An algorithm to calculate optimized coordinated trajectories of intelligent vehicles in a segment of road for any lane configuration

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Abstract—This paper presents a simulation algorithm to calculate optimized coordinated trajectories of intelligent vehicles in a road. The objective is to reduce travel time for each vehicle. The configuration of the road can be modelled regarding the number of lanes, direction preference and exclusivity. The calculation considers the main elements of the traffic system, such as topography of the lane, traffic rules and individual capacity of acceleration. It can deal with most traffic situations such as overtaking, obstacles, slopes, and speed reducers. An extension of the algorithm is also proposed. It improves results by providing ideal space for acceleration within a platoon, maintaining the order of priority of the vehicles in a queue. Experimental tests to evaluate the algorithm are presented.

I. INTRODUCTION

The autonomous car, also called driverless car or Intelligent Vehicles (IV), are vehicles that are assumed to be fully automated with throttle, braking, and steering commands being determined by automated on-board controllers. Recently, IVs are being considered so seriously that governments have even approved laws conceiving special driving license for prototypes. The fact that governments already trust IVs transiting among pedestrians means at least two important things: the IV correlated technologies are developed enough to minimize risks of traffic accidents and the existing demand from society to continue research for IVs. The many advantages of changing from a manual to a full automatic vehicle, mostly related to human errors, reinforce a probably upcoming scenario where there will be only IVs transiting on a traffic infrastructure. This scenario could be explored to improve traffic efficiency in many ways, such as safety, road flow capacity and fuel consumption. In this context, the objective of this study is to propose a mathematical solution to compute and optimize coordinated trajectories of IVs in a segment of road. The idea is that traffic evolutions could be microscopic predicted and oriented according to a dynamic model reflecting real traffic situations.

We consider available a traffic infrastructure provided with a Central Coordination System (CCS) capable to communicate to vehicles within a road segment. The CCS function is to monitor id and vehicle position, seeks correlated information of each vehicle, such as destination and vehicle characteristics. The CCS then collects additional real time information, such as weather condition and traffic accidents, and computes next displacements of all vehicles resulting in a plan called global coordinated trajectory planning (GCTP). The GCTP combines all individual plans which are finally communicated to the respective vehicles. The plan received by each vehicle is analyzed by the on-board control and can be adopted or not, depending on the others important information considered, such as an obstruction in front of the vehicle. This process can occur, for example, in every 60s as proposed by PATH information control architecture for a group of vehicles coordination [1]. The GCTP would provide better performance not only for the traffic control but also for drivers that can have their travel time assured.

The problem is especially difficult to solve due to the combinatorial possibilities of overtaking as can be seen in Fig. 1. Also, the temporal dynamics and vehicles acceleration constraints demand a lot of variables to model the problem. Other difficulty is to define priority between local and global optimization, in this case to define which vehicles should overtake first.

There are already some algorithms proposed, however each one for a specific road configuration. In this work it is proposed a generic algorithm for any road configuration regarding the quantity of lanes, direction preference and exclusivity of each lane.

Another improvement proposed is a scheme to optimize the travel time of some vehicles without compromising the travel time of the others. The idea is to force specific delays in some IVs running in a queue. These delays will force one IV to slow down, but rather than make it loose time it will allow the IV to reach maximum velocity during the overtaking moment and at the end of the trajectory there will be a travel time saving.

The organization of this paper is as follows. In Section II, areas of related work are discussed. Section III introduces the problem and presents the approximation considered for the model. Section IV summarizes the proposed algorithm and section V presents an optimization scheme. In Section VI, simulation tests using both the algorithms are presented. Finally, the conclusions can be found in Section VII.
II. RELATED WORK

The problem is treated in robotics as a special case of general motion planning for multiple robots, as for example multi-robot path planning and motion coordination problem. The robots are then called autonomous vehicles. Most of the approaches do not use central coordination and are more concerned with uncertainties in the movement of neighbouring vehicles [2], [3]. One common strategy is to reduce the configuration space of the vehicles. The free space for the vehicles is called roadmap [4]. One idea proposed in [5] is to turn a two dimensional roadmap representing a road into a one dimensional. Another common approach to robot path planning is routing algorithms based on decomposition in two steps. First is defined the path and then velocity [6].

A relevant approach proposed the Mixer Integer Linear Programming (MILP) optimization to calculate the path considering time discretization [7]. Similar approach was done for aerial and subaquatic vehicles [8]. However, these approaches do not take advantage of the specific aspects of the road, such as that longitudinal displacement is not relevant comparing to lateral displacement.

Recently some approaches introduce acceleration constraints for the vehicles and direct approach traffic evolution. In [5] the vehicle traffic dynamics was modelled as MILP model with travel time as criteria for optimization. The resulted trajectory plan called velocity profile plan, should correspond to the optimal velocity plan for each vehicle at each time and also should indicate the ideal overtaking moments. This solution is a probably optimal value but with no real time application due to high computational time. The solution proposed in the paper is to use instead an algorithm based on traffic simulator specific for a single lane road. The paper also exposed the problem of decision between local and global optimum. The solution proposed is to consider the current order of position in a road as the order of preference for overtaking. Another proposed algorithm is also based on traffic simulator and specific for a double lane road[9]. None of these algorithms can represent a generic configuration of a road regarding number of lanes, preferences and exclusivity. Another recent approach uses evolutionary genetic programming and can consider acceleration constraints, however is also limited in computational time and cannot be used in real application [10].

The conflict of decision in a road intersection is not the focus in this paper but can represent the decision for overtaking. Many ideas were proposed based on centralized and decentralized rules. The objective is to avoid a deadlock situation. The solution proposed in this work follows in some way the idea proposed in [11] in which the first robot to arrive in the intersection request the authority to decide which candidate will cross first. Actually, in this case, which vehicle on queue will overtake first (see Fig. 1).

III. THE TRAFFIC DYNAMICS APPROXIMATION

The traffic dynamics is typically non-linear therefore in order to enable mathematical modeling it was considered some approximations:

(a) Max acceleration curve for (b) Example of lateral displacement approximation.

Fig. 2: Example of discretization adopted in the model.

A. Time discretization

The model uses uniform discretization of time. Considering an initial traffic situation, where all respective positions and velocities are known, then all future displacements are calculated for defined intervals of time $\Delta t$ until a final period of time $T (t = 1, \cdots, T)$. The vehicle position is defined by coordinates $(P_x, P_y)$ and receives acceleration $(a_x, a_y)$ in each direction resulting on the respective velocities $(V_x, V_y)$. The state equations are:

$$
\begin{bmatrix}
P_x \\
P_y \\
V_x \\
V_y
\end{bmatrix}_{k+1} =
\begin{bmatrix}
1 & 0 & \Delta t & 0 \\
0 & 1 & 0 & \Delta t \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
P_x \\
P_y \\
V_x \\
V_y
\end{bmatrix}_k +
\begin{bmatrix}
\frac{(\Delta t)^2}{2} & 0 \\
0 & \frac{(\Delta t)^2}{2} \\
\Delta t & 0 \\
0 & \Delta t
\end{bmatrix}
\begin{bmatrix}
a_x \\
a_y
\end{bmatrix}_k
$$

B. Maximum acceleration curve discretization

The acceleration of a vehicle depends mainly on its velocity at the previous instant, also its weight, engine power and aerodynamics. The value of this acceleration can be determined by a specific maximum acceleration curve of the vehicle as the example in Fig. 2a. This curve represents the maximum velocity that the vehicle can achieve considering its characteristics (engine power, weight, ...). The discretization proposed can also be seen in Fig. 2a where the velocity range is divided into $k$ intervals.

C. Lateral displacement approximation

The lateral movement is related to the vehicle deviation from the center of the track and normally occurs during changing lanes maneuvers. The lateral displacement is not relevant if compared to the longitudinal, i.e. displacement along the track. Based on this fact, the idea for approximation of the lateral movement calculation is to simply indicate the lane that the vehicle is at each moment as can be seen on Fig. 2. The lateral position of the vehicle will always be considered in the middle of the width of the track. When a vehicle change the lane, the vehicle will be considered in the center of the width of a track at the previous moment and at the next instant considered is at the center of the width of the other track. The concept can be understood as a rail system where each lane of the road would be represented by...
a rail track. In this manner the focus of calculation will be the overtaking priority problem.

D. Road segment discretization

The road segment is divided into $S$ sections (see Fig. 3) in a way to represent topography and regulatory signals. One section $s$ ($s = 1, \cdots, S$) can represent, for example, a slope, or a prohibited pass area, or even both situations. Sections can affect calculation by modifying acceleration, limiting velocity or/and avoiding overtaking.

IV. The TIH algorithm

The proposed algorithm TIH (Trajectories of IVs in Highways) can calculate coordinated trajectories of IVs for any configuration of the lanes. The traffic dynamics was modeled similar to traffic simulator with a “car following” model (see [12]). Broadly speaking, each vehicle tries to develop his desired velocity, respecting the restrictions, and must decrease velocity as approaching the vehicle ahead. Drivers will have aggressive behaviour to reproduce the individual objectives of reducing travel time. This means drivers will always try to reach the desired velocity and will always try to overtake. The order of priority adopted for overtaking, including both directions, is according to respective distance to the end of segment, i.e. one vehicle has priority over the others behind.

The trajectory plan resulted from TIH calculation is a list of positions and velocities for each vehicle at each interval of time.

The model considers adjacent lanes ($l = 1, 2, \cdots, L$). In terms of traffic flow direction the lane can assume the following types:

1) $\uparrow$ exclusive for one direction (access permitted only for the vehicles in the respective direction),
2) $\uparrow\downarrow$ preferable for one direction (vehicles in opposite direction are allowed just when lane is not occupied),
3) $\downarrow\uparrow$ passing lane (can be used in both directions).

One assumption for the model is that drivers set only one desired, i.e. maximum, velocity for the whole trip.

Vehicles can simulate obstacles (lane interrupted) by setting their velocities to zero. The vehicle is considered to be a moving point. This is a classical approach taken in robot motion planning: the obstacles are enlarged with the size of the moving object, such that the vehicle itself reduces to a moving point [2].

A. Parameters

The parameters are (see also Fig. 3 and Fig. 4):

\begin{itemize}
  \item $AC_{i,s,k}$ acceleration (m/s$^2$) capacity corresponding to velocity range $k$ for vehicle $i$ in section $s$,
  \item $AS_s$ acceleration (m/s$^2$) due to topography in section $s$,
  \item $DL_{i,s}$ distance (m) from the beginning of the lane $l$ to the end of section $s$,
  \item $DI$ length (m) of segment of the road,
  \item $DM$ minimum distance space separating two vehicles,
  \item $DO$ min dist. (m) to another vehicle in opp. direction,
  \item $DR$ minimum distance (m) to the vehicle behind when a vehicle just changes to its lane,
  \item $DU$ min distance between vehicles after they crossed,
  \item $DV_i$ length (m) of vehicle $i$,
  \item $FD_{i,l,s}$ flux direction [0, 1] of lane $l$ on section $s$,
  \item $FV_i$ direction [0, 1] of vehicle $i$,
  \item $FP_i$ priority level of vehicle $i$, default=0,
  \item $FU_{i_1,i_2,s}$ if (=1) allow changes from lane $l_{i_1}$ to $l_{i_2}$ in sec. $s$,
  \item $WC_{i,s,k}$ velocity (m/s) range $k$ for vehicle $i$ in section $s$,
  \item $WD_{i,s}$ desired velocity (m/s) of vehicle $i$ in section $s$,
  \item $WP_{i,s}$ max permitted velocity of vehicle $i$ in section $s$,
  \item $Y$ period of time interval.
\end{itemize}

B. Continuous variables

The continuous variables are:

\begin{itemize}
  \item $d_1, d_\cdots$ distance (m) to neighboring vehicles (see Fig. 4),
  \item $D_1$ estimated $d_1$ necessary to change to lower lane,
  \item $D_2$ estimated $d_2$ necessary to move forward,
  \item $D_3$ estimated $d_3$ necessary to change to higher lane,
  \item $P(i,t)$ position (m) of vehicle $i$ on time $t$,
  \item $V(i,t)$ velocity (m/s) of vehicle $i$ on time $t$.
\end{itemize}

C. Discrete variables

The discrete variables are:

\begin{itemize}
  \item $C(i,t)$ current lane $l$ of vehicle $i$ on time $t$,
  \item $FC(l,i,t)$ if (=1) indicates previous colision when $i$ changed to lane $l$ on time $t$,
  \item $OI(i)$ time that vehicle $i$ last changed lane,
  \item $I_{1,2,3}$ index of next vehicles ahead (see Fig. 4).
\end{itemize}

D. Metric for traffic evolutions

The objective function can be modelled in two different ways: minimize the travel time on a certain distance, or to maximize the distance travelled in a given period of time. As suggested in [5] the last way is adopted because it is easier to implement considering time discretization. More precisely, the objective function maximizes the sum of the values corresponding to the final position of all vehicles. The metric ($Z$) to be used to evaluate traffic evolutions is the sum of the values corresponding to the final position of all vehicles:

\begin{equation}
Z = \sum_{i=1}^{l} |(P(i,T) - P(i,0))| 
\end{equation}
E. Procedure

Algorithm 1.0 TH2

set zero value for all variables; set initial values for $P(i, 0)$ and $V(i, 0)$

WHILE $t < T$

DO

FOR $i = 1$ TO $l$

$I = C(i, t-1); C(i, t) = I$ (define lane for $i$)

$s$: $D_{C(i, t)} \geq P(i, t-1) < D_{C(i, t)+1}$ (def. section)

$k$: $WC_{i, x} \geq V(i, t-1) < WC_{i, x+1}$ (range)

$V(i, t) = \min [WD_t; WA_t; V(i, t-1) + AC_{i, x} + AS_{i} \text{max vel.}]$

$d = Y \times (V(i, t-1) + V(i, t)) / 2$ (max possible replacement)

calculate $d_1, d_2, d_3, d_4, d_6, I_1, I_2, I_3$ (see Fig. 4)

$D_1, D_2, D_3 = DM$

IF $FV_i \neq FV_i$, THEN (opposite direction)

$D_1 = DO + (d_1(WD_t - WD_i) + 0.5) \ast WD_t$

IF $FP_i < FP_i$ THEN $D_1 = DO + FP_i$, ENDIF

ENDIF

IF $FV_{i, t-1} = 0$ OR $FV_{i, t-1} = 0$ THEN $D_1 = E$

ENDIF

IF $FV_i \neq FV_i$, THEN $D_1 = DO$

ENDIF

IF $FV_i \neq FV_i$, THEN

$D_2 = DO + (d_2(WD_t - WD_i) + 0.5) \ast WD_t$

IF $FP_i < FP_i$ THEN $D_3 = DO + FP_i$, ENDIF

ENDIF

IF $FV_{i, t+1} = 0$ OR $FV_{i, t+1} = 0$ THEN $D_3 = E$

ENDIF

*** identify collision

IF $d_2 < D_2$ AND (vehicle ahead too close)

$t_0 = O(i); FC(l, t, t_0) = 1$ (set flag)

clear all vectors for index $t \geq t_0$; $t = t_0$

LOOP (restart from $t_0$

ENDIF

CASE $FD_{i, i} = FV_i$ AND $d_2 > D$ AND $D > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(i, t-1)$

CASE $FD_{i, i} = FV_i$ AND $FD_{i, t-1} = FV_i$, AND

$l > D$ AND $d_2 > D$ AND $d_4 > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(i, t-1)$

CASE $FD_{i, i} = FV_i$ AND $FD_{i, i+1} = FV_i$, AND

$l < L$ AND $d_3 > D$ AND $d_4 > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(i, t-1)$

CASE $FD_{i, i} = FV_i$ AND $d_2 > D$ AND $d_4 > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(i, t-1)$

CASE $d_1 > d_2$ AND $d_2 < D_2$ AND $D_2 > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(t, t-1)$

CASE $d_3 > d_3$ AND $d_3 > D_3$ AND $D_3 > DU + DV_i$

THEN

$P(i, t) = P(i, t-1) + d; C(i, t) = C(i, t-1)$

ENDCASE

V. THE TH2 OPTIMIZATION SCHEME

The TH2 optimization scheme proposed consist in recalculating trajectories for some vehicles that could reach higher velocity. This is possible by identifying some vehicles that stayed obstructed for a relevant period of time, so they have a cumulative travel time lost, and force them to decelerate in previous intervals before leaving the platoon. This deceleration will cause a spacement to the vehicle ahead in such way that is possible to better accelerate in the following intervals and reach higher velocity at the end by the instant he change lane to escape the platoon. Note that without the forced deceleration, the vehicle would change lane at the same time interval but with lower velocity (see Fig. 5). This happens because there is not enough time to reach maximum velocity.

The parameters and variables additional to TH2 are:

A. Parameters

| FB | min quantity of intervals stayed in platoon necessary to a vehicle participate in TH2 |
| WW | min velocity ratio (to vehicle ahead) necessary to a vehicle participate in TH2 |
The algorithm was tested in random traffic situations in different configurations. It is important to note that the simulation algorithm tries to reproduce a rational decision and not a driver behaviour, so this study can not be compared with the many traffic simulators available.

The tests are divided in two parts.

A. Absolute comparison

The first part of tests compare results with the optimal solution provided by a MILP model described in [5]. The model suggested, and was adopted, only 6 vehicles in a straight and flat road with the following parameters: \( AC = 7m/s^2 \); \( AR = 30m/s^2 \); \( DV = 0m \); \( DL = 600m \); \( DM = 10m \); \( DO = 50m \); \( DR = 30m \); \( DU = 30m \); \( T = 11s \); \( UL = 1 \); \( Y = 1s \). The traffic situation (initial positions of vehicles and respective desired velocities) were generated considering random variables \( A_t \), ranging \([0,1]\) with normal distribution, and were calculated as follows: \( P(3,0) = A_1 \times 80 + P(2,0) = A_2 \times 80 + P(3,0) + 15 \), \( P(1,0) = A_3 \times 80 + P(2,0) + 15 \), \( P(4,0) = A_4 \times 80 + 260 \), \( P(5,0) = A_5 \times 80 + P(4,0) + 15 \), \( P(6,0) = A_6 \times 80 + P(5,0) + 15 \), \( V(3,0) = \text{AR} \times 3 + 10 \), \( V(2,0) = \text{A} \times 10 + V(3,0) + 6 \), \( V(1,0) = A_9 \times 5 + V(2,0) + 4 \), \( V(6,0) = A_10 \times 3 + 10 \), \( V(5,0) = A_{11} \times 10 + V(6,0) + 6 \), \( V(4,0) = A_{12} \times 5 + V(5,0) + 4 \). The tests were applied in two configurations: a single lane road “|||” and a three lane road (center line shared by both direction) “↑↓↓”. The graph of Fig. 6 shows the performance of TIH comparing two factors proposed to evaluate the traffic evolution [5]. The first factor is called complexity of the microscopic traffic: \( CX(T) = 1 - Z_2/Z_1 \). Where \( Z_1 \) represents the total of distances travelled alone in the road, i.e. considering only one vehicle travelling each time. Also, \( Z_2 \) represents the total of distances travelled regarding optimal evolution calculated by a MILP mathematical model. The other factor is called complexity of the microscopic congestion: \( CXC(T) = 1 - Z_2/Z_2 \). Where \( Z_2 \) represents the total of distances travelled in a proposed algorithm \( g \) to be evaluated. It represents how a certain traffic is congested compared to optimized traffic calculated by mathematical model \( Z_2 \). The same 3000 situations were applied for both configurations. The single lane resulted an average \( CXC(T) \) of 8.0%. This is 5.2% more than the value found in the SPVP algorithm proposed in [9]. The three lane resulted an average of 17.2%. This is 4.8% more than the value found in the SUPVP algorithm proposed in [5]. The results of the algorithm dedicated for specific configuration are better. The reason is probably because they considered the complete overtaking process, i.e. each vehicle that initiates an overtaking leaving one lane, is expected to return to this lane. This is only possible were there is maximum two lanes. Actually the three lane SUPVP algorithm simulates a two lane road with rules.

Comparing the two performances on Fig. 6 it is possible to conclude that regarding the chosen parameters, the three lane configuration has an average of 9.2% \((17.2 - 8.0)\) less congestion than the single lane.
B. Relative comparison

In the second part of tests the results of different configurations are compared with each other, as can be seen on Tab. I. After empirical tests, it was adopted a quantity of 30 vehicles, 15 running in each direction, to be displaced randomly along a segment of road. The length of this segment was varied and so traffic flow density was simulated. The 600 traffic situations tested were generated for vehicles $i$ considering random variables $A_i$ ranging [0,1] with normal distribution, and were calculated as follows: $P(i,0) = A_i \times LI, V(i,0) = \text{int}(A_i \times 22 + 9)$. Most parameters were maintained from the first part. Also, $T = 20s; FB = 4; WW = 1.2$. The Tab. I shows $Z$ calculated by TIH and TH2 algorithms according to road configuration and length of segment $DI$, that is equivalent of density of vehicles in the road.

The lane with preference in one direction performed very similar to the lane with preference in both directions. The configuration (1) was equal to config. (2) for TIH and very similar to TH2. The improvement of $Z$ from a two lane to a three lane $\downarrow\uparrow\downarrow$ varied within (13.4%, 9.09%), as expected similar to found in the first part tests. Overall, the configurations containing exclusive lanes did not performed as well as the others without these lanes. The reason is because there is less alternatives for changing lanes. Some configurations, such as (1) and (5) are not suitable for human drivers traffic due to safety reasons. The problem is that human drivers do not have rules neither speed of decision capabilities to deal with such a road configuration. For example, a situation where two vehicles, approaching in opposite direction, must change lanes to avoid crash (Fig. 1), and in this case all lanes are viable alternatives instead of the normal rule decision for the lane on right hand side.

Comparing the performances of different configurations dealing with the same volume of traffic (see Tab. I), it is possible to conclude that regarding the chosen parameters, the three lane configuration (6) has an average of 9.9% less congestion than the single lane (8). The four lane configuration (2) has an average of 4.0% less congestion than the three lane (6).

VII. CONCLUSIONS

The proposed algorithm can calculate, in real time application, an optimized solution for the problem of defining coordinated trajectories of IVs transiting in any road configuration. Although the algorithm performance was around 5% worse than the equivalent specific algorithm, it is useful for a centralized traffic management of IVs as it can model any road configuration. Also, an extension of the algorithm was proposed exploring the introduction of specific spaces for acceleration. The result increased in more than 3% depending on configuration. A comparison among many road configurations was presented and the best performance was related to those with more non-exclusive lanes.

REFERENCES


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\begin{array}{cccccccccccc}
\text{DI} & \text{TIH} & \text{TIH2} & \text{TIH} & \text{TIH2} & \text{TIH} & \text{TIH2} & \text{TIH} & \text{TIH2} & \text{TIH} & \text{TIH2} & \text{TIH} & \text{TIH2} \\
100 & 10017 & 10023 & 10066 & 10171 & 8787 & 9068 & 8787 & 8835 & 10017 & 10017 & 9457 & 9457 \\
200 & 10531 & 10537 & 10588 & 10632 & 9615 & 9872 & 9615 & 9663 & 10531 & 10531 & 10179 & 10179 \\
300 & 10778 & 10788 & 10797 & 10847 & 10120 & 10330 & 10120 & 10163 & 10778 & 10778 & 10483 & 10506 \\
400 & 10934 & 10944 & 10945 & 10987 & 10444 & 10614 & 10444 & 10475 & 10934 & 10934 & 10631 & 10656 \\
500 & 11044 & 11057 & 11049 & 11092 & 10684 & 10830 & 10684 & 10834 & 11044 & 11044 & 10784 & 10781 \\
600 & 11110 & 11125 & 11114 & 11152 & 10838 & 10963 & 10838 & 10873 & 11110 & 11110 & 10736 & 10738 \\
700 & 11162 & 11177 & 11164 & 11197 & 10956 & 10969 & 10956 & 10982 & 11162 & 11162 & 10773 & 10803 \\
\end{array}
\]