Time-varying LQR on Hypersonic Vehicle Profile-following

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Abstract— For reference profile-following guidance, the traditional method calculating weight matrices in linear quadratic regulator (LQR) is not applicable to hypersonic vehicles (HSV) which have particular dynamic characteristics. This paper proposes time-varying LQR to utilize time-varying parameters structuring weight matrices which Bryson method calculates. In this method, the current deviations of the flight states are substituted into the parameters calculation. Simulations demonstrate that the variation of parameters can significantly improve the performance of hypersonic vehicle profile following, and own stronger robustness against different disturbances.

I. INTRODUCTION

Reference profile-following guidance is one of the most important problems in aerial vehicle re-entry guidance. As a result of external and internal disturbances, the actual profile of the vehicle entry flight will deviate from the reference entry profile. Guidance compensatory signal which generates the next step control command can be gained by state feedback using the deviations between the real and reference profiles. And the command will control the vehicle to track the reference profile properly [1-2]. The chief challenge of following the reference profile lies in generating the proper compensation signal by deviations. Linear Quadratic Regulator (LQR) can solve this problem effectively [3-5].

In the case of linear system and quadratic performance index, LQR eventually transforms optimal control problem into solving Riccati equation [6]. In LQR optimization, weight matrices Q and R are the most important components and determine the solution effectiveness of Ricatti equation [7-10]. Trial-and-error method has been employed to construct these two matrices. This method is simple, but primarily depends on people’s experience and intuitive adjustment. Elements of weight matrices must be repeatedly experimented to get a proper value, and this method is not feasible for application in large scale system. Because of that, Bryson method can be applied to system which has plenty of state variables to obtain a better result in less time [11]. Bryson method has been successfully utilized in reference profile-following guidance of the aerial vehicle [12].

Since the Hypersonic Vehicles owns particular characteristics, including high flight altitude, large flight envelope, complex entry environment, and precise terminal guidance accuracy requirement, it’s difficult for HSV to track the reference profile properly using LQR with weight matrices obtained by Bryson method [13-14]. Genetic algorithm can be applied to find a global optimal solution with less computation [15]. However, it has invalid improvement to profile-following performances.

In this study, contrast to the traditional constant weight matrix, we propose a time-varying method to construct Q with varying parameter on the basis of Bryson method. This idea provides more accurate feedback gain to linear flight systems. The remainder of the study is organized as follows. Firstly, Section II represents LQR, Bryson method, and their applications in aerial vehicle tracking reference profile. Next, Section III analyzes the problem of hypersonic vehicle profile-following, and introduces the time varying method to construct weight matrix. Then, Section IV tests the performance of HSV profile following with the new method. Finally, Section V gives the conclusion of the proposed algorithms.

II. REFERENCE PROFILE-FOLLOWING GUIDANCE PROBLEM

A. Profile-following Guidance

After generating a feasible reference entry profile in the altitude and velocity plane, the bank angle $\sigma$ (guidance command) will control the aerial vehicle to track the reference profile properly. As a result of external and internal disturbances, we obtain the deviations between reference profile and the actual real-time data measured by navigation facilities. The deviations contain altitude $z_s$ and velocity $v_s$. In order to minimize the deviations and keep the vehicle tracking reference profile properly, the optimal feedback gain of guidance compensatory signal $\sigma$ can be calculated by LQR as follows:

$\begin{align*}
\delta x' &= A\delta x + B\delta u \\
\delta y &= C\delta x
\end{align*}$

(1)

Where $\delta x = [z_s, v_s]^T$, $\delta u = \sigma_s$

Then, construct the quadratic performance index as follows:
\[
J(t, t_f) = \int_{t}^{t_f} [\delta x^T(\tau)Q\delta x(\tau) + \delta u(\tau)R\delta u(\tau)]d\tau \tag{2}
\]

In order to minimize the index \( J \), we calculate the Riccati equation to obtain the optimal feedback gain \( K \). The Riccati equation is given:

\[
P A - PB R^{-1} B^T P + Q + A^T P = 0 \tag{3}
\]

After getting the solution \( P(t) \) corresponding to each time instant \( t \) by solving the equation, the feedback gain can be obtained:

\[
K(t) = R^{-1}B^T(t)P(t) \tag{4}
\]

Then, the compensatory signal can be acquired.

\[
\delta u = -K\delta x \tag{5}
\]

Finally, we can get the real guidance signal which consists of reference signal \( u \) and compensatory signal \( \delta u \):

\[
\sigma = u + \delta u = \sigma_{ref} - K\delta x \tag{6}
\]

**B. Bryson Method and Problem**

In order to obtain a proper quadratic performance index, weight matrices \( Q \) and \( R \) must be chosen properly, and Bryson method can solve this problem effectively.

The basic principle of Bryson method is to normalize the contributions, and then the states and the control term may behave effectively within the definition of the quadratic cost function. The normalization is accomplished by using the anticipated maximum values of the individual states and control quantities. The method can be explained as follows.

Firstly, select the weight matrices \( Q \) and \( R \) as diagonal matrices, namely

\[
Q = diag[q_1, q_2], \quad R = [r_1] \quad \tag{7}
\]

Then, develop the quadratic index in the following expression:

\[
J = \int_{t}^{t_f} (q_1 z_{\delta}^2 + q_2 v_{\delta}^2 + r_1 \sigma_{\delta}^2)dt \tag{8}
\]

Determine each maximum value of all the states and control terms

\[
z_{\delta_{\text{max}}}, \quad v_{\delta_{\text{max}}}, \quad \sigma_{\delta_{\text{max}}} \tag{9}
\]

Where \( z_{\delta_{\text{max}}} \) and \( v_{\delta_{\text{max}}} \) are anticipated maximum deviations between the actual profile and the reference profile, and \( \sigma_{\delta_{\text{max}}} \) is the maximum allowable modification of guidance signal \( \sigma \).

Normalize all the contributions to 1 with the help of all the maximum values

\[
q_1 z_{\delta_{\text{max}}}^2 = q_2 v_{\delta_{\text{max}}}^2 = r_1 \sigma_{\delta_{\text{max}}}^2 = 1 \tag{10}
\]

Then, the elements to construct the weight matrices can be obtained

\[
q_1 = \frac{1}{z_{\delta_{\text{max}}}^2}, \quad q_2 = \frac{1}{v_{\delta_{\text{max}}}^2}, \quad r_1 = \frac{1}{\sigma_{\delta_{\text{max}}}^2} \tag{11}
\]

Then we can get the weight matrices:

\[
Q = diag[\frac{1}{z_{\delta_{\text{max}}}^2}, \frac{1}{v_{\delta_{\text{max}}}^2}], \quad R = [\frac{1}{\sigma_{\delta_{\text{max}}}^2}] \tag{12}
\]

However, in the reference profile-following guidance of HSV because of its particular dynamics characteristics including high flight altitude, large flight envelope, and complex entry environment, new problems occur in the selection of weight matrices.

In the initial stage, it’s supposed that the deviations of altitude and velocity are \( z_{\delta_{0}} \) and \( v_{\delta_{0}} \), respectively. In this study, we set them as follows:

\[
[z_{\delta_{0}} = 3\text{km}, \quad Q_{0} = diag[\frac{1}{z_{\delta_{0}}^2}, \frac{1}{v_{\delta_{0}}^2}]]
\]

\[
[z_{\delta_{0}} = 0.5\text{km}, \quad v_{\delta_{0}} = 200\text{m/s}, \quad Q_{0} = diag[\frac{1}{z_{\delta_{0}}^2}, \frac{1}{v_{\delta_{0}}^2}]]
\]

Based on above assumption, \( z_{\delta_{0}} \) and \( v_{\delta_{0}} \) are bigger than \( z_{\delta_{1}} \) and \( v_{\delta_{1}} \) respectively. The weight matrix \( Q_{0} \) which is determined by \( z_{\delta_{0}} \) and \( v_{\delta_{0}} \), can effectively constrain the large deviations between the real and reference profiles in the initial stage. Nevertheless, the capacity of \( Q_{0} \) for resisting entry process disturbance is not strong enough to satisfy the accuracy metrics of the terminal guidance. On the contrary, the weight matrix \( Q_{0} \) constructed by \( z_{\delta_{1}} \) and \( v_{\delta_{1}} \), can resist the process disturbances effectively. However, facing the existence of large state deviations in the early entry flight, it is difficult to keep HSV tracking the reference profile properly which will further influence the terminal guidance accuracy. Therefore, compared to \( Q_{0} \), the weight matrix \( Q_{1} \) is not applicable to initial deviations, and has good robustness to process disturbance. The simulations for hypersonic vehicles following reference profile are shown in Fig. 1 to Fig. 3.

In order to solve above problem, we hope that LQR can not only minimize the initial deviations, but also enhance the capability that resists the process disturbance effectively. This study, with the help of Bryson method, constructs the weight matrix \( Q \) with varying parameters versus velocity. Following simulation figures indicate that, this method solves this problem effectively.
III. Time-Varying Linear Quadratic Regulator

In this study, $z_{\delta_{\text{max}}}$ and $v_{\delta_{\text{max}}}$ are a group of varying parameters versus changing velocity. The weight $Q$ determined by Bryson method, will change versus velocity correspondingly.

The reference profile-following guidance of HSV based on LQR is shown as the solid lines in Fig. 4.

The work flow is explained as follows:

Comparing the actual flight profile with the reference profile, we get the state deviations containing $z_{\delta}$ and $v_{\delta}$. With these deviations, we calculate the compensatory signal $u_\delta$ by multiplying feedback gain $K$. Then we can input the compensatory signal and the reference guidance signal into HSV guidance loop. In this way, we can get actual flight profile of the next step. The calculation of the feedback gain
LQR involves four matrices. As shown in the figure, the construction of system matrices A and B need actual state parameters. Weight matrices Q and R need to be determined and downloaded into the on-board computer before starting entry guidance of HSV.

Instead of presetting the specific elements in traditional method, the time-varying optimization method substitutes the flight state deviations $z_{\delta}$ and $v_{\delta}$ into the calculation of Q. The main idea of this method can be explained as the dashed line in Fig. 4.

With the help of Bryson method, the calculation of elements in weight matrix Q involves two parameters $z_{\delta \text{max}}$ and $v_{\delta \text{max}}$. These two parameters represent maximal allowable deviations in altitude and velocity between actual and reference profiles, respectively. In time-varying optimization method, we make a comparison between the actual real-time profile and the relevant reference profile, and get the current deviations $z_{\delta}$ and $v_{\delta}$. Then we substitute them into $z_{\delta \text{max}}$ and $v_{\delta \text{max}}$, that is,

$$z_{\delta \text{max}} = z_{\delta}, \quad v_{\delta \text{max}} = v_{\delta}$$  \hspace{1cm} (9)

Substituting $z_{\delta \text{max}}$ and $v_{\delta \text{max}}$ into (8), the weight matrix Q can be obtained. Furthermore, the feedback gain K can be calculated. The specific steps are given as follows:

- Measure the actual current flight profile which contains altitude $z$ and velocity $v$. Compare them with the relevant reference altitude $z_{\text{ref}}$ and velocity $v_{\text{ref}}$, we obtain the current deviations $\delta z$ and $\delta v$, respectively.
- Substituting $z$, $v$, and $\gamma$ into system (1), we calculate the linear system matrices A and B.
- Substituting $\delta z$, $\delta v$, and maximal allowable adjustment of guidance signal $\delta \sigma_{\text{max}}$ into (9), (7), and (8), the weight matrices Q and R can be constructed.
- Using A, B, Q, R, (3), and (4), the feedback gain K can be obtained.
- The compensatory guidance signal $\delta u$ can be calculated by K in (5). Then we can get the actual guidance signal in (6), namely bank angle $\sigma$.

IV. TESTING

The simulations for hypersonic vehicle following reference profile based on time-varying optimization method are shown in Fig. 5 to Fig. 7.
which constructs the weight matrix with varying parameters. In this way, the capability of hypersonic vehicle following reference profile is significantly improved. Simulations and comparisons demonstrate that it could be more robust than traditional methods under different initial deviations and process disturbances.

REFERENCES


