generators to stay synchronized during large disturbances [3]. Large disturbances are known to slow down certain generators and accelerate others, causing the protection to trigger and disconnect other components, leading to transient instability [4]. A power system can experience transient instability in a few tenths of a second in the case of severe faults.

In transient stability studies, FACTS devices are almost always modeled as slowly varying impedances [5]. One of the most recent nonlinear FACTS controllers, proposed in [6], uses a variation of the variable impedance model. This approach is based on the power system model which was originally developed for generator excitation control design [7]. Dynamic time-varying phasor models of FACTS devices have been developed [8] in the past, but were not used to design controllers of interconnected power system dynamics until recently [9]. The main premise of this paper is that a much better control performance can be obtained if the system-level effects of the fast FACTS dynamics are explicitly modeled.

B. Proposed Approach to Modeling and Control

An approach to systematic multi-scale modeling of complex power system dynamics including dynamics of FACTS is proposed in this paper. Further, a relevant reduced-order model suitable for the design of FACTS controllers is derived. The main feature of this model is that it captures the effects of the fast FACTS dynamics which consequently results in better controller performance.

The proposed FACTS controller is based on entropy [10]. Entropy is a dual notion to entropy which has been used to propose excitation controllers in the past [11]. In this paper, entropy is used to design a FACTS controller which stabilizes interconnected system dynamics using fast switching of power-electronics. This controller is inherently a cooperative controller because the rotation of generators is stabilized using controllers on FACTS devices. This cooperation is made explicit in terms of interaction variable dynamics. The interaction variables are state variables whose dynamics represent the rate of change of energy stored in cooperating parts of the system.

The proposed approach to modeling and control design is illustrated on a three-bus system without loss of generality.
The example system contains typical power system devices. Therefore, the modeling and control approach can be reciprocated on a system of arbitrary topology.

The rest of the paper is organized as follows. Section II reviews the problem of transient stabilization. Section III describes the power system model and shows how this multi-time scale model can be systematically reduced to capture only the time scales of interest. Section IV shows how the reduced-order model is altered to a form suitable for control design. Section V introduces the entropy-based controller. Section VI shows the simulation results.

II. TRANSIENT STABILIZATION PROBLEM

A power system can be modeled in the following standard state-space form when the dynamics of all components are accounted for [9]

\[
\frac{dx}{dt} = \dot{x} = f(x, u, p), \quad 0 = f(x^*, u^*, p^*) \quad (1)
\]

where \( x \in \mathbb{R}^n \) is the vector of all physical power system states, \( u \in \mathbb{R}^m \) the vector of its inputs and \( p \in \mathbb{R}^q \) the vector of its parameters. Note that in this paper, vectors appear in a boldfaced font. The problem of transient stabilization can be stated as the problem of stabilizing the system states toward a known equilibrium \( x^* \).

The large disturbances are in the majority of cases created by sudden changes in topology. These changes can be represented as a change in the system parameter vector \( p \). In particular, the parameter vector can have one of the following values \( p \in \{ p_a, p_b, p_c \} \), where \( p_a \) represents the topology of the pre-fault system, \( p_b \) the topology of the faulted system, and \( p_c \) the topology of the post-fault system. Without loss of generality, it is assumed that \( p_a = p_b \) and that a post-fault system equilibrium \( x^* \) exists for \( p^* = p_a = p_c \) and \( u^* \).

Usually, a system equilibrium does not exist for a faulted topology \( p_b \) and a power system will typically become transiently unstable for faults whose duration is in the range of hundreds of milliseconds. However, the controller actions during a fault will highly impact the value of the states at the moment the fault is cleared. An effective transient stabilizing controller will react quickly and keep the system states in the vicinity of \( x^* \) even during a fault. In order to design such a controller, the system model and the controller objective need to be stated explicitly, as discussed next. Without loss of generality, the modeling and control design on the system shown in Fig. 1 is illustrated.

III. POWER SYSTEM MODELING FOR CONTROL WITH FACTS

The actual dynamical model of an interconnected power system given in (1) is extremely complex. Given a very large number of components in real-world power grids, various assumptions are made when simulating and analyzing system response. However, for the purposes of this paper which seeks to design very fast FACTS controllers, it is critical to start with a full-order multi-scale dynamical model and derive the reduced-order models essential for this controller design.

A. Typical Structure and Different Time Scales of Power System Dynamics

A comprehensive model of an interconnected power system, given in (1), represents: 1) fast dynamics of transmission lines; 2) the natural dynamic response of FACTS reactive components; 3) dynamics of relatively small inertia generators; and 4) dynamics of large inertia power plants. This model is given in the time-varying phasor domain with a single carrier frequency in the appendix; see (28).

The physical states of this model can be separated into state vectors from \( x_1 \) to \( x_4 \) and inputs from \( u_2 \) to \( u_4 \) as shown in (2). Each of these state vectors evolves at a different time scale.

\[
x_1 = [I_{f_1}, ..., I_{f_{n_f}}, I_{b_1}, ..., I_{b_{n_b}}, q_1, ..., q_{n_q}]^T \in \mathbb{R}^{2(n_b + n_f)} \nonumber
\]

\[
x_2 = [I_{f_1}, ..., I_{f_{n_f}}, I_{f_1}, ..., I_{f_{n_f}}, q_1, ..., q_{n_q}, V_{f_1}, ..., V_{f_{n_f}}, V_{f_1}, ..., V_{f_{n_f}, Q}]^T \in \mathbb{R}^{4n_f} \nonumber
\]

\[
x_3 = [I_{b_1}, ..., I_{b_{n_b}}, I_{b_1}, ..., I_{b_{n_b}}, q_1, ..., q_{n_q}] \nonumber
\]

\[
\delta_1, ..., \delta_{n_b}, \omega_1, ..., \omega_{n_b}]^T \in \mathbb{R}^{7n_b} \nonumber
\]

\[
x_4 = [I_{g_{n_b}+1}, ..., I_{g_{n_b}+1}, i_{g_{n_b}+1}, ..., i_{g_{n_b}+1}, \nonumber
\]

\[
\delta_{n_b+1}, ..., \delta_{n_b}, \omega_{n_b+1}, ..., \omega_{n_b}]^T \in \mathbb{R}^{7(n_g-n_b)} \nonumber
\]

\[
u_2 = [\alpha_{f_1}, ..., \alpha_{f_{n_f}}]^T \in \mathbb{R}^{n_f} \nonumber
\]

\[
u_3 = [v_{g_1}, ..., v_{g_{n_g}}]^T \in \mathbb{R}^{n_g} \nonumber
\]

\[
u_4 = [v_{g_{n_g}+1}, ..., v_{g_{n_g}+1},]^T \in \mathbb{R}^{n_g} \nonumber
\]

where \( I_{g_k} \in \mathbb{R}^{2} \) and \( I_{g_k} \in \mathbb{R}^{2} \) are the vectors of the stator and rotor coil currents of generator \( k \), and \( \delta_k \) and \( \omega_k \) are its rotor angle position and frequency. Node voltages are denoted by D and Q phasor components \((V_{b_d}, V_{b_q})\) and transmission line currents by phasor components \((I_{f_d}, I_{f_q})\). The FACTS states are \((V_{f_d}, V_{f_q})\) and \((I_{f_d}, I_{f_q})\). Input switching signal of FACTS devices is denoted by \( \alpha_{f_m} \) and input generator excitation voltage by \( v_{g_k} \).

The dynamics of the physical states can then be modeled by a multi-time scale nonlinear dynamic model

\[
\mu_1 \dot{x}_1 = f_1(x_1, x_2, x_3, x_4, p) \quad (3a)
\]

\[
\mu_2 \dot{x}_2 = f_2(x_1, x_2, p, u_2) \quad (3b)
\]

\[
\mu_3 \dot{x}_3 = f_3(x_1, x_2, p, u_3) \quad (3c)
\]

\[
\mu_4 \dot{x}_4 = f_4(x_1, x_4, p, u_4) \quad (3d)
\]

where small parameters \( \mu_i \) denote different time scales.
In this model, states $x_1$ evolve at the scale of tens of milliseconds and include dynamics of transmission lines. States evolving at the time scale of hundreds of milliseconds, $x_2$, represent the natural dynamics of reactive elements in FACTS. A distinction is made between smaller generators, described by states $x_3$, whose dynamics evolve at the time scale of seconds, and the larger generators, described by states $x_4$, whose dynamics evolve at the time scale of tens of seconds. The number of smaller generators is denoted by $n_s \leq n_q$. In the case of our example system, only the states of the smallest generator are included in $x_3$, i.e., $n_s = 1$, while the states of the other two generators belong to $x_4$. Fig. 2 shows this range of time scales in any general power system.

The relationship between time scales can be represented by defining small parameters in which the fastest time scale corresponds to the smallest parameter.

$$
\mu_1 < \mu_2 < \mu_3 < \mu_4
$$

(4)

Time constants $\tau_i = \frac{1}{\mu_i}$ correspond to different time scales. The model in (3) is a very detailed model of high complexity. Designing a controller for this model proves to be a difficult task. In the most general case, the controller is a function of all states. Therefore, its implementation in practice is highly questionable. To overcome this complexity, a reduced-order model is derived next.

### B. Reduced-order Model for FACTS Control Design

The dynamic model of the power system given in (3) is simplified in a systematic way using the singular perturbation method. Singular perturbation [12] is a modeling technique which allows the designer to extract from the model only time scales relevant for the control design. A relevant model for transient stabilization using FACTS can be obtained by reducing the full-order model given in (3).

In this context, the dynamics of $x_4$ which evolve at the slowest time scale can be assumed constant.

$$
\dot{x}_4 = \bar{x}_4
$$

(5)

These dynamics represent the response of large generators with considerable inertia which will take longer to accelerate/decelerate in comparison with the faster generators. States $\bar{x}_4$ are added to the augmented vector of parameters $\tilde{p} = [p \ x_4]$. On the other hand, dynamics of faster generators $x_3$ are relevant for transient stabilization as these generators are the first to lose synchronization with the rest of the grid. These generators are referred to as the critical generators.

Dynamics of states $x_1$, which evolve at a much higher rate than the dynamic states of critical generators, are considered instantaneous for the problem of interest.

$$
\mu_1 = 0
$$

(6)

By solving (3a) for $x_1$ in terms of $x_2$ and $x_3$, a reduced-order model at the time scale of interest is derived. This reduced-order model takes on the form

$$
\begin{align*}
\mu_2 \dot{x}_2 &= \tilde{f}_2(x_2, x_3, u_2, \tilde{p}) \\
\mu_3 \dot{x}_3 &= \tilde{f}_3(x_2, x_3, u_3, \tilde{p})
\end{align*}
$$

(7)

which captures the relevant dynamics for transient stabilization. In what follows, vector of parameters $\tilde{p}$ is omitted from (7) for easier notation.

### IV. Two-level Model Using Interaction Variables

The reduced model (7) is transformed into a form convenient for control design by introducing interaction variables.

Interaction variables capture how parts of the system, in this particular case FACTS and generators, affect each other. The notion of linearized interaction variables for electric power systems was used in [13] for hierarchical modeling and control in large-scale electric power systems. More recently, it was shown in [14] how linearized interaction variables can be used to design the governor control needed to ensure the quality of frequency dynamics in electric power systems.

In this paper, interaction variables of nonlinear form introduced in [15] are used. Interaction variables are defined for modules, where modules represent parts of the system. System (7) is a system composed of two modules, a FACTS and a generator, whose states are $x_2$ and $x_3$.

In a more general case, an interaction variable of module $i$ has the following properties:

- It is a function of the module’s internal states.
- Its dynamics are governed by the module’s own inputs, disturbances, and by the couplings with other modules.

An interaction variable of module $i$ is defined as a scalar energy function of the module’s states $x_i$ of the form

$$
z_i = \nu_i(x_i)
$$

(8)

whose dynamics are represented as

$$
\dot{z}_i = \dot{\nu}_i(x_i) = g_i(\bar{x}_i, \bar{x}_j, u_i)
$$

(9)

where $x_i$ are the states of all neighboring modules, $j \neq i$.

For physical systems, the energy function can be chosen as the Hamiltonian or the total accumulated energy inside the module, i.e.

$$
z_i = \nu_i(x_i) = E_i(x_i)
$$

(10)

where $E_i$ stands for the accumulated energy in module $i$. For modules which are described by differential equations without a clear physical interpretation the interaction variable can be chosen as a Lyapunov function of the module.
Dynamics of the accumulated energy of module $i$, i.e. its interaction variable, are governed by its internal active power dissipation, active power injection through the inputs of the module, and the exchange of active power between module $i$ and the rest of the system.

The interaction variable of a generator module is

$$z_3(x_3) = H_k \omega_k^2 + \frac{1}{2} \mathbf{I}_k^T \mathbf{L}_k \mathbf{I}_{g_k}$$  \hspace{1cm} (11)$$

where $H_k$ is the rotor inertia and $\mathbf{L}_k \in \mathbb{R}^{5 \times 5}$ is the generator inductance matrix. For the FACTS module, the interaction variable is equal to

$$z_2(x_2) = \frac{1}{2} C_f (V_{f_0}^2 + V_{f_q}^2) + \frac{1}{2} L_f (I_{f_0}^2 + I_{f_q}^2)$$  \hspace{1cm} (12)$$

where $C_f$ and $L_f$ are the capacitance and the inductance of a FACTS device.

To explicitly represent interaction variables, a change of states is performed in the model given in (7). One of each of the original states $x_2$ and $x_3$ is replaced by the corresponding interaction variable in the following way

$$x_2 \rightarrow [\bar{x}_2 \ z_2]^T$$

$$x_3 \rightarrow [\bar{x}_3 \ z_3]^T$$  \hspace{1cm} (13)$$

where $\bar{x}_2$ and $\bar{x}_3$ are the vectors of all original module states except one. This one state is omitted in order to avoid state redundancy. The omitted states are the frequency of the generator and the voltage magnitude of the FACTS. The interaction variable-based power system model becomes

$$\mu_2 \ddot{x}_2 = \bar{f}_2(\bar{x}_2, \bar{x}_3, z_2, u_2), \quad \dot{z}_2 = g_2(\bar{x}_2, \bar{x}_3, u_2)$$

$$\mu_3 \ddot{x}_3 = \bar{f}_3(\bar{x}_2, \bar{x}_3, z_3, u_3), \quad \dot{z}_3 = g_3(\bar{x}_2, \bar{x}_3, u_3)$$  \hspace{1cm} (14)$$

The interaction variable dynamics of each module can be further expressed in terms of its own local states, its local inputs and the power exchanged between the two modules.

$$\dot{\bar{x}}_2 = g_2(\bar{x}_2, u_2, P)$$

$$\dot{\bar{x}}_3 = g_3(\bar{x}_3, u_3, P)$$  \hspace{1cm} (15)$$

The exchanged power $P$ is equal to the product of the FACTS voltage state belonging to $\bar{x}_2$ and the generator current state belonging to $\bar{x}_3$.

Fig. 3 illustrates a two-level structure of the system modeled as shown in (14).

V. COOPERATIVE CONTROL BY USING A TWO-LEVEL INTERACTION VARIABLE-BASED MODEL

A cooperative controller of power-electronic switching $u_2$ in the FACTS module is proposed. The FACTS helps stabilize generator module dynamics $\bar{x}_3$ by controlling its interaction variable $z_3$. This is done in two steps. First, the higher-level model at the slower time scale is derived in order to obtain the controller behavior which stabilizes the system. A desired rate of exchange of energy between modules is determined using this sub-model. Second, a controller is designed using the lower-level model at the faster time scale. The fast controller response is utilized to obtain the desired rate of energy exchange.

A. FACTS Cooperative Control Strategy at the Slower Time Scale

To only preserve the slow reduced-order model from the model (14), $\mu_2$ is considered as an infinitely small parameter $\mu_2 = 0$, and $\bar{x}_2$ is expressed as a function of slow variables. This leads to

$$\bar{x}_2 = \bar{h}_2(\bar{x}_3, u_2)$$

$$\bar{x}_2 = \bar{h}_2(\bar{x}_3, u_2)$$  \hspace{1cm} (16)$$

Now, the slow reduced-order model takes on the form

$$\frac{d\bar{x}_3}{d\tau_3} = \bar{f}_3(\bar{x}_3, z_3, u_2, P_{\tau_3}), \quad \dot{z}_3 = g_3(\bar{x}_3, u_3, P_{\tau_3})$$  \hspace{1cm} (17)$$

Note that the exchanged power is denoted by $P_{\tau_3}$ at this slow time scale $\tau_3$ where

$$P_{\tau_3} = \frac{1}{\tau_3} \int_0^{\tau_3} P dt$$  \hspace{1cm} (18)$$

The entropy controller is designed to stabilize interaction variable $z_3$. An entropy function at the time scale $\tau_3$ is defined as

$$\varepsilon = \frac{1}{2} (z_3 - z_3^*)^2$$  \hspace{1cm} (19)$$

The first derivative of entropy is thus equal to

$$\dot{\varepsilon} = -(z_3 - z_3^*) g_3(\bar{x}_3, u_3, P_{\tau_3})$$  \hspace{1cm} (20)$$

The exchanged power is chosen in such a way to guarantee that $\dot{\varepsilon} < 0$, $\forall t$. This is guaranteed if $P$ (at any time scale) is chosen as

$$P = \{P : g_3(\bar{x}_3, u_3, P_{\tau_3}) = -K_z (z_3 - z_3^*)\}$$  \hspace{1cm} (21)$$

Note that the last expression allows for $P$ to take those values at faster time scales which do not satisfy condition (21) as long as this condition is satisfied for average $P_{\tau_3}$.

Finally, the internal dynamics of the generator is expressed by using only its internal states, interaction variable and its derivative as

$$\frac{d\bar{x}_3}{d\tau_3} = \bar{f}_3(\bar{x}_3, z_3, u_2, P_{\tau_3}, \frac{dz_3}{dt})$$  \hspace{1cm} (22)$$

Note that the internal states of the generator are naturally stable in the region of the attraction of the equilibrium. Therefore, a controller with the objective (21) will also stabilize internal states while stabilizing the generator’s interaction variable.
B. FACTS Controller Logic at the Faster Time Scale

The controller is driven by the fast thyristor switching \( u_2 \) at the faster time scale. The boundary-layer model is obtained by setting \( \tilde{x}_3 = \bar{x}_3 \) and \( z_3 = z_3^2 \) and it takes on the form

\[
\frac{d\tilde{x}_2}{d\tau_2} = \tilde{f}_2(\tilde{x}_2, z_2, u_2, u_3, P_{\tau_3}), \quad \dot{z}_2 = g_2(\tilde{x}_2, u_2, P_{\tau_3}) \tag{23}
\]

The controller can be designed to minimize the difference between \( P_{\tau_3} \) in (21) and the active power \( P_{\tau_2} \) exchanged between the modules at the faster time scale.

\[
\nu = \frac{1}{2}(P_{\tau_2} - P_{\tau_3})^2 \tag{24}
\]

In other words, the power between the two modules is controlled at the faster time scale to achieve the objective defined at the slower time scale.

The controller is designed to guarantee that \( \dot{\nu} < 0, \forall t \). Slow average power \( P_{\tau_3} \) is considered constant at this time scale. It is shown in [9] that a controller can be designed to guarantee that this condition is fulfilled. The sketch of the controller derivation is shown next. The first derivative of \( \nu \) is expressed as

\[
\dot{\nu} = (P_{\tau_2} - P_{\tau_3}) \frac{d\nu_2(\tilde{x}_2, u_2, P_{\tau_2})}{dt} \tag{25}
\]

Reference [9] shows that (25) can be rewritten as

\[
\dot{\nu} = (P_{\tau_2} - P_{\tau_3})(a_2(\tilde{x}_2) + b_2(\tilde{x}_2)u_2) \tag{26}
\]

Finally, the controller expression takes on the form

\[
u_2 = \frac{1}{b_2(\tilde{x}_2)} (-a_2(\tilde{x}_2) - K_P(P_{\tau_2} - P_{\tau_3}))\tag{27}\]

VI. Simulation Results

Simulations are performed on the system shown in Fig. 1. A fault is simulated as a short circuit at bus 3 with a duration of 0.33 s. This fault causes the system with constant control inputs to become transiently unstable.

Fig. 4 shows the dynamic response of the two interaction variables \( z_2 \) and \( z_3 \) in the uncontrolled system.

As seen from the plots, the interaction variable of the generator becomes unstable while the interaction variable of the FACTS is stable and does not change considerably from its equilibrium. In other words, this simulation shows that the states of one device can become unstable while the states of other devices are stable. The cooperative controller should be installed on the naturally stable devices, in our case FACTS, in order to improve the stability of the unstable devices, i.e. critical generators.

Next, the response of the system with the controller given in (27) to the same fault is shown. Fig. 5 shows the stable interaction variables in the case with the cooperative controller. The plots show that the accumulated energy of the FACTS is much larger in this case than in the case without a controller. This temporary accumulation of energy helps the generator preserve stability and gives the system a longer critical clearing time.

Fig. 6 shows the power exchanged between the two devices at two different time scales. The yellow circle marks the place at which the power exchanged at the slower time scale \( \tau_3 \) is equal to the average of the power at the faster time scale \( \tau_2 \). This is a desired behavior described by (18).
VII. CONCLUSIONS

This paper introduces a novel approach to modeling and controller design for stabilization of power systems affected by large disturbances. The approach, based on singular perturbation theory and a change of states, reduces a complex power system model to keep only the relevant dynamics for transient stabilization using FACTS. A cooperative controller is designed to stabilize the interactions of the critical generator with the rest of the system. The performed simulations show promising results. The future work should address the question of generalization of the controller design in the case of multiple critical generators.

APPENDIX

The physics-based dynamic model of a power system, given in (28), is taken from reference [9]. This model considers a power system with $n_g$ nodes, $n_t$ transmission lines, $n_g$ generators and $n_f$ FACTS devices. For $i=1...n_g$, $j=1...n_t$, $k=1...n_g$ and $m=1...n_f$, the interconnected power system model can be expressed as:

\[
\begin{align*}
\dot{I}_{g_k} &= A_{sa_k}(\delta_k, \omega_k)I_{g_k} + A_{sr_k}(\delta_k, \omega_k)I_{r_k} + B_{sa_k}(\delta_k)S_g[k, i]V_1 + B_{sr_k}(\delta_k)v_{g_k} \\
\dot{I}_{r_k} &= A_{ra_k}(\delta_k, \omega_k)I_{r_k} + A_{rr_k}(\omega_k)I_{r_k} + B_{ra_k}(\delta_k)S_g[k, i]V_1 + B_{rr_k}(\omega_k)v_{g_k} \\
\dot{\delta}_k &= \omega_h \omega \hat{\delta}_k \\
\omega_k &= \frac{1}{2H_k}(M_k - \Gamma^T_{g_k}N_kI_{g_k} - D_k\omega_k) \\
\dot{\hat{V}}_{b_i} &= \frac{\omega_h}{C_{ch_i}} \sum_{j=1}^{n_t} S_{tl}^i[j, i] \dot{\hat{I}}_{t_{lj}} + S_{gl}^i[i, k] \dot{\hat{I}}_{g_k} \\
&- \frac{\omega_h}{C_{ch_i}} S_{hl}^i \hat{V}_{b_i} + j\omega_h \hat{\omega} \hat{V}_{b_i} \\
\dot{\hat{I}}_{t_{lj}} &= -\frac{\omega_h}{L_{t_{lj}}} \left( -\sum_{i=1}^{n_h} S_{hl}^i[j, i] \hat{V}_{b_i} - S_{tr}[j, m] \hat{V}_{f_m} + \hat{R}_{t_{lj}} \hat{I}_{t_{lj}} + j\omega_h \hat{\omega} \hat{I}_{t_{lj}} \right) \\
&+ \frac{\omega_h}{L_{f_{mj}}} \left( S_{tr}[m, j] \dot{\hat{I}}_{t_{lj}} - \dot{\hat{I}}_{f_m} \right) + j\omega_h \hat{\omega} \hat{V}_{f_m} \\
\dot{\hat{V}}_{f_m} &= \frac{\omega_h}{L_{f_{mj}}} \left( S_{tr}[m, j] \dot{\hat{I}}_{t_{lj}} - \dot{\hat{I}}_{f_m} \right) + j\omega_h \hat{\omega} \hat{V}_{f_m} \\
\dot{\hat{I}}_{f_m} &= \frac{\alpha f_{r_h}}{L_{f_{mj}}} \hat{V}_{f_m} + j\omega_h \hat{\omega} \hat{I}_{f_m}
\end{align*}
\]

Generator parameter matrices $A_{sa_k}$, $A_{sr_k}$, $A_{ra_k}$, $A_{rr_k}$, $B_{sa_k}$, $B_{sr_k}$, $B_{ra_k}$ and $B_{rr_k}$ are described in detail in [9]. Their dependence of $\delta_k$ and $\omega_k$ comes from the Park/Blondel transform. Matrices $S_{tl} \in \mathbb{R}^{n_t \times n_t}$, $S_g \in \mathbb{R}^{n_g \times n_g}$ and $S_f \in \mathbb{R}^{n_f \times n_f}$ are incidence matrices which show the connectivity between dynamical components.

FACTS Dynamics of Interest

The natural dynamics of the FACTS device shown in Fig. 7 are governed by the very fast thyristor switching [8]. To model these dynamics, the FACTS voltage and current are subject to the Fourier transform [16]. Of particular interest are two components of the Fourier transform: the fundamental component at the time scale of the nominal grid frequency, and a higher-order component at the thyristor switching frequency. These components are described by

\[
\begin{align*}
\hat{V}_f &= h_V(\hat{I}_{t_{lj}}, \hat{I}_{f_m}, \hat{V}_f) \\
\dot{\hat{I}}_f &= h_f(\hat{V}_f, \hat{I}_{f_m}, \alpha_f) \\
\dot{\hat{V}}_{f_{kh}} &= h_{V_{f_{kh}}}(\hat{V}_f, \hat{I}_{f_m}, \alpha_{f_{kh}}) \\
\dot{\hat{I}}_{f_{kh}} &= h_{I_{f_{kh}}}(\hat{V}_f, \hat{I}_{f_m}, \alpha_{f_{kh}})
\end{align*}
\]

where $\alpha_{f_{kh}} = \frac{1}{\tau_0} \int_{t_0}^{t} \alpha_f(t) dt$ and $\alpha_f = \frac{1}{\tau_0} \int_{t_0}^{t} \alpha_{f_{kh}} d\tau_0$ and $\tau_0 = 1ms$ and $\tau_f = \frac{\pi}{2kHz}$. The faster time scale in the range of kHz is eliminated from this model preserving only the relevant dynamics of the fundamental component in (28).

Fig. 7. An example of a FACTS device.

REFERENCES